BRITTLE-FRACTURE PROPAGATION IN WIDE STEEL PLATES

-2

by W. J. Hall S. T. Rolfe F. W. Barton and N. M. Newmark

SHIP STRUCTURE COMMITTEE

SHIP STRUCTURE COMMITTEE

MEMBER AGENCIES:

BUREAU OF SHIPS, DEPT. OF NAVY MILITARY SEA TRANSPORTATION SERVICE, DEPT. OF NAVY UNITED STATES COAST GUARD, TREASURY DEPT. MARITIME ADMINISTRATION, DEPT. OF COMMERCE AMERICAN BUREAU OF SHIPPING

ADDRESS CORRESPONDENCE TO:

SECRETARY SHIP STRUCTURE COMMITTEE U. S. COAST GUARD HEADQUARTERS WASHINGTON 25, D. C.

October 3, 1961

Dear Sir:

As part of its research program related to the improvement of hull structures of ships, the Ship Structure Committee sponsored an investigation of Brittle Fracture Mechanics at the University of Illinois. Herewith is a copy of the Final Report, SSC-131, <u>Brittle-Fracture Propagation in Wide Steel Plates</u> by W. J. Hall, S. T. Rolfe, F. W. Barton, and N. M. Newmark.

The above project was conducted under the advisory guidance of the Committee on Ship Structural Design of the National Academy of Sciences-National Research Council.

This report is being distributed, without charge, to individuals and groups associated with or interested in the work of the Ship Structure Committee. Please send any comments on the report to the Secretary, Ship Structure Committee.

Sincerely yours,

Rear Admiral, U. S. Coast Guard Chairman, Ship Structure Committee Serial No. SSC-131

Final Report of Project SR-137

to the

SHIP STRUCTURE COMMITTEE

on

BRITTLE-FRACTURE PROPAGATION IN WIDE STEEL PLATES

by

W. J. Hall, S. T. Rolfe, F. W. Barton, and N. M. Newmark

University of Illinois Urbana, Illinois

under

Department of the Navy Bureau of Ships Contract NObs-65790 BuShips Index No. NS-731-034

transmitted through

Committee on Ship Structural Design Division of Engineering and Industrial Research National Academy of Sciences-National Research Council

under

Department of the Navy Bureau of Ships Contract NObs-72046 BuShips Index No. NS-731-036

Washington, D. C. National Academy of Sciences-National Research Council October 3, 1961

ABSTRACT

This fundamental fracture mechanics investigation has been concerned with studies of the propagation of brittle fractures in wide steel plates in an attempt to delineate many of the parameters associated with a propagating fracture, and in particular to study the strain field surrounding a propagating brittle fracture.

The experimental phases of the program involved tests of 3/4-in-thick structural-steel plate specimens, either 2 ft or 6 ft in width. In most cases the plate specimens were stressed uniaxially to about 19,000 psi, cooled to about 0° F, and a fracture was started with the notch-wedge-impact method of fracture initiation. Measurements of the strain distribution on the surface of the plate and the crack speed were made as the fracture traversed the plate. Recorded strains generally remained elastic even though the peak magnitudes, in some cases, exceeded 5000 microin./in. The majority of the recorded fracture speeds ranged from 2000 to 4000 fps. For the particular specimen geometry used and associated test conditions, the strain field surrounding the tip of an advancing fracture appears to remain essentially unchanged after traversing about one-third the width of a 6-ft-wide plate.

Exploratory studies were also made of the propagation of brittle fractures in prestressed plates. Measured fracture speeds ranged from about 4000 fps in the region of high tensile strain near the initiation edge to as low as 50 fps in the compressive strain region. The intensity of strain in the field associated with the tip of the moving crack diminished as the speed of the fracture decreased. The visual appearance of the fracture surface was unusually smooth when the crack had run at low velocity, with no evidence of the familiar herringbone pattern. The results indicate clearly that a residual strain field can have a marked ϵ or the ease of initiation and propagation of a brittle fracture.

Analytical studies of plate response were undertaken by representing the plate as a series of initially perpendicular, rigid bars connected at their points of intersection by a deformable node and interconnected at their midpoints by a shear element. Although the grid used was rather coarse, studies with this plate analog indicate that the lattice representation is a promising method of studying plate response during fracture propagation.

CONTENTS

_ - -

-

- - -

_...

- ---

- -----

-

Page

INTRODUCTION	1							
EXPERIMENTAL STUDIES	2							
Initial Exploratory Tests	2							
Six-Foot Wide Plain Plate Tests	5							
Prestressed Plate Tests	13							
STUDIES OF PLATE RESPONSE BY A LATTICE REPRESENTATION								
ACKNOWLEDGMENT	28							
BIBLIOGRAPHY OF REPORTS, PAPERS AND THESES ORIGINATING FROM THE INVESTIGATION	29							

.

BRITTLE FRACTURE MECHANICS ADVISORY COMMITTEE

for the

COMMITTEE ON SHIP STRUCTURAL DESIGN Division of Engineering & Industrial Research National Academy of Sciences-National Research Council

Chairman:

N. J. Hoff Head of Department of Aeronautical Engineering Stanford University

Members:

D. S. Clark Professor of Mechanical Engineering California Institute of Technology

Morris Cohen Department of Metallurgy Massachusetts Institute of Technology

F. J. Feely, Jr. Esso Research & Engineering Company

Martin Goland Vice President Southwest Research Institute

G. R. Irwin Head, Mechanics Division Naval Research Laboratory

Egon Orowan Department of Mechanical Engineering Massachusetts Institute of Technology

W. R. Osgood Department of Civil Engineering The Catholic University of America

M. P. White Head, Civil Engineering Department University of Massachusetts

INTRODUCTION

The widespread acceptance of steel as a structural material can be attributed largely to its mechanical properties. One of the most important of these properties is ductility, which permits inelastic deformation while retaining the useful load-carrying capacity of the member. Under certain conditions the normal ductility of steel may not be developed, and the structure may suddenly, and without previous warning, fracture in a brittle manner. In many applications today, the probability of brittle fracture may be minimized by giving proper attention to such variables as type of material, geometrical layout, fabrication procedure, temperature, and stress level. Also, in selected applications, it is now possible to effectively use crack arrestors, which may retard or stop the propagation of a brittle fracture. The above progress is a result of many years of extensive research; nonetheless, a clear understanding of the brittle-fracture mechanism is still unavailable and needed for the eventual development of satisfactory design procedures.

The objective of this investigation was to study the propagation of brittle fractures in wide steel plates. More specifically it was planned that the investigation would delineate many of the parameters associated with a propagating fracture by studying the strain field surrounding a propagating brittle fracture. Such measurements were of particular interest because little of the brittle-fracture work reported previously in the literature had included high-speed measurements during fracture propagation.

The experimental phases of the investigation involved tests of 3/4-in. thick structural-steel plates either 2 ft or 6 ft in width. In the later stages of the program, a few plates were tested in which there was a residual strain field of varying magnitude across the plate width. With the exception of a few of the latter tests, the test procedure consisted of stressing the plates uniaxially to about 17,000 to 20,000 psi, cooling the plates to approximately 0°F, and initiating the fracture by means of the notch-wedge-impact method. Measurements of crack speed and surface strain were measured electronically and recorded during fracture propagation to provide fundamental data of value in arriving at a better understanding of the phenomena associated with brittle-fracture propagation.

The analytical phase of the program consisted of developing an approximate method for studying plate response by representing the plate as a latticetype structure. This plate analog divides the plate into a series of rigid bars, each representing one degree of freedom, and deformable connections. Computer codes were developed for the calculation of static and dynamic response resulting from body forces or external forces. The studies showed that the model was able to represent a crack-type of discontinuity in a plate and was able to provide a picture of the strain redistribution resulting from the release of internally stored strain energy as a crack was initiated.

This final report of Project SR-137 summarizes the major findings of the investigation. The results of the studies as a part of this program have been reported in six SSC Reports, four published papers, and six theses, as listed in the bibliography. For more detailed information on any particular phase of the program, the reader is referred to these references.

EXPERIMENTAL INVESTIGATION

Initial Exploratory Tests

The initial phase of this investigation consisted of developing a test that would enable strain and speed measurements to be made on the surface of a plate while a brittle fracture propagated across a plate, and, at the beginning, was a joint effort with another related program concerned with the evaluation of crack arrestors (Ship Structure Committee Project SR-134). Four items that required early consideration were crack initiation, specimen cooling, instrumentation, and specimen geometry.

Two general methods of crack initiation were investigated. One involved an explosive device (the powder-detonator bolt) and the other involved a notch-wedge-impact system. The detonator bolt method failed to initiate any fractures and thus was dropped from further consideration. Therefore the successful notch-wedge-impact method of fracture initiation has been used for all plate tests in this program. In this

-2-

scheme a gas-pressurized piston drives a wedge into a prepared saw-cut notch in the edge of the stressed plate specimen. As a result of the early test work with this impact device, it was decided to use a piston energy input sufficient to insure initiation but low enough so that the plate fracture behavior was not markedly affected. The actual energy input used for all major tests made as a part of this program was about 1000 ft-lb.

Studies revealed that the most efficient method for cooling a plate specimen to the desired test temperature consists of placing crushed dry ice in wooden containers that are fitted alongside the specimen. This cooling procedure was used for all the major tests.

In order to obtain a measure of the crack speed, it appeared that the best available method would consist of placing detectors (conducting strips or wires mounted perpendicular to the expected crack path) on the surface of the specimen. The breaking time of the detectors would be measured by a recording instrument, and, with a knowledge of the time of breaking and the distance between the detectors, the crack speed could be computed. Several types of detectors were investigated, and it was finally decided that Baldwin SR-4 Type A-9 strain gages (single wire, 6-in. gage length) would be used for crack detectors. Although they were somewhat more ductile than other. detectors studied, it was felt that the greater ease of application and smaller variation of dimensions and properties of these strain gages made their use desirable.

Baldwin SR-4 Type A-7 strain gages (1/4-in. gage length) were used for the measurement of strain on the surface of the plates during the time the crack was propagating; where strain rosettes were used, these were made-up from individual strain gages. In the early tests several different types of strain gages were used, but studies indicated that the easiest and most desirable arrangement resulted from using gages of the type noted.

The details of the various types of sensing devices and recording instrumentation tried may be found in an earlier report on the project.¹ In general, the recording was done on oscilloscopes using either fixed- or movingfilm cameras.

-3-

Test No.	Avg. Net Applied Stress (ksi)	Avg. Test Temp. (F)	Remarks	Test No.	Avg. Net Applied Stress (ksi)	Avg. Test Tomp. (F)	Remarks	
Bests conducted with 3/4-in. thick by 2-ft wide specimens of Instrumented tests resumed on 6-ft wide rimmed steel specimens.								
l	25.0	13	Complete fracture. Good record.	30	18.0	78	Room temp Cooa record. No submerged cracks.	
2	16.0	-1	Complete fracture. No instrumentation.	31	18.0	-3	Complete fracture. Good record.	
3	14.0	0	Submerged crack 4.5 in. long. No instrumentation.	32	18.0	-1	Prestrained specimen. Com- plete fracture. Good record.	
4	16.0	20	Submerged crack 1 in. long. No instrumentation.	With the exception of Test 35, which was tested in conjunction with Project SR-134, all tests were conducted on $5/4$ -in. thick by 6-ft-wide semikilled steel plate specimens (See Reference h).				
	17.0	20	Submerged crack 2 in. long. No instrumentation.	33	19.0	0	Complete fracturegood	
5	18.0	1	Complete fracture. Record good for the part that exists.	=h	. 19.0	0	from 7 rosettes.	
6	18.0	0	Complete fracture. Record poor.	74	19.0	0	strain records obtained	
7	18.0	1	Complete fracture. Good record.	56	28.0	-15	Plate spectmen composed of	
8	18.0	-3	Complete fracture. Fair record.		20.0	/	36-in. starter strake of rimmed steel. 4 in. strake	
9	18.0	-2	Complete fracture. Record lost with the exception of one strain channel.				of T-1, 20-in. strake of rimmed steel, and a 12-in. strake of T-1 steel. Specimen was 27 in long. Fracture	
10	18.0	-5	Complete fracture. Good record.				arrested at leading cdge of final T-l strake. Initial load-	
11	18.0	-5	Complete fracture. Fair record.				1475 lbs. Final load85 kips. Good strain records obtained	
rinmed	Tests cond steel (For Te	lucted vites to the states of the second sec	th 3/4-in. thick by 6-ft wide specimens of mough 52, see Reference 2).				iron 4 rosettes.	
12	20.0	-10	Complete fracture. Record lost.	36	19.0	-10	Complete fracturefair strain record obtained from 11 rosettes. Double fracture last two-thirds	
13	20.0	0	Complete fracture. Good record.				of plate width.	
14	18.0	-8	Complete fracture. Record extremely poor, considerable noise.	37	19.0	-8	Complete fracturegood strain records obtained from 11 rosettes.	
15	18.0	-5	Complete fracture. Fair record.	38 č	19.0	-9	Complete fracturegood strain records obtained from 11 rosettes.	
16	18.5	74	Room temp. No record obtained. Submerged crack 1/2-in. long.	39	19.0	-6	Complete Tracturegood strain	
17	18.5	74	Room temp. Good record. Crack 3/8-in. long on east side and 1/8-in. long on west side.		The resid	lual strai	records obtained from 1) rosettes. n field in these specimens was produced	
18	18.5	8	Good record. Submerged crack 2-in. long. Essentially a	by welding tapered slots cut in the edges of the plate. These tests were conducted on $3/4$ -in, thick by 2-ft wide steel plates (See Reference 3 or δ).				
			striking test at low temp.	40	12.0	-32	Killed and normalized steel;	
19	20.5	-7	Complete fracture. Good record.				low fracture speeds recorded.	
20	18.0	٥	Complete fracture. Record lost. Evidence of branching	4 <u>1</u>	2.0	-9	Rimmed steel; 10-in. brittle fracture.	
21	18.0	85	Room temp. Good record. No	42	2.0	-5	Rimmed steel; 9 1/2-in. brittle fracture.	
22	18.0	-10	submerged cracks. Complete fracture. Good	These tests were conducted on $3/4$ in. thick by 6-ft wide semikilled steel plates (See Reference 6).				
			record.	43	0	-12	Tested on laboratory floor,	
23	18.0	-11	Complete fracture. Good record, except part was lost.				brittle fracture.	
24	18.0	5	Complete fracture. Record quality excellent, validity questionable.	1+1+	0	-12	Plain plate with no residual strain field tested on laboratory floor; no fracture.	
25	18.0	2	Complete fracture. Record quality excellent, validity questionable.	45	3.0	-20	Complete brittle fracture; low fracture speeds record- ed in central portion.	
wide	The follo semikilled ste	wing seri eel speci	ies of four tests was conducted on 6-ft- mens with no instrumentation.	46	3.0	0	Complete brittle fracture; low fracture speeds record- ed in central portion.	
26	17.0	-1	No submerged cracks.	47	o	-8	25-in. brittle fracture; arrested in compressive	
27	20.0	5	Complete fracture.				strain region.	

TABLE 1 OUTLINE OF TESTS - INITIAL EXPLORATORY TESTS - SIX FOOT WIDE PLAIN PLATES - PLATES WITH A RESIDUAL STRAIN FIELD

Avg. Net

28

29

18.0

17.0

-4

-2

Complete fracture.

Complete fracture.

Avg.

-4-

--

- -- -

. —

_.~ .

-

- -

. .

- -

-- -

_ .

_ _

0 36-in. brittle fracture; arrested in compressive strain region.

-- -- -- - - - -- --

_...

48 о

When the development work had progressed to a satisfactory stage, a series of tests of 2-ft-wide specimens was made. These tests are listed as Tests 1 through 11 in Table 1 and involved the surface measurement of crack speed and strain distribution while the fracture was propagating across the plate. The results have been reported in detail¹ and may be summarized brief-ly as follows:

- a. Crack speeds were recorded in the range of 1150 to 5900 fps.
- b. Absolute peak elastic strains as high as 2500 microin./in. in tension were recorded in the vicinity of the fracture.
- c. For the vertically oriented strain gages, as the vertical distance from the fracture to the gage increased, the magnitude of the peak strain decreased and the pulse time increased.
- d. No particular correlation of crack speed with texture was noted in this series of tests.
- e. A sizeable delay in the time necessary to start propagation was noted for cracks initiated in the vicinity of a sheared edge.

<u>Six-Foot-Wide</u> <u>Plain-Plate</u> <u>Tests</u>

After the general test procedure and reliable measuring techniques were developed from tests of 2-ft-wide plates as just described, tests of 6-ft-wide plain plates were conducted in order to obtain speed and strain records for a crack propagating across a greater width of plate. The test conditions and results are summarized in Table 1 (Tests 12 through 39). In general, the plates were tested with an average applied stress of 15,000 to 20,000 psi, at a temperature of about 0° F, and with the notch-wedge-impact method for fracture initiation. The specimens were prepared from several different types of steels; the type of steel and reference containing specific details are listed in Table 1. An overall view of a 6-ft-wide specimen in the 3,000,000-lb testing machine with the cooling tanks and initiation device in place is shown in Fig. 1.

A typical instrumentation layout and the strain-time records for one specimen are shown in Fig. 2. In the strain-time plot, zero time corresponds approximately to the time of fracture initiation. The initial strain values correspond to

-5-



FIG. I SIX-FOOT WIDE SPECIMEN MOUNTED IN 3,000,000-LB TESTING MACHINE



STRAIN - TIME RECORDS -- TEST 23

72"

the strain resulting from the applied test load. Vertically oriented strain gages in front of the crack indicated that there was negligible strain redistribution in the section of the plate ahead of the crack; also, beyond the center of the plate, the maximum strain values did not appear to increase with increasing crack length.

FIG. 2

In this series of tests, the majority of the strain and speed measurements were made in the immediate vicinity of the fracture path. Strain magnitudes as high as 3600 microin./in. were measured on the plate surface near the fracture with negligible permanent set remaining after fracture, that is, the exhibited response was elastic. In general, the nearer a vertically oriented strain gage was to the fracture path, the sharper and greater the magnitude of the strain pulse; as the distance increased, the strain pulse extended over a longer period of time, but the precise shape of the pulse depended on the distance from the fracture path. Fracture speeds ranging from 1800 to 7550 fps

-6-

were measured in these tests, with 75 per cent of the speed values within the range of 2100 to 3900 fps.

Striking tests conducted at room temperature, in which the specimens were subjected to the full impact energy of the notch-wedge-impact method of initiation but did not fracture because of the high test temperature, indicated that the strains resulting from the wedging action was relatively small when compared to the transient strains associated with a running crack. Thus, although the records indicated the impact was felt throughout the entire plate, it appears that the strain in regions other than the immediate vicinity of the notch was not materially affected by the wedging action.

With the acquisition of an additional 25 channels of cathode-ray oscilloscope recording equipment on loan from the Naval Research Laboratory, it was possible to study in more detail the strain field associated with a propagating brittle fracture. In this phase of the investigation, eleven rectangular strain rosettes were mounted on the surface of each plate specimen in order to determine the magnitudes and directions of the principal strains in the vicinity of the fracture. From Tests 33 through 39 (Table 1) sufficient data were obtained to establish the characteristics of the strain fields surrounding the propagating fracture. The results discussed below are based primarily on Tests 33, 34, 37, 38, and 39.

A typical instrumentation layout and a photograph of the fractured portion of one of these specimens are presented in Fig. 3. In general, the fracture paths in each test curved upward from the initiation edge and then remained essentially straight while traversing the rest of the plate; rarely did the fracture rise above the notch line (an imaginary horizontal line connecting notches on opposite edges of the plate) as much as the fracture shown in Fig. 3. Portions of the fracture surfaces exhibited a coarse texture with the usual herringbone pattern, while in other regions, usually near the initiation edges, the texture was quite smooth.

Strain-time traces for three rosettes of the specimen shown in Fig. 3 are presented in Fig. 4. In this figure zero time corresponds approximately to

. ...

. .

-7-





FIG. 3 INSTRUMENTATION LAYOUT AND FRACTURE

PATH -- TEST 39





-8-

the time of fracture initiation. The initial strain values correspond to the strain resulting from the applied test load. The vertical and diagonal component gage traces of each rosette exhibit a general shape characterized by a slight decrease and then a fairly steady and rapid increase in strain to a maximum value as the fracture propagates past the gage. The peak is followed by a decrease to a strain level associated with the removal of external load. As may be seen in Fig. 4, the pulse is very sharp for gages located close to the fracture; for gages located away from the fracture, the peak strain is of a lower magnitude and the pulse extends over a longer time.

The horizontal-gage (oriented parallel to the crack path) traces are characterized by three major changes in strain. The strain trace first exhibits an initial relaxation of compressive strain followed by a compressive pulse corresponding to the tensile peak of the vertical gage. Finally, the trace exhibits another relaxation of compressive strain before leveling off at the final strain value. The strain-time traces shown in Fig. 4 are representative of the strain records obtained as a part of this investigation.

The crack-speed detectors were used to measure the approximate surfacefracture speed and to aid in determining the location of the surface fracture at any time. For the five major tests cited, the recorded fracture speeds ranged from 1850 to 4750 fps, with an average fracture speed of 2700 fps in the region where the rosettes were located.

Typical principal strain curves for three rosettes are presented in Fig. 4. In general, the maximum principal strain trace for each rosette is of essentially the same shape and magnitude as the vertical component gage trace shown in the same figure; this merely indicates that the strain in the vertical direction is dominant. Most peak strain values fall in a range between 800 and 3000 mi-croin./in., although several values as high as 5000 microin./in. were recorded. The minimum principal strain traces are of much lower magnitude and resemble the horizontal gage strain traces.

For rosettes located close to the fracture path, the magnitude of the strain peak is relatively large. As the vertical distance between a rosette and

-9-



MAXIMUM PRINCIPAL STRAIN CONTOURS FOR VARIOUS CRACK LENGTHS

-10-

°0

000

60<u>9</u>

2200

°/

<u>o</u>g

6

the fracture surface increases, the peak principal strain magnitude decreases rapidly up to a distance of about 2 in. This may be seen clearly in Fig. 4 and in Fig. 5, the latter showing the peak maximum principal strains plotted against vertical distance between the rosette and the fracture. It should be noted that in all cases, even in the case of very high vertical component gage strains, the exhibited response was elastic with negligible permanent set remaining after fracture.

The maximum strain rates, as measured by the slope of the strain-time trace of a gage during peaking, were computed for all vertically oriented gages. Strain rates ranged from 1 to 109 in./in. sec, and, as would be expected, the highest strain rates were obtained for gages closest to the fracture. Although the scatter was fairly large, typical strain rates were 20 and 5 in./in. sec for gages located 1 in. and 3 in. from the fracture, respectively.

In order to portray the strain distribution on a plate surface during the time a crack is propagating, data from several tests that were conducted under identical test conditions were superimposed to permit the plotting of contours of maximum principal strain for various surface-crack lengths. The procedure used in superimposing the results from several tests is discussed in detail elsewhere.⁴

Maximum principal strain contours for various crack lengths are presented in Fig. 6. In the contour for a 22-in. crack, the magnitudes and directions (shown as short straight lines) of the maximum principal strains are shown at their respective locations on the plate layout. The individual strain values are not shown for the other crack lengths in order to simplify the drawings. The contour interval selected for the plots was 200 to 400 microin./in., and the strain contours are plotted in terms of absolute strain, that is, with respect to the as-rolled condition. Thus, the 600 contour for maximum principal strain corresponds approximately to the initial applied stress of 19,000 psi. The data shown in Fig. 6(c) for the 22-in. crack were obtained from strain data similar to those of Fig. 4 and are discussed more fully elsewhere.⁴

Since the strain-time traces for single vertically oriented gages and

-11-

maximum principal strains are similar. as noted earlier in the discussion of Fig. 4, it was possible to use the results of all earlier tests conducted as a part of this investigation in developing the strain contour patterns. Based on the large amount of data available, it is believed that the SE strain contours are representative of the strain distribution surrounding the tip of a brittle crack propagating in a wide steel plate for the test conditions used, in spite of the limitations imposed by the procedure of superimposing data from similar tests and by the variations in strain data, as may be noted in Fig. 6(c).

For the majority of the rosettes, as the fracture approaches the rosette, the direction of the maximum principal strain rotates slightly so that it points toward the approaching fracture and then follows the fracture for a short time as it goes by the gage position. The direction of the maximum principal strain more or less retains a vertical orientation. Values of θ , the angle of orientation of ϵ_{\max} as indicated by the slope of the short straight lines in Fig. 6(c), illustrate this general trend.

For an 8-in. crack (Fig. 6), the shortest crack length for which contours have been plotted, it will be noted that the increase in strain directly above and below the fracture tip is small in comparison with strain changes for longer crack lengths. The small changes in strain exhibited by the rosettes located only 8-in. from the initiation edge attest to the fact that the notch-wedge-impact method of fracture initiation has little effect on the strain distribution in a wide plate in regions away from the notch.

The change in strain distribution as the crack length increases may be seen clearly by comparing the contours for crack lengths of 8 in., 15 in., and 22 in. As the crack length increases, it will be noted that the magnitude and extent of the strain field associated with the crack tip likewise increase. For crack lengths in excess of 22 in., the extent of the strain contours ahead of the fracture increases only slightly with increasing crack length, and the strain field surrounding the advancing crack tip remains essentially unchanged. Strain contours for crack lengths between 22 in. and 43 in. are not shown but are essentially the same as the contours for either the 22-in. or 43-in. crack length.

-12-

Thus, for this particular specimen geometry, the propagating brittle fracture apparently reaches a more or less "steady-state" condition after traversing a distance of about 22 in.

The major portion of the strain field associated with the propagating brittle fracture extends only about 8—10 in. directly ahead of the fracture. The main changes in strain occur above and below the fracture tip and slightly ahead of it; behind the fracture front the strains decrease rapidly. All strain contours appear to converge toward the fracture front. Although extensive strain measurements were not obtained at distances far above and below the fracture, the available data indicate that the contours are symmetrical about the fracture path.

A set of typical maximum principal strain contours for a crack length of 22 in. to 50 in. is presented in Fig. 7; these contours are based on the results presented in Fig. 6 for a 22-in. crack and a 43-in. crack and for strain contours for other crack lengths between 22 in. and 50 in., as discussed in an earlier report on this project.⁴

Contours for vertical strain and maximum shear strain also were plotted as a part of this study but have not been presented. As would be expected, the vertical strain contours are quite similar to the principal strain contours. The variation in maximum shear strain with distance from the crack path is less clearly defined than the variation in the principal strains, because the maximum shear strain is more dependent upon the somewhat erratic horizontal strain readings than is the principal strain. However, in spite of the somewhat nonuniform nature of the shear strain contours, they are similar in shape to the principal strain contours, and likewise similar to those observed in photoelastic studies.

Prestressed Plate Tests

All of the tests described previously were conducted on specimens in which the only major stress present was that resulting from the externally applied load. The series of tests described in this section, identified as Tests 40 through 48 (Table 1), were conducted on both 2-ft and 6-ft wide plates

-13-



field on the propagation of a brittle fracture.

Preliminary studies were carried out on 2-ft-wide steel plates in which the residual strain field was produced both by flame heating and water quenching areas along both edges of the plate and also by welding tapered slots cut perpendicular to the edges of the plate. Although a high residual compressive strain could be produced in the central portion of the plate by heating and quenching, the resulting longitudinal strain gradient across the plate was not considered to be desirable for this series of tests. However, ____



the method of welding tapered slots did produce a satisfactory residual strain pattern consisting of high tensile strains reaching yield point magnitude at the edges of the plate and a fairly uniform compressive strain field throughout the central portion. The longitudinal residual strain distribution for Tests 40, 41, and 42, produced by welding tapered slots, is shown in Fig. 8.

Brittle-fracture tests were conducted on three plates (Tests 40-42) in which the residual strain field was produced by welding tapered slots. The specimens were instrumented with strain gages and crack-speed detectors to permit determination of strain response and fracture speed during the test. By use of the standard test procedure described earlier, brittle fractures were successfully initiated in all specimens; a complete fracture of the plate resulted in one specimen tested at 12,000 psi, while in the other two specimens, tested at only 2000 psi, the fracture arrested in the compressive strain region.

The results of these preliminary tests show that brittle fractures can be initiated in a region of high residual tensile strain with an extremely low applied stress. Also of interest is the fact that the residual compressive strain field causes an appreciable decrease in the fracture speed as the fracture propagates into the compressive strain region, as well as causing a marked reduction in the strain field associated with the tip of the moving crack as evidenced by the lower peak strain magnitudes recorded in the compressive strain region. The instrumentation layout and strain-time records from Test 40 are shown in Fig. 9.

In the series of tests on 2-ft specimens, the instrumentation was not sufficient to allow a thorough study of the speed and strain response. Also the magnitude and extent of the strain field produced in the plates were necessarily limited by the size of the specimens, and it was not possible to easily separate the effects of the various parameters involved. It was believed, as later results have verified, that similar tests conducted on 6-ft wide plates would facilitate a study of the factors influencing the propagation of a fracture through a residual strain field.

For the next phase of the investigation, six specimens were prepared from a semikilled steel plate material 3/4 in. thick and 72 in. wide. These specimens were designated as Tests 43 through 48 (Table 1). Five of these specimens (Tests 43, and 45-48) were prestrained by welding tapered slots cut in the edges of the plate. A photograph of the tapered slots at one edge of a plate may be seen in Fig. 10. The sixth specimen, Test 44, was a plain-plate specimen tested in conjunction with the prestrained specimens to aid in evaluating the effect of the initiation procedure. The measured longitudinal residual strain distribution in each specimen resulting from the welding of the tapered slots is shown in Fig. 11. As may be noted from the figure, the residual strain patterns for all specimens are similar as to extent and magnitude; at the edges of the specimens, the residual tensile strain is of yield magnitude, and across the central portion of the plate, there is a fairly uniform compressive strain region.

Brittle fractures were successfully initiated in all prestrained specimens tested, even those in which no external load was employed. In Tests 45 and 46, in which a small load was applied, complete fracture resulted, while in the remaining three prestrained specimens tested at zero applied load, the fractures

-16-



FIG. 10 TAPERED SLOTS PRIOR TO WELDING -- TEST 45



arrested in the compressive strain region. For comparison, in the earlier 6ft-wide plain-plate tests in which there was no residual strain field, an average applied stress in excess of 15,000 psi was necessary to insure initiation under similar test conditions. The fact that brittle fractures could be consistently initiated in the prestrained specimens with no external applied stress seems to indicate that residual tensile strains are an important factor in the mechanics of initiation and propagation. Even though the impact of the wedge contributed somewhat to initiation of the fractures in all of the 6-ft-wide prestrained-plate tests, it should be noted that in the absence of a residual strain field and with no applied load, the impact of the wedge is insufficient in itself to start a fracture. This fact was illustrated by the results of Test 44, in which a plain-plate specimen, similar in geometry to the prestrained specimens, was tested under conditions identical to those of Test 43, except that no residual strain field was present. Using the same initiation technique, no evidence of any fracture was observed.

While the residual tensile stresses materially aided fracture initiation and propagation, the residual compressive stress field in the central portion of the test specimen had just the opposite effect on the fracture characteristics. In addition to causing greatly reduced speeds throughout the central portion of the plates and a contraction of the strain field surrounding the moving crack tip, the compressive stress field also acted as an arrestor, completely arresting the fracture in Tests 43, 47, and 48 and slowing the fracture almost to the arrest point in Tests 45 and 46.

The fracture speeds in the compressive strain region were considerably lower than those recorded earlier on tests of 6-ft-wide plain-plate specimens. A plot of detector location versus detector breaking time for a typical prestrained specimen and, for comparative purposes, a plot for a typical nonprestrained plain test are given in Fig. 12. The prestrained plate, Test 46, had an external applied stress of approximately 3000 psi, while the plain-plate specimen, Test 37, had an external applied stress of 19,000 psi. In Fig. 12 the slope of the curve is a measure of the fracture speed at any particular time. The recorded

-18-



FIG. 12 TYPICAL FRACTURE SPEEDS -- TESTS 37 AND 46

fracture speeds ranged from 3000 to 4000 fps in the initiation region to as low as 50 to 100 fps in the compressive strain region.

In general, a brittle fracture propagated across the entire width of a non-prestrained plate in approximately 3 millisec, while a complete fracture of a prestrained specimen required as much as 30 millisec. It seems likely that the extent and magnitude of the residual compressive stress field in its initial state was sufficient for arrest in every test after the fracture had propagated well into the compressive strain region; however, for the two specimens that were tested under a small external load, the length of time required for fracturing was apparently long enough to permit redistribution of load and thus sustain propagation.

The effect of the residual strain field on the propagation characteristics also was apparent from the dynamic strain records obtained during the tests. The strain-time traces recorded from the various tests were similar and a typical set of strain-time traces, taken from the records of Test 46, are shown in Fig. 13. The traces shown are for vertically oriented strain gages located at the positions indicated in Fig. 14; in Test 46 the fracture propagated







FIG. 14 PLATE LAYOUT -- TEST 46

completely across the plate. The traces for gages located in the initial tensile region exhibited the usual response of vertical gages: a sharp elastic tensile strain peak as the fracture propagated by the gage and an immediate return to the approximate zero level of the gage. A trace of this type is illustrated by the response of Gage 5 in Fig. 13. As the fracture propagated into the compressive strain region, the peak magnitude of the recorded strain decreased. Strain gages located in the central region of relatively low compression exhibited a response similar to that of Gage 5 but with greatly reduced peak magnitudes; a relaxation was often noted before the gage peaked into tension. For those gages located in the com-

pressive strain region, the width of the strain pulse increased noticeably, averaging approximately 1.5 millisec, as compared to a pulse width of about 0.1 millisec for gages in the region of high tensile strain. The width of the strain pulses became progressively wider for gages located toward the far edge of the plate, sometimes having a width as great as 5 millisec; the pulse shapes were similar to that of Gage 14 in Fig. 13. The fact that the gages farthest from the initiation edge exhibited the greatest relaxation indicates that there was a marked bending relaxation in the plane of the plate which was associated with the low-velocity fractures. This may be seen from the traces of

-20-

Gages 12 and 14 in Fig. 13.

The fact that the peak strain magnitude decreased as the propagating fracture slowed down indicates that the intensity of strain in the field associated with the tip of the moving crack was diminished by the residual compressive strain field in the plate. The strain-time records from the specimens in which the fracture propagated completely across the plate show this effect most clearly, since in these tests, dynamic measurements were recorded across the entire plate width; however, this effect also was evident even in the case of the specimens in which the fracture arrested. Gages located 2 in. above and 3 in. below the fracture tip in the region where the fracture arrested showed no noticeable response, and gages located less than 1 in. from the fracture tip in the same region showed only a very small peak as a result of the fracture.

The crack path for all specimens was similar to that observed in the earlier plain-plate tests in that it remained essentially flat in its travel but exhibited a symmetrical variation which may have been influenced by the direction of the principal strain; the crack path for Test 46 is shown in Fig. 14. The texture of the fracture surface was noticeably different in regions of high and low speeds. In every test the surface texture had a rougher appearance in the vicinity of the initiation edge where the highest speeds were recorded. Throughout the compressive strain region (low-speed zones), the texture was very smooth and remained smooth as long as the speed was low; there was no evidence of the familiar herringbone pattern. The smooth texture observed in the region of low speed was even more evident when compared to the texture of high-speed fractures in the 6-ft-wide plain-plate specimens. Photographs of typical fracture textures in regions of both high and low speed are shown in Fig. 15. On the basis of this series of tests it may be stated that the smoother crack textures seem to be associated with the slower speeds, while the rougher textures occur in the region of higher speed.

The specimens in Tests 47 and 48, in which the fracture arrested, were cut longitudinally beyond the tip of the fracture and subsequently pulled apart to permit examination of the fracture texture and particularly the region of arrest. The

-21-



FAST FRACTURE - TEST 38 SLOW FRACTURE - TEST 45 FIG. 15 TYPICAL FRACTURE TEXTURES



FIG. 16 ARREST REGION -- TEST 48

texture of the fracture surface was essentially the same as that observed in the specimens that fractured completely, with the rougher texture occurring near the initiation edge where the higher speeds were recorded. Examination of the arrest region revealed that the interior fracture extended only a fraction of an inch beyond the visible surface fracture. The arrest region of Test 48 is shown in the photograph of Fig. 16.

STUDIES OF PLATE RESPONSE BY A LATTICE REPRESENTATION

The behavior of a continuous medium can be approximately represented by a lattice system consisting of a combination of rigid bars, deformable connections, and springs. Systems of this type are often helpful in arriving at an approximate solution for problems in which it is difficult or impossible to apply classical methods of analysis. Such a model can be used for investigating strain-wave propagation in two dimensions and for investigation of the tran-



sient strain redistribution associated with a crack that propagates in finite jumps. The type and arrangement of the lattice system are determined by the type of problem to be solved and the requirements of compatibility, equilibrium, and boundary conditions. In general the accuracy with which the lattice system reproduces the actual behavior of the continuous medium improves as the number of units in the lattice increases. The practical limit on the number of lattice units depends on the computational facilities at hand.

The particular lattice system used in this study was suggested by Professor N. M. Newmark and provides an easily visualized model for dynamic analysis. This lattice, shown in Fig. 17, is obtained by considering the plate to be replaced by a series of perpendicular rigid bars connected at their points of intersection by a deformable node and interconnected at their mid-points by a shear-type device which provides restraint against changing the original square configuration to a parallelogram. The elastic or inelastic properties of the material are concentrated in the deformable nodes in the case of the normal strains and in the shear connection in the case of shear strains. The mass of each of the rigid bars is equal to that of a piece of the plate having dimensions λ by λ in the interior and λ by $\lambda/2$ at the edge. It is thus seen that the total mass is actually represented twice in order to define inertial body forces in two directions.

Displacements of the lattice are only defined at the mid-points of the rigid bars. Normal stresses and resulting forces are confined to nodes and act on the ends of the bars, while shear forces resulting from shear stresses act at the mid-points of the bars. Equations expressing the elastic behavior

-23-

of the model under plain stress conditions were developed for both static and dynamic conditions.

Several statically loaded plates were analyzed with lattice models having different numbers of subdivisions. One case, a square plate loaded with parabolically distributed end tension, was solved by an energy method and with a lattice model having three different sizes of subdivisions. Averaged energymethod stresses were compared to stresses calculated in the lattice model, and it was found that the lattice model gave excellent results. One of the greatest difficulties encountered with the lattice model was that of duplicating the boundary deformation where the end loads were applied. The finer subdivision of the lattice model produced a better approximation of the loaded boundary deformation as well as giving more accurate results at other points of the lattice model.

Two examples were worked with concentrated effects, one a plate with a pair of concentrated loads, and another a plate containing a crack or pair of cracks; the results of these studies showed that the lattice model represents the strain distribution associated with such effects but that if steep strain gradients are involved, a finely divided lattice model is required to give satisfactory representation of the strain gradients. These studies also were made for the purpose of getting some indication in the change of representation of strain distribution as the number of model subdivisions changed.

The differential equations expressing the dynamic behavior of the lattice model were developed by adding a time-dependent term to the static equilibrium equations of the lattice model. Application of these equations to a steady-state condition and the calculation of natural frequencies of lattice models were studied. Two methods of numerical integration were used in the analysis of crack propagation in lattice models, both of which gave essentially the same results after a small number of time intervals. The computer used for the integration of the differential equations expressing the transient behavior of the lattice model could not be used for lattices having more than forty bars without the calculation time becoming excessive. A lattice model with only forty bars is much too coarse to adequately represent the steep strain gradient associated with the crack, and

-24-

1200 NODE 3 NODE 4 800 ε_v ϵ_y STRAIN - microinches/in 400 0 ε. €x - 400 - 800 0.2 0.4 0 0,6 0.8 0 0.2 0.4 Q,6 0.8 TIME - milliseconds

STRAIN-TIME TRACES FROM LATTICE

FIG. 18



FIG. 19 STRAIN-TIME TRACES FROM LATTICE SOLUTION



FIG. 20 STRAIN-TIME TRACES FROM LATTICE SOLUTION

therefore the forty-bar lattice provided only a rough qualitative picture of the transient strain redistribution associated with crack propagation.

Some of the results obtained with the latticemodel representation for unsymmetrical crack propagation are shown in Figs. 18-20. The crack propagated in four jumps, starting at the lefthand edge of the lattice shown in Fig. 17 and extending to the right-hand edge of the lattice model. The crack was started by separating the first node in Fig. 17 at time zero. After successive intervals of 0.18 millisec, the second, third and fourth nodes were separated. Typical strain time curves in the x- and ydirections are shown in the figures for Nodes 3, 4, 7, 8, 13 and 14, as noted on Fig. 17. It will be seen by examining Fig. 18 that in the nodes ahead of the crack in the lattice model the vertical strain increased in value as the crack approached the node

-25-

SOLUTION

until the node was separated to extend the crack. Nodes 3 and 4, shown in Fig. 18, exhibited a slight relaxation of the initial positive vertical strain prior to the increase resulting from the approaching crack. For points situated in the interior of the lattice model and above the crack path, the vertical strain traces showed only a slight peaking, and oscillations began shortly after the first small peaks occurred. After the crack had passed beneath one of the nodes, the strain values began to relax and release stored strain energy in the form of strain waves that eventually reflected from the lattice model boundaries. After a period of time, the strain values all oscillated within the spectrum of the natural frequencies of the lattice model as strain waves reflected and re-reflected at the lattice model boundaries.

The horizontal strain, as may be seen in Figs. 18-20, went from a negative value to a positive value as the crack approached and passed one of the nodes. This created a zone of biaxial tension ahead of the crack. Thereafter there were definite oscillations noted in the horizontal strain traces in much the same manner as noted for the vertical strain traces.

The lattice model indicates a general picture of the strain redistribution associated with crack propagation as follows: A tension pulse is generated when strain energy is released as a result a crack formation, and this pulse propagates away from the source, creating a new zone of biaxial tension ahead of the crack. As the crack approaches a particular point, the shear strain becomes positive in value and then decreases in value as the crack goes past. Because the lattice model used is purely elastic and dissipates no energy, the strain energy released by the crack formation must be stored in the lattice model as kinetic energy of the bars and changes in strain of nodes and shear points. After the initial pulse resulting in separation of a node is passed, the bars of the lattice model develop an oscillatory motion because of strain reflection from the boundaries.

Another problem solved by this method was that corresponding to symmetrical crack propagation, and in general, the results of this study were similar to those for the one just noted.

As a result of these studies it was concluded that the lattice representation can be used for the investigation of strain-wave propagation problems in

-26-

two dimensions. The use of the lattice-model method of analysis is contingent upon the availability of a large high-speed digital computer. It was not possible with the computer available to use lattice models having many bars, and only a crude picture could be formed of the transient strain distribution in a plate with a propagating crack. With larger and faster computers, the lattice analog for investigating transient strains associated with a propagating crack can be used to theoretically investigate different aspects of the fracture process in a plate by assuming fracture criteria for nodes in the lattice model and determining whether a fracture would propagate or arrest. The fracture criteria assumed could include internal damping, strain rates, strain magnitudes, non-linear stress-strain relationships and previous deformation history. A complete description of the theory and the results of the lattice model studies are presented elsewhere.⁵

ACKNOWLEDGMENT

This program was conducted in the Structural Research Laboratory of the Department of Civil Engineering, University of Illinois, under sponsorship of the Ship Structure Committee through the Bureau of Ships, U. S. Navy, Contract NObs 65790. The authors wish to acknowledge especially the assistance provided by the late Dr. Finn Jonassen, Mr. F. C. Bailey, Dr. D. K. Felbeck and Mr. R. W. Rumke of the National Academy of Sciences-National Research Council, and Captains R. D. Schmidtman and D. B. Henderson, Commander F. C. Munchmeyer and Lieutenant Commander J. D. Crowley who successively held the position of Secretary, Ship Structure Committee, during the administration of this program. The authors also are indebted to the Brittle Fracture Mechanics Advisory Committee and particularly to its chairman, Professor N. J. Hoff, which was created under the cognizance of the Committe on Ship Structural Design of the National Academy of Sciences-National Research Council, for continuing assistance and advice throughout the course of the program.

The authors gratefully acknowledge the assistance of the many persons who have contributed to the various phases of the program, particularly Professor W. H. Munse, Professor R. J. Mosborg, Professor V. J. McDonald, Dr. W. G. Godden, Dr. M. P. Gaus, and Messrs. O. A. Fettahlioglu, R. Lazar, and T. M. Lynam.

-28-

BIBLIOGRAPHY OF REPORTS, PAPERS AND THESES ORIGINATING FROM THE INVESTIGATION

<u>Reports</u>

- Hall, W. J., Godden, W. G., and Fettahlioglu, O. A., <u>Preliminary</u> <u>Studies of Brittle Fracture Propagation in Structural Steel</u> (Ship Structure Committee Report Serial No. SSC-111), Washington: National Academy of Sciences-National Research Council, May 15, 1958.
- Lazar, R. and Hall, W. J., <u>Studies of Brittle Fracture Propagation in</u> <u>Six-Foot Wide Structural Steel Plates</u> (Ship Structure Committee Report Serial No. SSC-112), Washington: National Academy of Sciences-National Research Council, September 17, 1959.
- Rolfe, S. T., Hall, W. J., and Newmark, N. M., <u>Brittle-Fracture</u> <u>Tests of Steel Plates Containing Residual Compressive Strain</u> (Ship Structure Committee Report Serial No. SSC-117), Washington: National Academy of Sciences-National Research Council, July 2, 1959.
- Rolfe, S. T., Lynam, T. M., and Hall, W. J., <u>Studies of the Strain</u> <u>Distribution in Wide Plates During Brittle Fracture Propagation</u> (Ship Structure Committee Report Serial No. SSC-118), Washington: National Academy of Sciences-National Research Council, December 31, 1959.
- 5. Gaus, M. P., <u>A Numerical Solution for the Transient Strain Distribu-</u> <u>tion in a Rectangular Plate with a Propagating Crack</u> (Ship Structure Committee Report Serial No. SSC-129), Washington: National Academy of Sciences-National Research Council, September 5, 1961.
- 6. Barton, F. W., and Hall, W. J., <u>Studies of Brittle Fracture Propagation</u> <u>in Six-Foot Wide Steel Plates With a Residual Strain Field</u> (Ship Structure Committee Report Serial No. SSC-130), Washington: National Academy of Sciences-National Research Council, April 3, 1961.

Published Papers

- 7. Hall, W. J. Mosborg, R. J., and McDonald, V. J., "Brittle Fracture Propagation in Wide Steel Plates," <u>The Welding Journal</u>, 36:1, Research Supplement, pp. 1s-8s (January 1957).
- Rolfe, S. T., Hall, W. J., and Newmark, N. M., "Brittle-Fracture Tests of Steel Plates Containing Residual Compressive Strain," <u>The</u> <u>Welding Journal</u>, 38.4, Research Supplement, pp. 169s-175s (April 1959).

- Barton, F. W., and Hall, W. J., "Brittle Fracture Tests of Six-Foot Wide Prestressed Steel Plates," <u>The Welding Journal</u>, 39:9, Research Supplement, pp. 379s-384s (September 1960).
- 10. Rolfe, S. T., and Hall, W. J., "The Strain Field Associated With a Brittle Fracture Propagating in a Wide Steel Plate." (Accepted for publication in the <u>Proceedings</u>, <u>Society for Experimental Stress</u> <u>Analysis</u>)

<u>Theses</u>

- 11. Fettahlioglu, O. A., <u>A Preliminary Investigation of Brittle Fracture</u> <u>Propagation in Structural Steel</u> (M. S. Thesis submitted to the Graduate College, University of Illinois), 1956.
- 12. Lazar, R., <u>Studies of Brittle Fracture Propagation in Six-Foot Wide</u> <u>Structural Steel Plates</u> (M. S. Thesis submitted to the Graduate College, University of Illinois), 1957.
- Lynam, T. M., <u>Strain Patterns During the Propagation of a Brittle</u> <u>Fracture in Steel Plates</u> (M. S. Thesis submitted to the Graduate College, University of Illinois), 1957.
- 14. Rolfe, S. T., <u>Studies of the Strain Distribution in Wide Plates Dur-</u> ing <u>Brittle Fracture Propagation</u> (M. S. Thesis submitted to the Graduate College, University of Illinois), 1958.
- 15. Gaus, M. P., <u>A Numerical Solution for the Transient Strain Distribu-</u> <u>tion in a Rectangular Plate With a Propagating Crack</u> (Ph. D. Thesis submitted to the Graduate College, University of Illinois), 1959.
- Barton, F. W., <u>Studies of Brittle Fracture Propagation in Six-Foot</u> <u>Wide Steel Plates With a Residual Strain Field</u> (M. S. Thesis submitted to the Graduate College, University of Illinois), 1959.

_____ · ___

COMMITTEE ON SHIP STRUCTURAL DESIGN Division of Engineering & Industrial Research National Academy of Sciences-National Research Council

<u>Chairman</u>:

12.

-

.

 $|\cdot|_{1}$

N. J. Hoff Head of Department of Aeronautical Engineering Stanford University

<u>Vice</u> <u>Chairman</u>:

M. G. Forrest Vice President-Naval Architecture Gibbs and Cox, Inc.

Members:

C. O. Dohrenwend Provost and Vice President Rensselaer Polytechnic Institute

J. H. Evans Department of Naval Architecture & Marine Engineering Massachusetts Institute of Technology

J. M. Frankland Consultant, Mechanics Division National Bureau of Standards

J. W. Miles Department of Engineering University of California

William Prager "L. Herbert Ballou University Professor" Brown University

Dana Young School of Engineering Yale University