SSC-123

AN INTERPRETATION OF LOWER YIELD POINT PLASTIC FLOW IN THE DYNAMIC TESTING OF MILD STEEL

by

J. M. Krafft

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June 26, 1961

Dear Sir:

As part of its research program related to the improvement of hull structure of ships, the Ship Structure Committee sponsored an investigation at the Naval Research Laboratory to determine what occurs when steel is stressed at a high strain rate, and how this may relate to brittle fracture.

Herewith is a copy of the first progress report of this investigation, Serial No. SSC-123, entitled An Interpretation of Lower Yield Point Plastic Flow in the Dynamic Testing of Mild Steel by J. M. Krafft.

The project was conducted under the advisory guidance of the Committee on Ship Steel of the National Academy of Sciences-National Research Council.

Please address any comment concerning this report to the Secretary, Ship Structure Committee.

Yours sincerely,

E. H. Thiele

E. H. Thiele
Rear Admiral, U. S. Coast Guard
Chairman, Ship Structure
Committee

Serial No. SSC-123

First Progress Report
of
Project SR-142

to the

SHIP STRUCTURE COMMITTEE

on

AN INTERPRETATION OF LOWER YIELD POINT PLASTIC FLOW IN THE DYNAMIC TESTING OF MILD STEEL

by

J. M. Krafft

U. S. Naval Research Laboratory
Washington, D. C.

under

Bureau of Ships
Department of the Navy
Index No. NS-011-078

transmitted through

Committee on Ship Steel
Division of Engineering and Industrial Research
National Academy of Sciences-National Research Council

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Washington, D. C.
National Academy of Sciences-National Research Council
June 26, 1961

Abstract

Evaluation of the role of rapid plastic flow in the fracture process necessitates some measure of elastic and plastic strength which will be suitable for high strain rates. For a limit to the static elastic strength of mild steel, the lower yield stress is a generally accepted criterion. Its use for the dynamic test, although common in the literature, is more difficult to justify because here neither the lower yield strain along the specimen, nor the lower yield stress, are even approximately constant. The variability of both is a consequence of the fact that in the high strain rate test, a "steady state" form of the Lüders strain front is not obtained, nor is it then contained within the length of the specimen.

Once load is applied to a rate sensitive material a time must elapse before an equilibrium strain level is obtained. If plastic strain is initiated by a rapidly moving Lüders band front, this flow equilibrium time may be longer than the time required for the front to traverse the entire specimen. Receding from such front, increasing levels of strain will have had time to accumulate; thus the strain cannot be uniform. Correspondingly, as greater length of the specimen is brought into plastic flow by traverse of the front, an increasing fraction of head speed is diverted to support this "behind the front" flow; less is available to move the front. As front velocity thus decreases, the stress required to drive it correspondingly decreases.

For this paper, strain distribution is measured after stopping a dynamic machine during the lower yield. From head displacement and load records obtained during the test and also from plastic flow rate sensitivity measured in other high-speed tests on the same material, the strain distribution is predicted. The predicted strain distribution behind the Lüders band front is found in good agreement with that measured. When the velocity of the band front is corrected for its diminution owing to continued flow behind the front it is related to driving stress with a sensitivity corresponding to that for the upper yield point--or the delayed yield.

CONTENTS

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INTRODUCTION

When a material is characterized by an abrupt drop of load at the yield point, both upper and lower yield stresses are available measures of elastic strength. The maximum or upper yield strength, although it appears to be advantageously well defined, is rarely relied upon because initiation of yield is extremely sensitive to test conditions and thus difficult to reproduce. As an alternative, the lower yield strength is commonly characterized by the stress required to drive the initiated band of plastic deformation through the length of the specimen. If the testing speed is slow and reasonably uniform, the lower yield stress--and corresponding strain--will be relatively constant and thus will provide a reproducible strength parameter which is readily measured. This favorable situation is not realized at higher head speeds where even the lower yield stress is not generally constant and assignment of a definitive strength parameter requires a more critical analysis of the deformation process.

As a basis for discussion of the case of high testing speed, it is helpful to review a picture of slow speed deformation. Assume that yield has been initiated completely across one end of a long, slender, cylindrical specimen by compressive loading. The load level required to move the front of the deformation zone, or Lüders band, into yet unyielded material will in general be less than that which was required to initiate that band (i.e., less than the upper yield point), and thus less than that to initiate another. Consequently propagation of a single band is likely to occur. Encroachment of this band upon each segment of the specimen will require it to compress the amount of the lower yield strain. In the gross, this transition from nil (or elastic) to lower yield strain appears abruptly in both time and distance along the specimen. But in actual fact the abruptness is limited. Timewise it is limited by the rate sensitivity of plastic flow. Equilibrium strain is approached only at finite speed after suddenly applied constant stress. In distance, a perfectly stepwise deformation front is prevented by the lateral shearing stresses that such a transition would imply. The segments of our hypothetical bar are laterally, as well as longitudinally, coupled.

When the Lüders band front moves slowly there is ample time for equilibrium "static" deformation to be reached close behind the front. The abruptness of the strain transition at the front is then limited only by the lateral restraint which in slender specimens has a rather small length of influence on "width" of the front. But if the band moves rapidly so as to traverse a considerable length in a time less than that required for the rate sensitive material to reach static equilibrium strain, then the strain level at positions receding from the front will become progressively larger with the correspondingly increased duration of load after onset of plastic flow. Thus, quite in contrast to the rather abrupt strain transition of the slow Lüders strain propagation, a much more gradual strain transition characterizes the rapid deformation. Indeed, at any given time during the Lüders band propagation, every segment of the specimen behind the front may be continuously flowing. In other words, with increasing nominal strain rate, the "width" of the front may become larger than the specimen length. In such cases it is the transient state or development of the front which is of primary interest.

Obviously the motion of the head must be divided between that required to continue the flow in each segment behind the front and that required to expand the band front into unyielded material. As the band lengthens, an increasing proportion of the head speed will be diverted to this "behind the front" flow. Therefore, the speed of advance of the band front must diminish. But it is the rate of advance of the front that largely determines the required "lower yield stress." If the band front velocity decreases, then so will the lower yield stress, even though head speed remains constant. As a consequence, neither the lower yield stress nor the lower yield strain are even approximately constant in the dynamic tensile (or compression) test. And when the Lüders band front has reached the end of the specimen and general plastic flow commences, the specimen is not uniformly strained so that strain subsequent to this may not be expected to be immediately uniform.

The decreasing abruptness of Lüders band strain profile with increasing testing speed was first measured by Miklowitz who termed the length of the transition region "working length."[1] Hart subsequently developed an analysis for a case in which (a) the "working length" is contained within the specimen length and (b) the band-front velocity (and load) is constant.[2] The present paper, while based on existence of a mechanical equation of state as assumed by Hart, provides a basis for numerical calculation of the Lüders strain transition for the more realistic situation when neither conditions (a) nor (b) are realized.

Brief of Procedure

The procedure for calculating the strain distribution is largely numerical, and is based on empirical data. Certain simplifying assumptions are required. For the sample of Lüders strain propagation, the head motion of a high-speed testing machine is arrested just before the lower yield strain is completed. The profile of the specimen then gives the empirical measure of strain distribution. Load- and head-displacement history are measured for this machine cycle. Both measurements are used as given quantities in the calculation. The load is considered as acting simultaneously on all segments of the specimen to continue plastic flow once it is initiated by arrival of the Lüders band front. The front is considered as lying perpendicular to the longitudinal axis of the specimen even though there is a tendency for it to occur on planes of high shear stress at an intermediate angle. The head displacement is used to calculate the position of the band front by considering the displacement as divided between that used to continue plastic flow behind the front and that remaining to extend the band into unyielded material.

To complete the necessary information, a measure of the rate sensitivity is required so that the flow in each segment behind the front may be calculated for the varied load occurring during the lower yield. Basic data are available in the form of stress-strain curves covering a range of testing speeds. From these, curves of stress versus strain at constant strain rate can be interpolated and these extrapolated back to strain levels obscured by the lower yield strain itself. These curves can in turn be converted to curves of strain versus time at constant stress. These are directly applicable to calculation of the post yield flow in each segment of the specimen, considered as loaded by a succession of steps of load approximating the measured lower yield stress pattern.

Apparatus

A gunpowder-driven press was employed both for rapid deformation terminated during the lower yield strain as well as for that permitted to continue into the region of general plastic flow as required to obtain rate sensitivity data. An analysis of this machine has been published elsewhere[3] so only essential features will be summarized here. Basically, a pressure source produced by burning a properly selected charge of gunpowder (Fig. 1) is applied to a piston which presses in turn upon the specimen. The rate of gas pressure application is controlled by interposition of a restrictive orifice between the pressure source and piston. Strain gages on a hard steel anvil, which is interposed between piston and specimen, provide a measure of load while those gages on a tapered cantilever beam, which is deflected by contact with the anvil, give a measure of head displacement. Suitable cathoderay oscilloscopes provide records of the strain signal (displayed in Fig. 2) either independently as a function of time or against each other with superimposed timing signals on the trace. As noted on Fig. 2, the head speed actually increases as the lower yield stress drops because of the relative softness of the gas machine; however, the machine does not speed up enough to maintain the lower yield stress even approximately constant.

The extent of plastic strain in each test is limited by arresting the piston with an internal sleeve stop (not shown in diagram) interposed between the piston and the threaded base. By appropriate selection of the sleeve length, strain may be terminated before the lower yield strain is complete. Records comparable in all particulars with those of Fig. 2 but so limited in deformation stroke are shown in Fig. 3. The shape of specimens recovered after such test can then be compared with the prediction based on the record and other data on plastic flow strain-rate-sensitivity obtained from records similar to that of Fig. 2.

Specimen Material

Choice of material for these experiments is not critical. Any reasonably

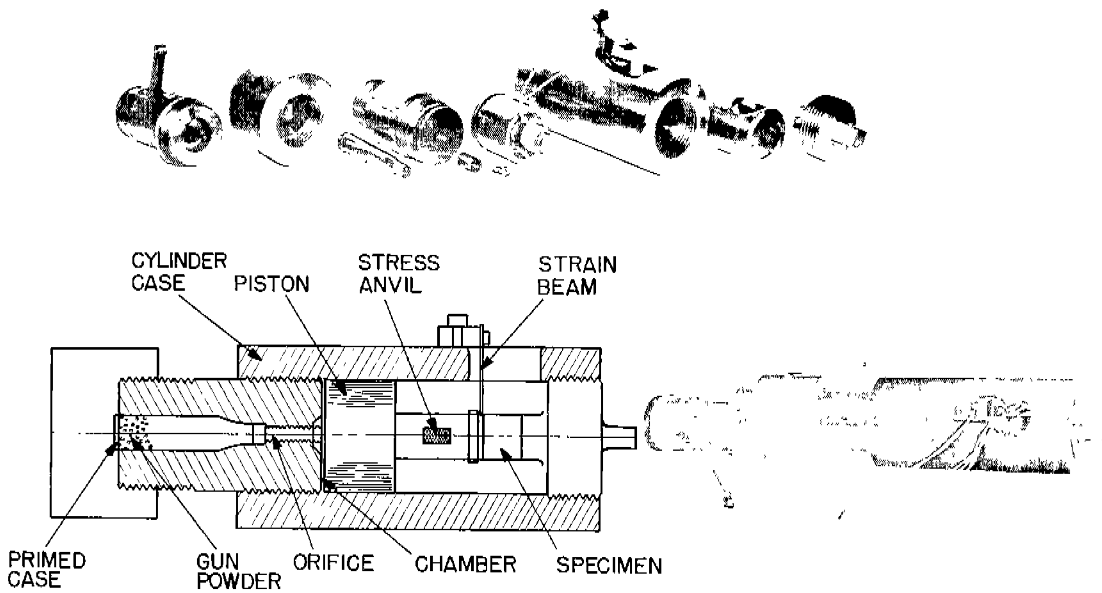

FIG. 1. GUNPOWDER DRIVEN, HIGH STRAIN RATE COMPRESSION MACHINE

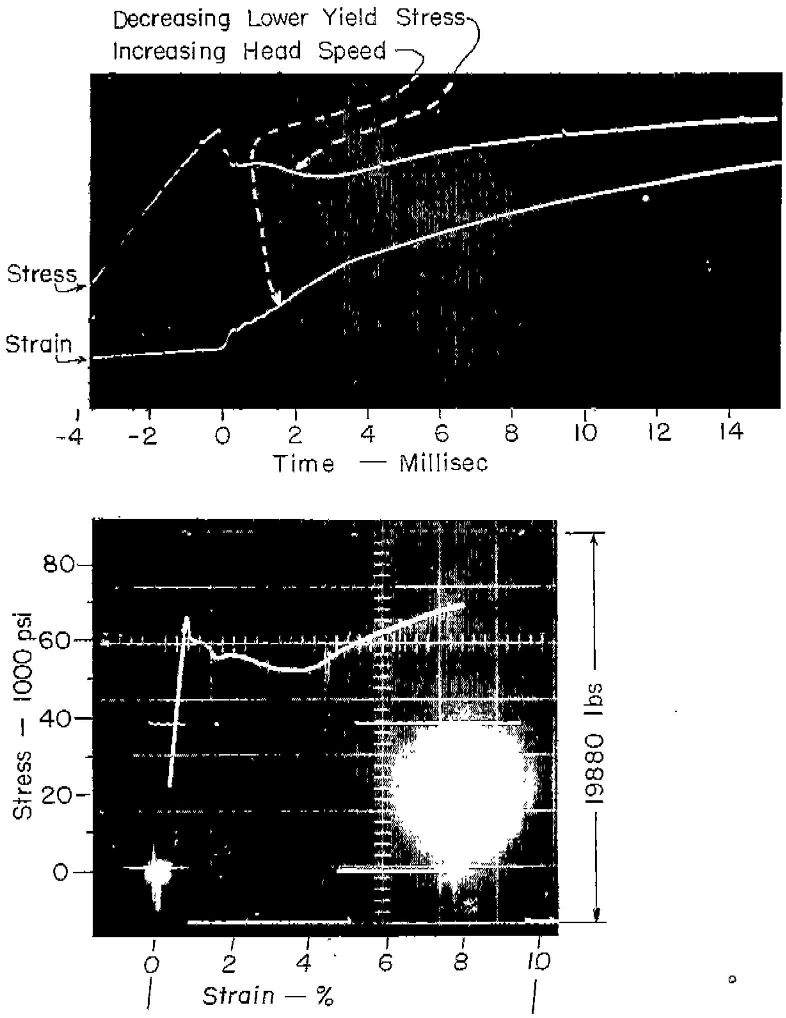

FIG. 2. OSCILLOGRAPH RECORDS TYPICAL OF LOWER SPEED RANGE OF MACHINE OF FIG. 1. INDEPENDENT LOAD AND HEAD DISPLACEMENT RECORDS ON A COMMON TIME BASE ARE SHOWN AT THE TOP; THE TWO SIGNALS ON X-Y PRESENTATION WITH SUPERIMPOSED TIMING MARKS AT THE BOTTOM

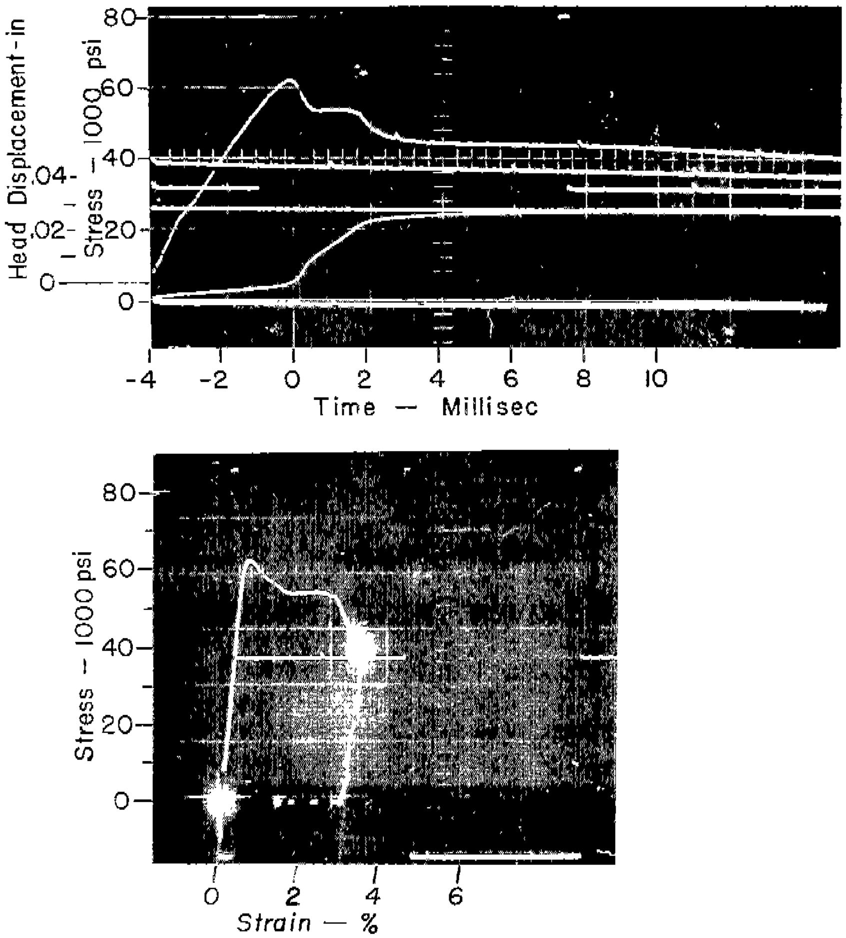

FIG. 3. OSCILLOGRAPH RECORDS FOR A CASE OF SINGLE LÜDERS BAND PROPAGATION. MOTION WAS ARRESTED BEFORE COMPLETION OF LOWER YIELD STRAIN.

uniform sample of a steel possessing upper and lower yield points would have been satisfactory. Available in sufficient quantity, a plain carbon, semikilled steel of 0.11% carbon, 0.62% manganese was used. A heat treatment to obtain relatively fine grain size included normalization from 900°C (1650°F) and annealing at 870°C (1600°F) for one hour followed by a slow furnace cooling. This steel is designated as 1010 in a previous publication[4] which contains additional information about it.

<u>Determination of Plastic Flow Rate Sensitivity</u>

When plastic strain was permitted to extend to about 10 per cent (as in Fig. 2) the data on flow rate sensitivity, which is needed to calculate Lüders strain distribution, could be obtained. The presentation of such data in the

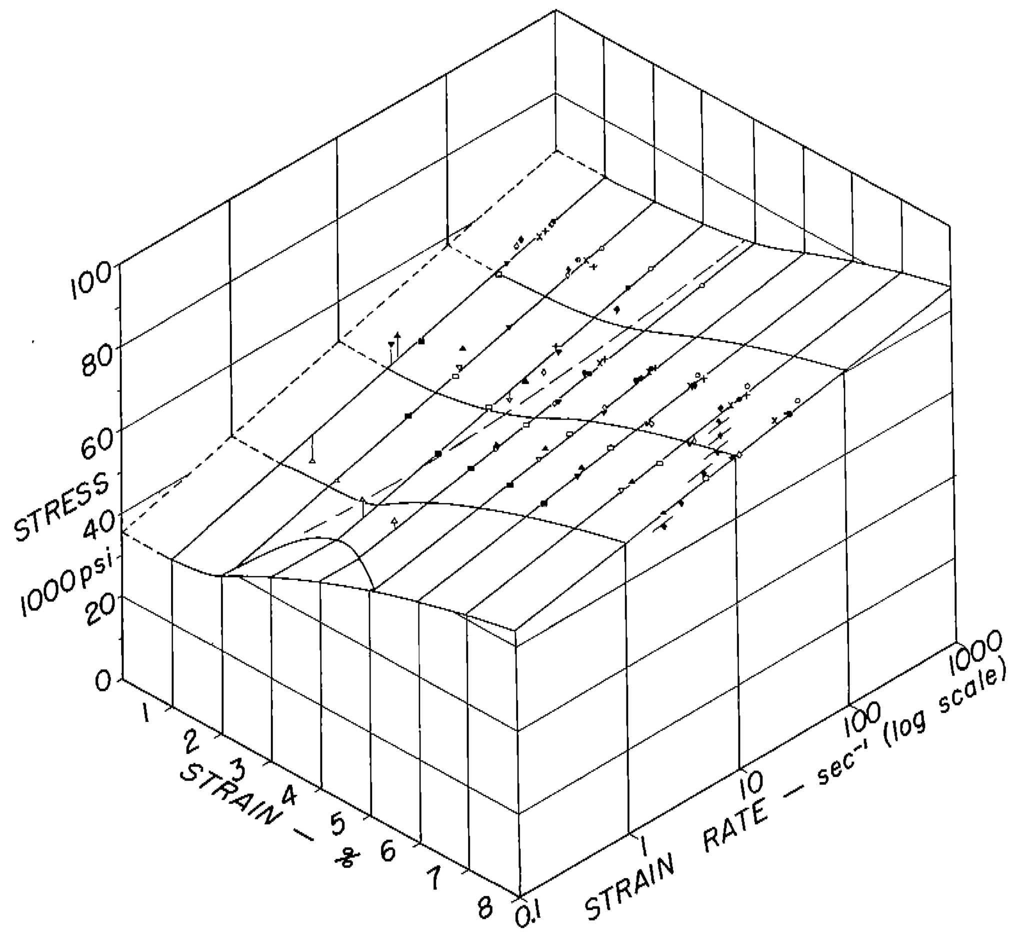

FIG. 4. NOMINAL STRESS VS STRAIN VS STRAIN RATE DATA FOR A 0.10% CARBON STEEL FOR WHICH STRAIN DISTRIBUTION DURING THE LOWER YIELD IS TO BE CALCULATED

three dimensional graph of Fig. 4 permits effects of variation in machine head speed during the test to be viewed. Actually rather substantial variations in head speed seem to be permissible without serious deviation from the stress-strain-strain rate surface defined by tests at relatively constant head speed. Note for example, the diamond-shaped symbols representing data after the piston had been intercepted by its stop. At this point residual elastic energy from the machine is available to continue plastic flow as the load drops. Through more than a factor of 10 decrease in strain rate, close conformity to the flow surface is observed. It is a necessary condition, of course, that such insensitivity to flow history (i.e., equation of state: $F(\sigma_1\ \epsilon_1\ \dot{\epsilon}) = 0$) exist in the material (as this must be assumed in order to make the calculation) in order to transform coordinates from stress versus strain at constant strain rate to strain versus strain rate at constant stress and thence to strain versus time at constant stress. It should be noted that nominal stress and strain in compression are used in Fig. 4 and in curves based on it, so no area correction is required in the calculations for the compressive Lüders band.

The data plotted in Fig. 4 in the lower yield range are not expected to define a unique surface as each case of Lüders strain propagation must be treated individually. This is a consequence of the fact that the velocity of the band front relative to average strain rate will vary inversely with the number of bands simultaneously growing. While the number of bands could not be determined, some improvement in the correlation was obtained by correcting for specimen length. The band velocity is related mainly to the head speed and not to the average strain rate. Thus strain rate was plotted as 1/0.50 (in.) or 2.0 times head speed for both 1/2-in. and 0.925-in. specimen length even though this provides a nominal strain rate correct only for the half-inch length. Although both 1/2-in. and 0.925-in. lengths were employed for the rate sensitivity data of Fig. 4, only the longer ones were used for Lüders strain examination. In retrospect, even longer specimens would have been desirable.

Transformation of Plastic Flow Data

As previously noted the present calculation is based on the assumption that once the band front passes an incremental section of the specimen, the material in that section must support the subsequently applied stress and still be governed by the flow surface defined by Fig. 4. The load will be sustained by combined effects of straining speed and strain hardening. As a simplification, a one-dimensional model is used so that the lateral intercoupling or shearing tendency between adjacent sections of the specimen is neglected.

The stress that must be supported during Lüders strain propagation is directly measurable from the load time record of the testing machine. The problem is to calculate the strain as a function of time as this measured stress varies. To do this the stress-strain relation at constant strain rate must be converted to a strain-time relation at constant stress level. Numerical methods were employed for this case although direct integration would be possible in some of the steps. First, the stress strain relationships at constant strain rate (nominal compressive) shown in Fig. 5 are interpolated from the data shown in Fig. 4. As expected, a linear relationship between stress and strain is obtained by use of logarithmic scales. These lines are readily extrapolated into the low strain region where the lower yield phenomenon itself obscures the needed relationship. Validity of this extrapolation will, of course, affect success of calculations based upon it.

Values of strain and strain rate may be read from Fig. 5 for selected stress levels. The strain at constant stress is then plotted against the reciprocal of the strain rate (not shown), which is equivalent to the time required to reach unit strain if the strain rate unique to a particular combination of stress and strain were continued indefinitely. Intergration of this relationship (i.e., $dt/d\epsilon$ vs ϵ at constant stress) provides a relation between the strain and time as shown in Fig. 6. Each line represents the strain which would accrue as a function of time after application of the stress noted. It would be assumed that after sufficient time passes, the static or equilibrium strain value for a given flow stress should be reached. The dashed line on the right-hand side is drawn to indicate this intersection. Location of this intersection is somewhat arbitrarily defined, however, not only because few data in the transition range are available but also because the static deformation is essentially isothermal while the dynamic is adiabatic. Thus at higher strain levels, temperature of the rapidly deformed specimen will be significantly increased.

Calculation of Strain for a Case of Single Lüders Band Propagation

The distribution of Lüders strain was observed with the present apparatus by arresting the head motion before the yield point strain is completed. For the record shown in Fig. 3, the gunpowder loaded was charged with 1000 milligrams of powder and restricted by an orifice, 0.0135 in. in diameter and 0.25 in. in length. The specimen, initially 0.925 in. in length (and 0.500 in. outside diameter), was compressed 0.0283 in., or 3.06 per cent. The diametral profile was scanned before and after deformation to obtain the strain distribution.

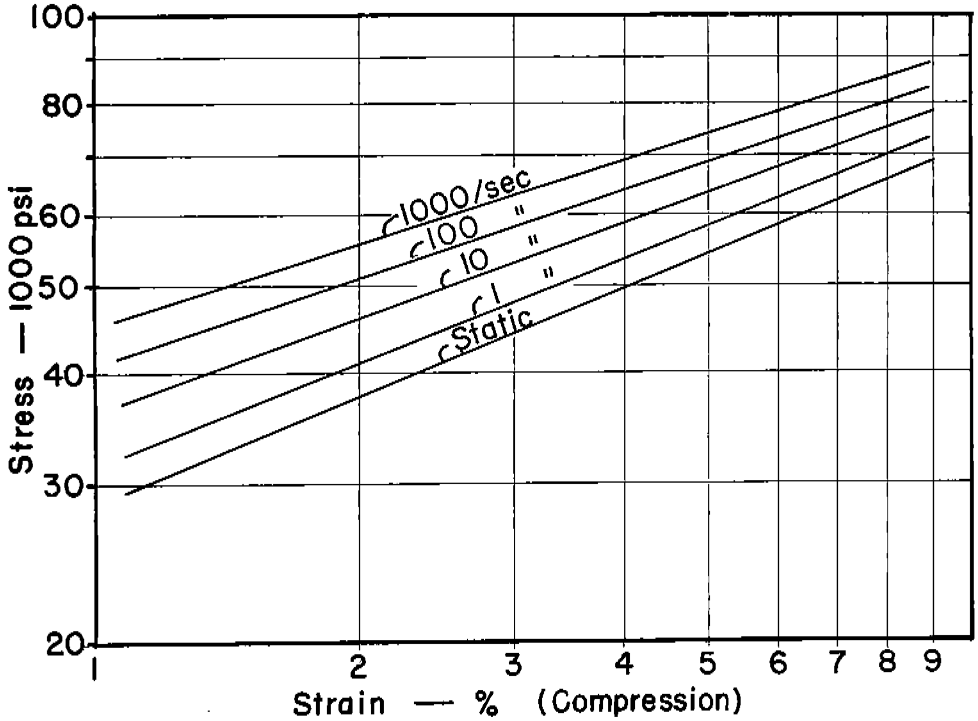

FIG. 5. REPRESENTATION OF THE PLASTIC FLOW REGION OF FIG. 4 AT VARIOUS CONSTANT STRAINING SPEEDS.

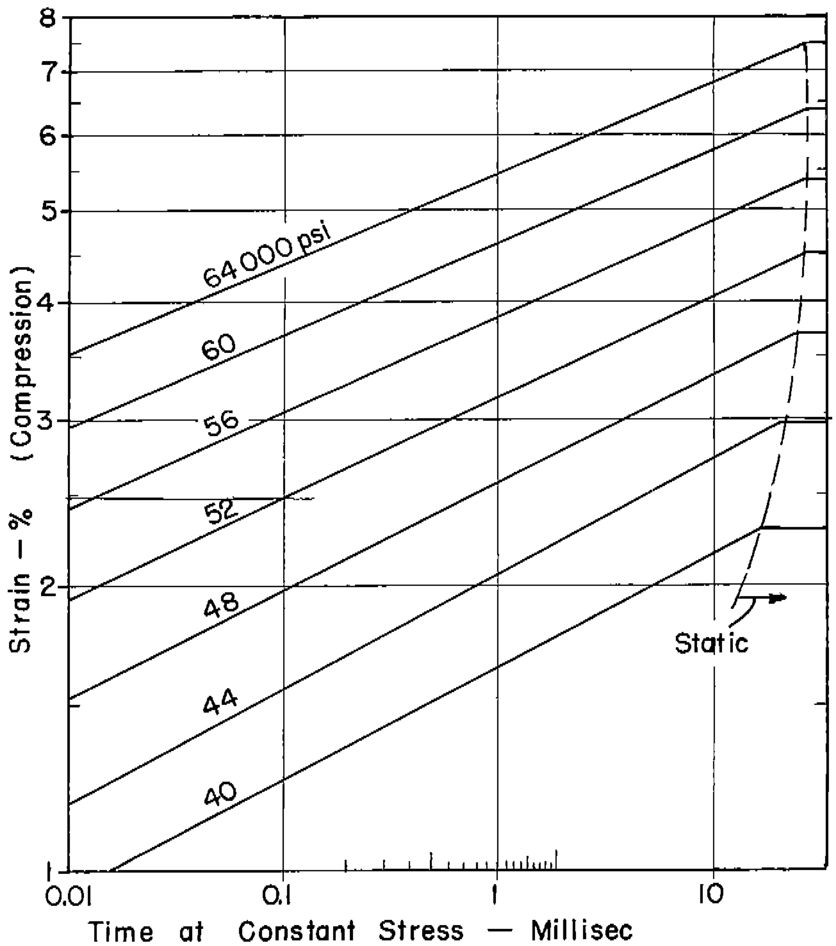

FIG. 6. DEVELOPMENT OF STRAIN AS A FUNCTION OF TIME AT CONSTANT STRESS

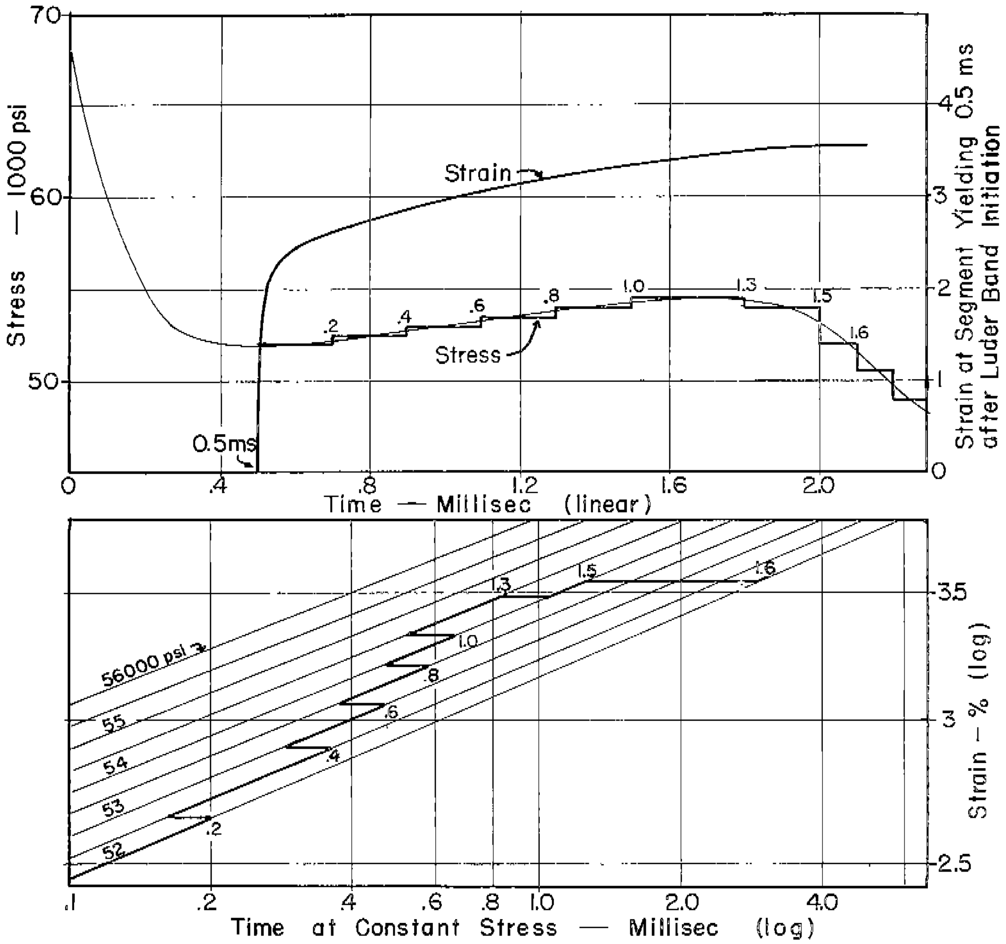

FIG. 7. LOWER PORTION SHOWS AN ENLARGED SECTION OF FIG. 5, THE PATH OF INTEGRATION FOR STRAIN AS A FUNCTION OF STRESS AND TIME IS OUTLINED. STRESS HISTORY AS WELL AS CALCULATED STRAIN TIME HISTORY IS SHOWN AT TOP.

For the numerical integration, stress and strain values were measured from the oscillogram (Fig. 3) at equal time increments of 0.10 milliseconds. Values of the head displacement for each 0.10-millisecond time increment were graphically smoothed from a plot of head position vs time. Assuming the Lüders band to begin at one end, as indicated by the measured strain distribution, the first length increment traversed by the band will be subjected to the complete stress pattern; subsequent increments to successively shortened residual portions of the load history. An example of the strain history for the length increment reached by the zone front 0.50 milliseconds after yield is shown in Fig. 7. The strain in each length increment is obtained by following the strain-time lines as illustrated in the lower part of Fig. 7. As the stress varies, one moves horizontally (i.e., at constant strain) from one stress line to another. At the end of each time increment, the strain is recorded in a table set up to facilitate numerical integration. The sequence of recorded strain values is then taken to represent the strain history of that particular length increment. Such a strain pattern must be tabulated for each segment of the specimen.

<u>Calculation of Band Front Positions</u>

As noted earlier, as the Lüders band progresses, an increasing fraction of the head displacement is absorbed in continued flow behind the front. Thus the rate of band advance Δx for increment of head displacement $\Delta \ell$ generally decreases; it may be calculated from the expression:

$$\Delta x_t = n + 1 \left\{ \frac{\Delta \ell_t - \sum_{p=1}^{p=t-1} \Delta x_p \, \Delta \epsilon_{pt}}{\epsilon_t} \right\} .$$

where Δx_t is the distance the band advances in the t^{th} time interval, n is the slope of the log ϵ vs log T plot (Fig. 6) at constant stress, $\Delta \ell_t$ is the machine head displacement in that interval, Δx_p is the incremental distance the band advanced in previous time increments p, ϵ_{pt} is the strain continuing in those prior incremental lengths during the t^{th} time interval, and ϵ_t is the strain during the t^{th} time interval in the t^{th} distance increment.* The specimen may be thought of as divided into incremental specimen of length Δx_t. These increments vary, generally decreasing with the rate of advance of the front as indicated in Fig. 8 by the decreasing separation of the points on a given curve representing band position at equal 0.1-millisec. time increments. The continued contraction of each incremental specimen, $\Delta x_p \, \Delta \epsilon_{pt}$, must be counted in every succeeding time interval until its contribution becomes negligible. The required summation, although it appears cumbersome, is readily tabulated with the cumulative multiplication facility of a desk calculator.

<u>Comparison of Calculated with Measured Band Shape</u>

The calculated values of strain for a case of single Lüders band propagation

*See Appendix A

(Fig. 3) are plotted in Fig. 8. The correspondence with the measured shape shown as the heavy solid line is thought to be quite satisfactory in view of the rather approximate nature of the calculation and particularly the fact that the specimen is shorter than ideal for this type of test. Strain distribution patterns obtained on longer specimens by D. B. C. Taylor[5] do in fact show much closer similarity to patterns here calculated.

Certain prominent features of the Lüders band strain distribution are readily explained by the proposed model. Generally the greatest strain is expected in the region traversed by the band front just after its initiation. Here the effects of stress elevated from the upper yield point are still retained as the machine specimen system tries to unload. Once the stress drops from the upper yield, the region which was then excessively strained should undergo very little additional deformation as the reduced stress levels are now quite ineffective at the strain levels already present. Regions subsequently reached by the band front at lower, relatively constant, stress levels, should, however, continue to flow as illustrated by shapes calculated for intermediate positions in the test, as shown in Fig. 8. As the stress rises after Lüders band strain covers the specimen, the regions last covered by the band should flow more rapidly than those traversed during the recession from the upper yield stress. This will tend eventually to equalize strain along the specimen, although it is not expected that traces of the unequal early deformation profile will ever be completely obliterated.

The Double Lüders Band

If two Lüders bands are simultaneously initiated at opposite ends of the specimen, the shape can be calculated as for a single band except that only half of the total head displacement for each time increment should be considered as applicable to each band. Results of such a calculation are compared to profile strain measurements in Fig. 9 where in accordance with the assumption of initiation, the calculated curve is drawn symmetrically from opposite ends. Fair agreement in the general features of the two curves is evident. However, details of the calculated profile appear to be lost in the measured shape, an effect probably caused by greater smoothing as a result of relatively more severe specimen continuity requirements

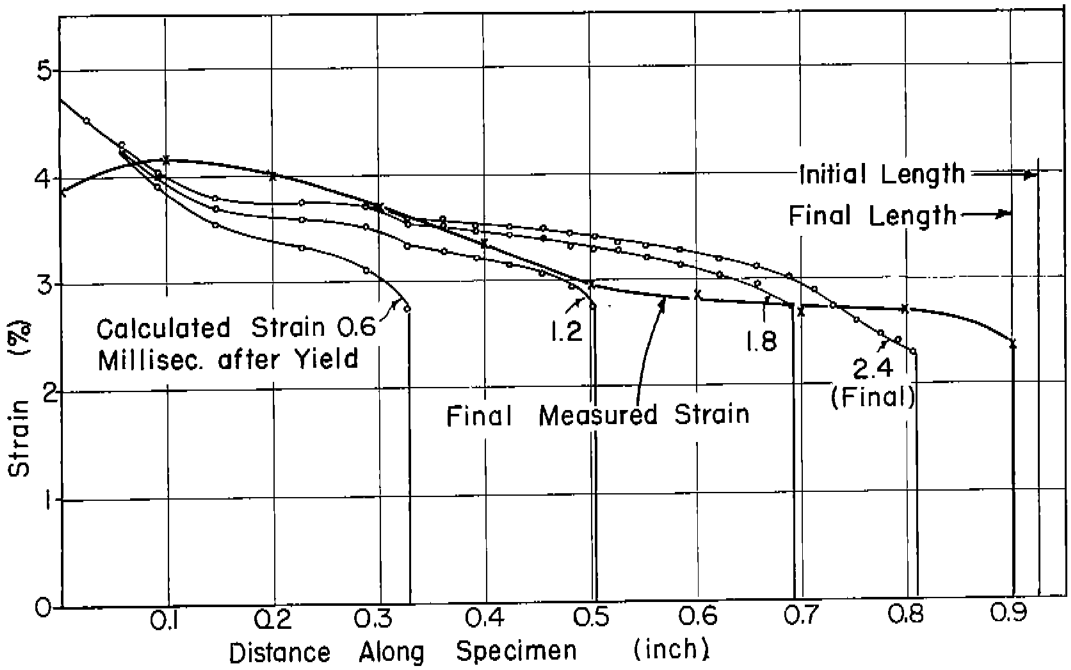

FIG. 8. CALCULATED SPECIMEN STRAIN DISTRIBUTION AT VARIOUS INTERVALS AFTER THE UPPER YIELD COMPARED TO MEASURED STRAIN AFTER THE TEST FOR SINGLE BAND CASE OF FIG. 3

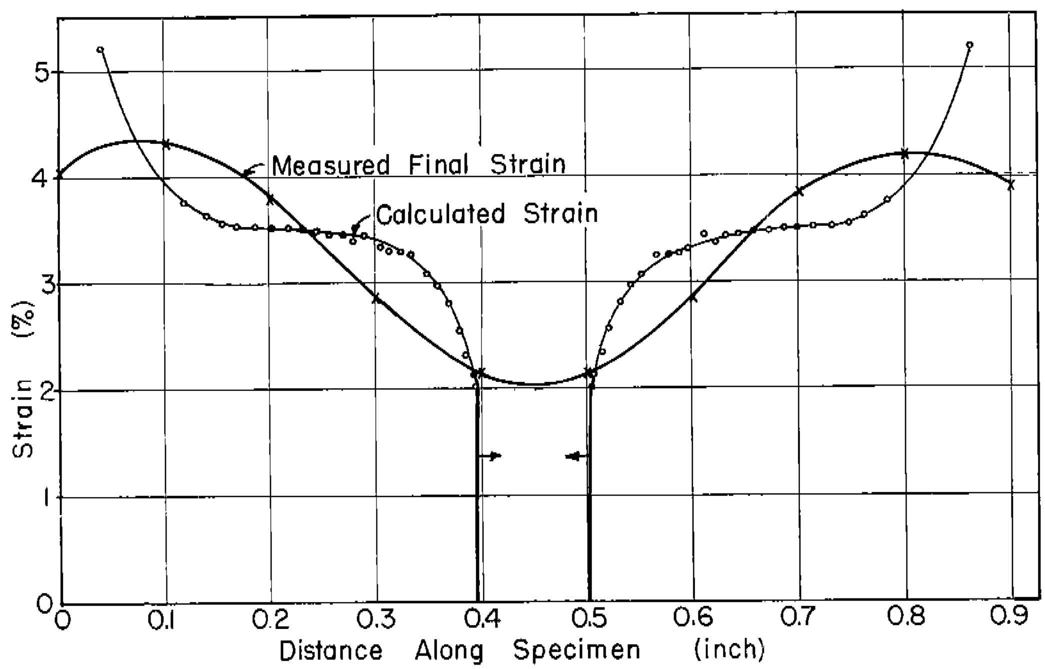

FIG. 9. DOUBLE BAND PROPAGATION STRAIN CALCULATED AND MEASURED

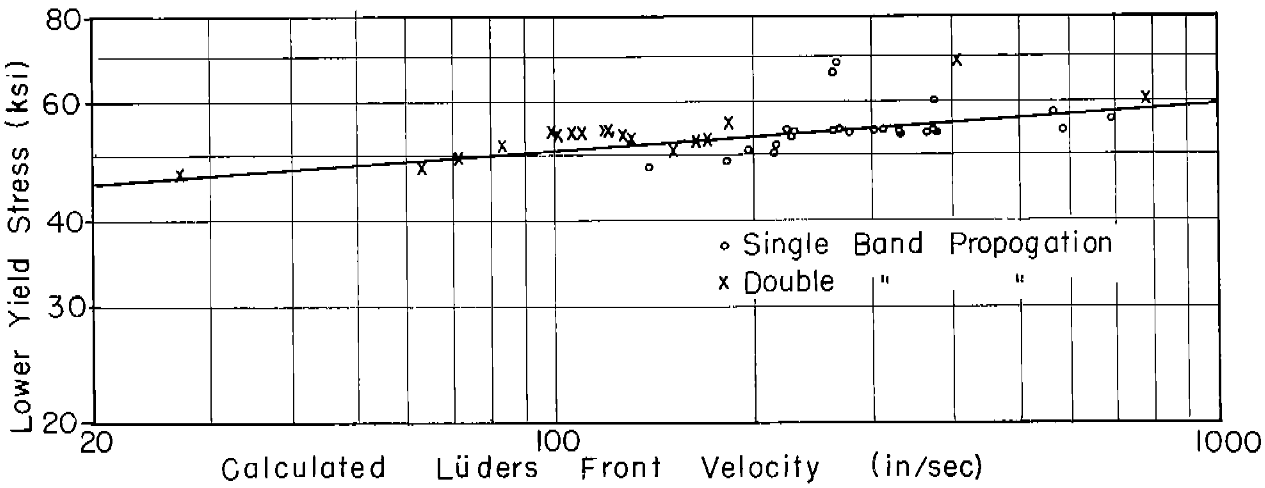

FIG. 10. STRESS DURING LOWER YIELD AS A FUNCTION OF CALCULATED LÜDERS FRONT VELOCITY. RATE SENSITIVITY IS THAT OF UPPER YIELD POINT OR DELAYED YIELD FOR THIS MATERIAL

and end constraints.

<u>Band Velocity to Lower Yield Stress Relationships</u>

With the analysis at hand, it is possible to compare the calculated velocity of the Lüders band front with the stress required to drive it so as to obtain an estimate of rate sensitivity for the process. The stress here plotted is, of course, only the nominal stress acting on the specimen as a whole. The actual combined stress condition in the elastic material immediately ahead of the plastic band is certainly more severe, and should generally increase as the front approaches a given point in the material. While the distribution of this stress field is uncertain it would seem reasonable to assume that over narrow limits of average stress, the actual stress intensity varies linearly with distance. This implies a rate of stress rise directly proportional to band front velocity. Considered in this way, the relationship of driving stress to band velocity should resemble that of stress for yield initiation to loading rate when both are considered in relative terms, or as ratios represented by log log plots of the variables. Points taken from both single and double Lüders band cases are shown in Fig. 10 with both the instantaneous lower yield stress and the front velocity plotted on logarithmic scales. A line of slope 0.070 has been drawn through the points, a value equal to the measured sensitivity of upper yield stress to loading stress rate for this particular material. The closeness of fit of the data points is taken to support the thesis advanced by Taylor[5] and by Fisher and Rogers[6] that the delayed yield in elastic material adjacent to the band front is primarily responsible for speed sensitivity of Lüders band propagation.

Conclusion

The reasonable success in predicting Lüders band strain distribution argues for validity of basic assumptions. The primary assumption is that plastic flow in the material behind the front has a response to stress and time equivalent to that measured in post-lower-yield general plastic flow. A result of calculation based on this assumption is the indication that the stress required to drive the band into unyielded specimen has a speed response characteristic of that for yield initiation; that is of the upper or delayed yield. These two speed sensitivities are generally quite different and moreover not simply related to each other. The lower yield measurement provides a rather unresolvable mixture of both effects. As a basic measure of rate sensitivity, it is certainly less valuable than a measure of either of the sensitivities that combine to determine it. Use of the lower yield as a dynamic strength parameter is thus of limited usefulness.

REFERENCES

1. Miklowitz, J., "The Initiation and Propagation of the Plastic Zone in a Tension Bar of Mild Steel as Influenced by Speed of Testing and Rigidity of Testing Machine," Journal of Applied Mechanics, vol. 14, p. A-31 (1947).

2. Hart, E. W., "A Uniaxial Strain Model for a Lüder's Band," Acta Met., vol. 3, No. 2 (March 1955).

3. Krafft, J. M., "Instrumentation for High Speed Strain Measurement," Proceedings, AIME Conference Response of Metals to High Velocity Deformation, Estes Park, Colo., July 1960. (To be published)

4. Krafft, J. M. and Sullivan, A. M., "Effect of Grain Size and Carbon Content on the Yield Delay-Time of Mild Steel," Transactions of the American Society for Metals, vol. 51, pp. 643-659 (1959).

5. Taylor, D. B. C., "Non-uniform Yield in Mild Steel under Dynamic Straining," Proceedings, Conference on Properties of Materials at High Rates of Strain," London: IME, pp. 229-238, 1957.

6. Fisher, J. C. and Rogers, H. C., "Propagation of Lüder's Band in Steel Wires," Acta Met., vol. 4, pp. 180-185 (1956).

APPENDIX A

*In an initial attempt to calculate the advance of the band in a given time increment, the strain was assumed constant in each newly covered region and equal to the maximum value attainable in that time increment. This is in effect an overestimate of the strain ϵ_t, or, from the above expression, an underestimate of the advance rate of the front. Correction to the original expression, the quantity in brackets $\{\;\}$, by the factor (n + 1) was suggested by Dr. H. Wiedersich of Westinghouse Research Labs, Pittsburgh. His derivation is given as follows:

The length change $\delta \ell$ during a time δt is

$$\delta \ell = \int_{0}^{x(t)} \delta\epsilon\ (\xi, t)\ d\xi \tag{1}$$

$$= \underbrace{\int_{0}^{x(t-\delta t)} \delta\epsilon\,(\xi, t)\ d\xi}_{\text{Length change behind "front".}} + \underbrace{\int_{x(t-\delta t)}^{x(t)} \delta\epsilon\,(\xi, t)\ d\xi}_{\phi} \tag{2}$$

For small enough δt it is safe to assume for calculation of ϕ: σ = const., $\frac{dx}{dt}$ = const. and $\delta\epsilon = \epsilon(\tau) = \alpha(\tau)^n$ from Fig. 6. τ is the time for which a length element within the length increment $x(t-\delta t) \longleftrightarrow x(t)$ is behind the "front."

$$\phi = \int_{x(t-\delta t)}^{x(t)} \alpha\,(\tau)^n\ d\xi \tag{3}$$

$$\xi = x - \frac{dx}{dt}\tau \tag{4}$$

$$\tau = \frac{x - \xi}{\frac{dx}{dt}} \tag{5}$$

$$x(t - \delta t) = x(t) - \frac{dx}{dt}\,\delta t \tag{6}$$

$$\phi = \frac{\alpha}{\left(\frac{dx}{dt}\right)^n} \int_{x - \frac{dx}{dt}\delta t}^{x} (x - \xi)^n \, d\xi \tag{7}$$

$$\phi = \frac{\alpha}{n+1}\,\frac{dx}{dt}\,(\delta t)^{n+1} \tag{8}$$

$(\delta t)^n$ can now be replaced by $\frac{\epsilon_t}{\alpha}$ (ϵ_t is the strain suffered during the time δt).

$$\phi = \frac{\epsilon_t}{n+1} \cdot \frac{dx}{dt}\,\delta t \tag{9}$$

$$\text{or } \phi = \epsilon_t \frac{\delta x}{n+1} \tag{10}$$

Therefore

$$\Delta x_t = (n + 1)\,\frac{\Delta \ell_t - \Sigma \Delta x_p \epsilon_{pt}}{\epsilon_t}$$

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