

# Reliability-Based Methodology for Life Prediction of Ship Structures

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## ABSTRACT

*Recent advances in structural analysis technology such as the finite element method, improvements in the ability to model realistic sea loads, development of fatigue and fracture mechanics methods, development of reliability analysis methods, and the rapid advancements in computing technology, make it possible to develop methods to evaluate the structural strength of a ship, considering the requirements of its mission. In assessing the life expectancy of a ship structure, one must consider the through life degradation of the structural system and its potential failure modes. This paper describes the fundamentals of a reliability-based method for life expectancy analysis of ship structures, considering first-passage failure modes and fatigue failure mode, in an intact or degraded condition. As for fatigue analysis, a method based on probabilistic linear fracture mechanics and capable of modeling the effects of the weld induced residual stress is presented. The paper includes a detailed example of fatigue analysis of a typical ship structural joint using the probabilistic fracture mechanics method. The effect of the residual stresses and corrosion on the joint reliability is evaluated through a parametric study.*

## Presentation

### 1. INTRODUCTION

The design of a ship hull structure needs to be performed within the framework of the ship system design. In the conception and design stages decisions pertaining the form of the vessel and details of the structural systems are made. These decisions are ideally based on the design life of the vessel and thus directly impact its life expectancy.

The structural life of a system can be loosely defined as the period during which repairs are more economical than replacement. When repairs are required in such a magnitude and frequency that running costs rise without any effective improvement of the vessel, the end of useful life is in sight.

This method of quantifying structural life expectancy requires that there be a precise definition of the end of structural life.

Traditionally, the design criteria of ship structures are typically codified in the form of simple equations or charts that are meant for a particular application or material. They usually contain some empirically derived factors of safety that may not be evident to the user. These factors of safety beyond others uncertainties related to ship structural design take into account the estimated life of the ship. The use of these criteria can possibly lead to overdesigning the structure or, even worse, underdesigning due to improper estimation of safety, reducing the structural life expectancy.

Consequently, improved design criteria and analysis methods need to be developed. These methods should be capable of handling the new technologies and materials as well as existing ones. Because the loading of a ship structure is mainly the sea, a truly random system, the most appropriate new method should be one that takes into account the randomness of both loading and structural properties uncertainties to estimate the risk of unacceptable response.

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Based on this need, in recent years, ship structural design has been moving toward a more rational and probability-based design procedure referred as limit states design, which are based on a reliability-based design philosophy. The reliability-based design approaches for the ship structural system starts with the definition of the mission and the environment for the ship. The general dimensions and arrangements, structural member sizes, scantlings, and other structural details must be assumed as known.

Using a pre-defined ship operational profile, and considering the sea states that might be acting on the ship according to this operational profile, the time dependent wave induced loads acting on the ship structure can be defined according to analytical techniques, such as the one proposed by Sikora et al (1983). The resulting load profiles can be adjusted using modeling uncertainty estimates that are based on any available results of full-scale or large-scale hull testing.

The reliability-based design procedure also requires defining performance functions that correspond to limit states for significant failure modes. A limit state may be viewed as the boundary between acceptable and unacceptable structural behavior. The failure of the structural component occurs when its strength is less than the loading acting on it. The reliability of the component is achieved when the strength is greater than the load. Engineering models comparing the applied load effect ( $S$ ) and the structural strength or resistance ( $R$ ) are used to develop algebraic performance equations of the form:

$$Z = R - S \quad (1)$$

where  $Z$  = limit state function,  $R$  = structural strength, and  $S$  = loading acting on the structural component. The failure of the component occurs when  $g < 0$ .

Uncertainties in the limit state model are modeled in terms of the mean, the variance, and the probability density and distribution functions of the structural strength and loading. Due to the variability in both strength and loads, there is always a probability of failure that can be defined as

$$p_f = P(Z \leq 0) = P(R \leq L) \quad (2)$$

As the probability of failure for structural members is small, the safety of a structural component under a given external loading is expressed in terms of a reliability index, that reflects the probability of failure of the structural component. The higher the reliability index, the greater the structural safety. The required level of

structural safety for the structural component is expressed in terms of a target reliability index.

The reliability index associated with a given structural member can be defined in terms of time, considering the exposure of the ship to a loading condition. The end of the structural component life, in a simplified point of view, corresponds to the instant of time where the reliability index associated with the member is lower than the target reliability index.

Therefore, a reliability-based procedure is adequate to estimate the life expectancy of a vessel, once it deals with the uncertainties in the variables that affect the ship operational life, such as the structural strength and external loading, and allows the modeling of progressive degradation mechanism, that can affect the structural strength.

### 1.1. Structural Limit States

In order to examine the safety of ship structures, it is important to identify the major modes of hull failure. In general, these modes can be grouped as follows:

- a) Failure due to yielding or plastic flow of deck or bottom,
- b) Failure due to elastic-plastic buckling of deck or bottom panels, and
- c) Failure in a fatigue and fracture mode.

When considering the primary hull structure, reference is usually made to the midship section, and checks on the capability of secondary structures were only made in some studies. Basically the ship hull is considered to behave globally as a beam under transverse load subjected to the stillwater and wave induced load effects. In general the governing variable is the vertical bending moment which will induce the bending of the hull. The resulting stresses are distributed linearly across depth of the hull and their intensity at the bottom and deck is the ratio of the applied moments by the respective section modulus.

The moment to cause first yield of the midship section either in the deck or in the bottom is a common limit state. This moment is equal to the product of the minimum section modulus by the material yield stress. It tends to be conservative in that typical ship structural steels have a reserve strength after initial yielding in terms of their ductility and related work hardening capabilities. In addition, when first yield is reached at one hull girder extreme fiber, the other likely has not yielded and the hull girder will remain in this partially plastic state until plastic collapse.

Another limit is the plastic collapse moment, which is reached when the entire section becomes fully plastic. This moment is calculated considering that all the material is at yield stress. Thus, the plastic neutral axis is in a position that the total areas of material above and below the neutral axis are equal. This limit state is generally unconservative because some of the plates that are subjected to compression may buckle locally decreasing their contribution to the overall moment. The ultimate moment will in general be between the first yield and the plastic collapse moments.

The study of the ultimate resistance of midship section has been subject of study in many papers, such as those presented by Casella et al (1996), Ren et al (1996) and Mansour (1972).

The buckling failure of bottom or deck structures is more complex. Bottom and deck structures are generally grillages stiffened longitudinally, but still presenting transverse structures, named frames, so that different buckling modes can occur: failure of plates between stiffeners, interframe flexural buckling of the stiffeners, interframe tripping of the stiffeners, interframe buckling of the panel and overall grillage failure, usually between bulkheads, involving deflection of both longitudinal and transverse stiffeners. The elastic-plastic buckling strength of this type of structural elements is the objective of active research, and there are various adequate expressions to quantify their strength. Among these the methods proposed by Faulkner (1975), Adamchak (1975) and Hughes (1988) are oriented to marine structures.

The failure of deck or bottom panels structures under compressive loads can affect such a large portion of the cross-section that it is sometimes considered equivalent to a hull failure mode, Mansour (1972), and can represent the loss of ship safety.

The failure due to yielding, plastic flow or buckling of deck or bottom is associated with the failure of the ship under an extreme sea load to be faced during the operational life of the ship. So for the failure analysis it is often necessary to know the maximum of the combined value of stillwater and wave-induced loads effects. There is some correlation between the two load process in that significant changes of deadweight may imply some changes in the wave-induced load effects.

Although many reliability studies are related to the analysis of midship section, the ship transverse frame can also present yielding, plastic flow or buckling collapse. These failure processes were analyzed by Wang (1996) in a similar fashion as that used for midship section analysis.

Fatigue damage accumulation is associated to the total history of the cyclic wave bending moment rather than just the extreme value of the moment per se; thus the random nature of this history needs to be considered.

Failure by fatigue is a progressive cracking and unless it is detected this cracking can lead to a catastrophic rupture. When a repeated load is large enough to cause a fatigue crack, the crack starts at the point of maximum stress. This maximum stress is usually associated with a stress concentration (stress raiser). After a fatigue crack is initiated at some microscopic or macroscopic level of stress concentration, the crack itself can act as an additional stress raiser causing crack propagation. The crack grows with each repetition of the load until the effective cross section is reduced to such an extent that the remaining portion fails with the next application of the load (fracture). For a fatigue crack to grow to such an extent to cause rupture, it usually takes thousand or even million stress applications, depending on the magnitude of the load, type of the material used, and on other related factors.

Usually, the fatigue life of a structural component, subjected to the action of cyclic stress, is defined as the total number of stress cycles required to initiate a dominant fatigue crack added to the number of stress cycles required to propagate this crack until the final failure. This total life, in a simplified view, is a function of the geometry of the structure (local and global), applied stress range, the mean stress and the environment where the structure is located.

The stress-based fatigue analysis methodologies, represented by the classical *S-N* diagram, embody the damage evolution, crack nucleation and crack growth stages of fatigue into a single, experimentally characterized continuum formulation. However, it should be noted that the *S-N* curves are developed experimentally based on relatively small structures and their failure does not necessarily correspond to ship structural failure, which is based on the behavior of very large highly redundant structure.

A detailed evaluation of engineering structures and their construction processes shows these processes can induce the presence of small flaws, despite structural inspection for quality assurance purposes. Therefore, a modern defect-tolerant design approach to fatigue are based on the premise that engineering structures are inherently flawed, and the useful fatigue life then is determined by the time or number of cycles to propagate a dominant flaw of an assumed or measured initial size to a critical dimension.

Table 1. Classification of the Structural Failures as a Function of the Extension of Damage to the Ship Structure

<i>Failure</i>	<i>Failure Degree of Importance</i>		
	<i>Primary</i>	<i>Secondary</i>	<i>Tertiary</i>
<i>Ultimate</i>	1) Midship cross section plastic flow 2) Buckling of panel structures 3) Fatigue fracture.	Stiffened panels buckling between frames.	Unstiffened panel buckling.
<i>Serviceability</i>	First yield of the midship cross-section.	1) Cyclic load induced through thickness crack. 2) Stiffened panel permanent set	Unstiffened panel permanent set.

Because of the limitation on the control of the properties of steel and other materials used in shipbuilding and because of limitations on production and fabrication of ship components, the strength of apparently identical ships will not be, in general identical. In addition, uncertainties associated with residual stresses arising from welding, the presence of small holes, etc may also affect the strength of the ship. These limitations and uncertainties indicate that variability in strength about some mean value will result. This will in turn introduce an element of uncertainty as to what is the actual strength of the ship that should be compared with the loads and their uncertainties in order to define the reliability index associated with the structure.

The foregoing brief discussion indicates that it is convenient to split the analysis of failure into: 1) ultimate failures that will represent the loss of the ship, and 2) serviceability failures that will decrease the operational performance the ship structure, perhaps making it unsuitable for service.

Table 1 suggests a possible classification of ultimate and serviceability failures as for reliability analysis.

According to this table, the ultimate failure modes include flexural strength and buckling, and the serviceability failure modes include permanent deformation and first yield. The fatigue failure is included in both modes, depending on the extent of fatigue damage.

The importance of a failure is classified according to the degree of deterioration of ship safety or extension of the ship structure affected by a given failure mode. For this study, the failures are classified as:

- a) Primary: a failure mode that may affect great part of the structure and cause the loss or great major degradation of the structure performance,
- b) Secondary: a failure mode that may affect a part of the structure and cause damage or degradation of the structure performance, and

- c) Tertiary: a failure mode that may affect a small part of the structure and cause minor damage or degradation of the structure performance.

As for reliability analysis, ideally one could use different target reliability levels or margins of safety in the evaluation of the primary, secondary and tertiary failure modes, thus embedding a level of conservatism commensurate with their structural significance or consequence of failure.

## 1.2. Ship Structural Degradation

An ultimate strength failure of a ship structure is generally a result of an extreme load event and/or the reduction in structural resistance due to progressive degradation. For example, cumulative structural wastage due to corrosion will reduce local scantlings and thus the hull girder section modulus and render the ship more susceptible to local buckling or hull girder failure in response to an extreme loading event or fatigue crack growth will increase the risk (probability) of fracture. For these reasons, one should consider the long-term effects of progressive degradation in design. Design rules have generally incorporated corrosion allowances or the idea of net scantlings to preclude the deleterious effects of corrosion.

In a reliability-based design approach it is necessary to express these degradation rates explicitly. A reasonable amount of work has been conducted to identify degradation rates of ship structures. Information is available to estimate corrosion rates based on the quality of the initial construction and protection systems as well as the vessel service and structural location. For example the pitting corrosion of the bottom plating of a tanker structure has been expressed as shown in Figure 1. This type of information could be used to develop the statistical distribution necessary for reliability analysis.

For fatigue damage, the crack growth rate is dependent on the vessel operational profile, since the crack growth depends on the wave-induced load.

Furthermore, the crack growth is dependent on the presence of structural defects, induced during the hull fabrication, including weld defects and stress raisers, such as misalignment between structural parts.

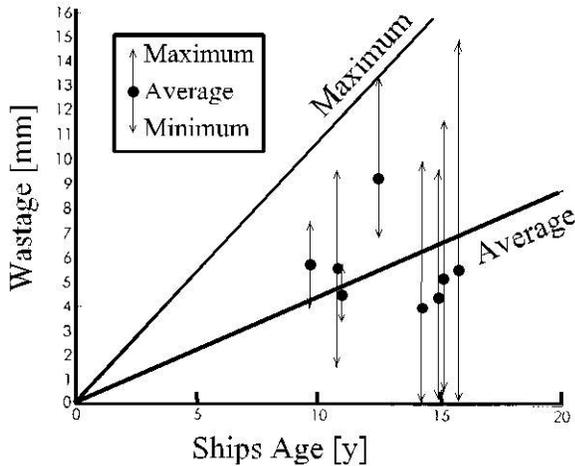


Figure 1. Pitting Corrosion of Bottom Shell Plating under Bellmouth, in Cargo Tank.

The quality of the fabrication and materials used in the construction of a vessel play a significant role in defining the life expectancy of a vessel. Misalignment, poor weld toe profiles, poor quality of paint application and cathodic protection systems, amongst other factors, can accelerate the degradation of a structural system. The degradation of the structural system can increase the probability of an ultimate structural failure, thus reducing the life expectancy of the structure, and consequently, reducing the vessel operational life.

### 1.3. Objectives and Scope

Time-dependent reliability assessment can be performed on new or existing ship structures when the structural resistance and the loads are described by probability distributions. It is expected in general, that as the service life of a structure progresses, expected extreme loads effects increase and structural strength decreases. For ship structures, the increase in load is based on the random nature of the wave loading. The longer the period of exposure, the greater the expectation of a high wave load. The decrease in strength is usually associated with the wastage of hull plating or stiffeners due to corrosion.

The time-dependent failure modes of ship structures can be classified in two main categories: (1) the first-passage failure modes, and (2) the cumulative failure modes. The first-passage failure mode includes the

failures due to yielding or plastic flow and due to elastic-plastic buckling. The fatigue failure is classified in the second category.

The objective of this paper is to propose a methodology for reliability-based ship structural life assessment, considering both categories of time-dependent failure modes above presented. The methodology takes into account the stochastic nature of the loads acting on the ship structure induced by environmental conditions and ship operational profile, randomness in strength and degradation resulting from environmental action.

Taking in view the great amount of fatigue failure in ship structures, the engineering models for estimating the fatigue resistance of ship structures are analyzed, taking in view the data requirements and relative level of approximation inherent in each approach. The fatigue analysis of a typical ship structure welded joint is performed, considering the use of the *S-N* and Fracture Mechanics approaches for fatigue reliability analysis.

## 2. RELIABILITY-BASED STRUCTURAL DESIGN AND ANALYSIS

The reliability of an engineered structure can be defined as the likelihood of it maintaining its ability to fulfil its design purpose for some time period under specified environmental conditions. The theory of probability provides the fundamental bases to measure the probability of satisfactory performance according to some performance functions under specific service and extreme conditions within the stated time period. In estimating this probability, system uncertainties are considered using random variables with associated mean values, variances, and probability distribution functions. Many methods have been proposed for structural reliability analysis, such as first-order second moment (FOSM) method, advanced second moment (ASM) method, computer-based Monte Carlo simulation (e.g., Ayyub and McCuen 1997, Ang and Tang 1990, Ayyub and Haldar 1984, White and Ayyub 1985), and conditional expectation simulation (e.g., Ayyub and McCuen 1997). These reliability analysis methods may be used to estimate the time-dependent or conditional reliabilities.

### 2.1. Instantaneous Reliability

The reliability of a structure may be estimated based on a limit state equation (or performance function) that can be expressed in terms of the basic design variables, some of which are represented as random variables ( $X_i$ ) to express their perceived variability. The limit state

equation,  $Z(X_i)$ , defines the boundary between acceptable and unacceptable performance (safe operation and failure) and can be presented as:

$$Z(X_i) = \text{Structural Capacity} - \text{Load Effects}$$

Unsatisfactory performance (or the occurrence of a *limit state*) is identified when the limit state equation is less than zero (i.e.  $Z(X_i) < 0$ ). If the joint probability density function for the basic random variables  $X_i$ 's is  $f_{X_1, X_2, \dots, X_n}(x_1, x_2, \dots, x_n)$ , then the unsatisfactory performance (i.e., failure) probability  $p_f$  of a structure is estimated based on the convolution integral:

$$p_f = \int \dots \int f_{X_1, X_2, \dots, X_n}(x_1, x_2, \dots, x_n) dx_1 dx_2 \dots dx_n \quad (3)$$

where the integration is performed over the region in which  $Z < 0$ . In general, the joint probability density function is unknown, and evaluating the convolution integral is a formidable task. For practical purposes, alternate methods of evaluating  $p_f$  are necessary.

One alternative is the use of a Taylor series approximation, which can express a first order (FORM) estimate of reliability by expressing the joint probability density function parameters (mean and standard deviation) as follows:

$$U_Z = Z(U_{X_i}) \quad \text{and}$$

$$\sigma_Z^2 = \sum_{i=1}^n C_i^2 \sigma_{X_i}^2 + \sum_{i=1}^n \sum_{j=2}^n \rho_{ij} C_i C_j \sigma_{X_i} \sigma_{X_j} \quad [i \neq j] \quad (4)$$

where  $C_i = \left. \frac{\partial Z}{\partial X} \right|_{X=U_{X_i}}$ ,  $\rho_{ij}$  is the correlation coefficient for

variables  $i$  and  $j$  which can be assumed to be zero for independent variables (uncorrelated) and  $U_{X_i}$  and  $\sigma_{X_i}$  are the random design variable means and standard deviations, respectively. With this information a FORM estimate of the reliability is developed using the concept of the reliability index ( $\beta$ ) and its assumed relationship to the inverse standard normal probability distribution ( $\Phi^{-1}$ )

$$\beta = \frac{U_Z}{\sigma_Z} = \frac{U_R - U_S}{\sqrt{\sigma_R^2 + \sigma_S^2}} \quad (5)$$

and the probability of failure is defined as  $p_f = \Phi^{-1}(\beta)$

As shown in Eq. 5, the reliability index may be expressed in terms of the structural resistance ( $R$ ) and loading ( $S$ ) distribution parameters for convenience. More advanced Taylor series expansion techniques such as that based on the work of Hasofer and Lind (Madsen,

Krenk and Lind 1986) could also be applied, defining the Advanced Second Moment method (ASM).

Simulation techniques offer a numerical alternative to the estimation of reliability or failure probabilities. These approaches basically continuously select random values for each random design parameter from their respective probability distributions and evaluate the limit state equation. The total number of limit state evaluations indicating unsatisfactory behavior divided by the number of evaluations is the probability of failure ( $=1 - \text{Reliability}$ ). This process becomes time consuming and computationally expensive for high reliability cases since the number of limit state equation evaluations required to converge on an accurate estimate of the failure probability is large. Advanced more efficient simulation techniques including importance sampling or Latin-Hypercube sampling (Dinovitzer 1992) may be used to reduce the computational cost and render simulation techniques viable.

## 2.2. Time-Dependent Reliability

The strength (or resistance)  $R$  of structural component and the load effect  $L$  are generally functions of time. Therefore, the probability of failure is also a function of time. The time effect can be incorporated in the reliability assessment by considering the time dependence of one or both of the strength and load effects.

Ayyub and White (1990a and 1990b) and White, Ayyub and Purcell (1989) developed a methodology for assessment of structural life of marine structures using the basic concepts of probabilistic analysis, and statistics of extremes. According to these authors, it is expected in general, that as the service life of a structure progresses, expected extreme load effects increase. Then the resulting extreme value probability distribution can be used in the reliability assessment. These authors also considered that the structural strength decreases, usually due to wastage of the hull plating due to corrosion.

The proposed reliability analysis is based on the determination of the probability of failure for a given time  $t$ . On the basis of the time-dependent load effect  $L(t)$  and the structural strength  $R(t)$ , the probability of failure for the time  $t$  is computed for a specified failure mode using one of the applicable methods described in section 2.1. These functions represent the instantaneous probability density function for the load effect and structural strength at a given time, considering both the extreme value distribution for the load effect and the degradation of the structural resistance at this time. By varying the time period  $t$  from zero to the design structural life, a plot of the probability of failure as a function of time can be developed. This probability of failure is defined as the

instantaneous probability of failure at time  $t$ , without regard to previous or future performance.

This method is suitable for using in reliability and structural life assessment according to certain failure modes, for example, plastic deformation and buckling. For failure modes, such as fatigue, that the failure event occurs because of the accumulation of damage due to repeated application of cyclic loads of variable amplitudes with varying frequencies, the reliability is defined according to the methods described in section 2.3.2.

Ayyub, White, Bell-Wright and Purcell (1990c) applied this methodology in a comparative analysis between two different patrol boats. The comparison is based on the identification of two critical failure modes, plastic plate deformation and fatigue.

Most of these concepts were reviewed by Ayyub and White (1995) in order to generalize them for any structural system.

Also for marine structures, Soares and Ivanov (1989) discussed a model to quantify the time variation of the reliability of a primary ship structure. The variation of resistance is assumed to be a decreasing function due to the corrosion effect. The basic equation is

$$R(L) = R_0 \exp \left[ - \int_{t_0}^L h(t) dt \right] \quad (6)$$

where  $R(L)$  represents lifetime reliability,  $h(t)$  represents the hazard function which is the probability that the structure will fail during interval  $t$  and  $t + dt$ ,  $t_0$  is the time at which the structure is put in service, and  $R_0$  is the reliability at that time. For high levels of reliability another relation is suggested by Soares and Ivanov (1989) as

$$R(L) = R_0 \exp[1 - n(1 - R_i)] \quad (7)$$

where the lifetime is equal to  $n$  years, and

$$R_i = 1 - p_f \quad (8)$$

$$p_f = \Phi(-\beta) \quad (9)$$

where  $\Phi$  is the Gaussian distribution function.  $\beta$  is the standard safety index. The assumption in Eq. 9 of a standard normal distribution is not always true.

Ellingwood and Mori (1993) developed a time-dependent methodology for the deterioration of concrete structures at nuclear power plants. This method models significant structural loads as a sequence of pulses which can be described by a Poisson process with mean occurrence rate,  $\lambda$ , random intensity,  $S_j$ , and duration,  $\tau$ .

Ellingwood and Mori (1993) define the limit state of the structure at any time as:

$$R(t) - S(t) < 0 \quad (10)$$

where  $R(t)$  is the strength of the structure at time  $t$  and  $S(t)$  is the load at time  $t$ . The probability of failure can then be defined at time  $t$  as  $P[R(t) < S(t)]$ . Ellingwood and Mori (1993) define the reliability function,  $L(t)$  as the probability that the structure survives during interval of time  $(0, t)$ . The equation for reliability function becomes

$$L(t) = \int_0^\infty \exp[-\lambda t [1 - \int_{t_0}^t F_S(g(t)r) dt]] \cdot f_R(r) dr \quad (11)$$

where  $f_R(r)$  is the pdf of initial strength,  $R$  and  $g(t)$  is the time-dependent degradation in strength.

Ellingwood and Mori (1993) express the reliability in terms of the conditional failure rate or hazard function,  $h(t)$  as

$$h(t) = - \frac{d}{dt} \ln L(t) \quad (12)$$

which can be expressed as

$$L(t) = \exp \left[ - \int_0^t h(\xi) d\xi \right] \quad (13)$$

Ellingwood (1995) later notes that the reliability and limit state functions  $L(t)$ , or conversely the probability of failure,  $p_f(t)$ , are cumulative, i.e., they should be used to define the probability of successful performance during a service life interval  $(0, t)$ . Ellingwood (1995) emphasizes that the  $p_f(t) = 1 - L(t)$  is not equivalent to  $P[R(t) < S(t)]$ , the latter being just an instantaneous failure at time,  $t$ , without regard to previous or future performance. This is a very important point that is lacking in much of the literature that is available.

Although the method developed by Ellingwood and Mori (1993) was used to analyze the reliability of concrete structures, it can be used to calculate the time-dependent reliability of ship structures. The main advantage of this methodology is the development of a closed function expressing the structure reliability, considering the time dependency of structural strength degradation. The probabilistic characteristics of the loading are considered as time invariant.

### 2.3. Methodology for Time-Dependent Reliability of Ship Structures

Based on the analysis presented on section 1.2 of this paper, the time-dependent failure modes of ship structure can be classified in two main categories: (1) first-passage failure modes, and (2) cumulative failure modes. The

methodologies for time-dependent reliability analysis for these two categories are presented in this section.

### 2.3.1. First-Passage Failure Modes

Structural safety under a single load is measured by the probability of survival,  $L$ , represented by

$$L = P(R > S) = \int_0^{\infty} F_S(r) f_R(r) dr \quad (14)$$

in which  $F_S(x)$  is the probability distribution function of the structural response due to the applied load,  $S$ , and  $f_R(r)$  is the probability density function of the resistance of the structure,  $R$ , expressed in units that are dimensionally consistent with  $S$ .

Structural loads and strength vary in time, making structural reliability time dependent. Assume, for the present that the initial strength of the structural component,  $R_0$ , is deterministic and equal to  $r$ . Suppose that  $N=n$  load events occur within time interval  $(0, t_L)$ , and that there is no change in component resistance,  $r$ . The loads  $S_j$ ,  $j = 1, 2, \dots, n$ , are assumed to be statistically independent random variables described with distribution function  $F_S(x)$ . The reliability function,  $L(t_L)$ , defined as the probability that the structural component survives during time interval  $(0, t_L)$  is expressed as

$$L(t_L | R_0 = r, N = n) = \quad (15)$$

$$P[(r > S_1) \cap (r > S_2) \cap \dots \cap (r > S_n)]$$

or

$$L(t_L | R_0 = r, N = n) = [F_S(r)]^n \quad (16)$$

However, strength of the component may deteriorate with time due to environmental action, and this deterioration may be expressed as

$$r(t) = rg(t) \quad (17)$$

in which  $r(t)$  is the strength at time  $t$ , and  $g(t)$  is the degradation function. Ellingwood and Mori (1993) assumed that the function  $g(t)$  is independent of the load history. Therefore, the formulation presented subsequently can be used to express time-dependent reliability due to deterioration induced by corrosion and similar environmental effects. However this methodology cannot be applied to fatigue analysis, where the time-dependent behavior in strength is affected by stresses induced by environmental loading.

If  $n$  loads occur within the time interval  $(0, t_L)$  at deterministic times  $t_j$ ,  $j = 1, 2, \dots, n$ , the reliability function is represented as follows:

$$L(t_L | R_0 = r, N = n) = P[(rg(t_1) > S_1) \cap (rg(t_2) > S_2) \cap \dots \cap (rg(t_n) > S_n)] \quad (18)$$

or

$$L(t_L | R_0 = r, N = n) = \prod_{j=1}^n F_S(rg(t_j)) \quad (19)$$

In general, the loads occur randomly at a vector of times  $\underline{T} = \{t_1, t_2, \dots, t_n\}$ , described by the joint probability density functions  $f_{\underline{T}}(t)$ . In this case, the time-dependent reliability function becomes

$$L(t_L | R_0 = r, N = n) = \int_0^{t_L} \int_{t_1}^{t_L} \int_{t_2}^{t_L} \dots \int_{t_{n-1}}^{t_L} \left[ \prod_{j=1}^n F_S(rg(t_j)) \right] f_{\underline{T}}(t) dt \quad (20)$$

The number of loads acting on a structure during a given time interval can be considered random, so the reliability function is also dependent on the random variable  $n$ . Ellingwood and Mori (1993) suggested that the number of load occurrences in a time interval can be modeled by a Poisson distribution, with characteristic parameter  $\lambda$  representing the mean rate of occurrence of loads.

Considering that the number of load occurrences is modeled with a Poisson distribution, the time to occur one load is a random variable modeled by an Exponential distribution. In the time interval  $(0, t_L)$ , the time to occur the  $j$  load is represented by the following probability density function

$$f_T(t_j) = \exp(-\lambda(t_L - t_{j-1})), \text{ for } t > t_{j-1} \quad (21)$$

where  $t_{j-1}$  is the time that the  $(j-1)$  load occurs.

As the loads are statistically independents, the joint probability function of the time of occurrence of  $n$  loads is expressed as:

$$f_{\underline{T}}(t_1, t_2, \dots, t_n) = \lambda^n \exp[-\lambda((n-1)t_L - t_n - t_{n-1} - \dots - t_2 + t_1)] \quad (22)$$

for the interval  $(0, t_L)$  and considering  $t_1 < t_2 < \dots < t_{n-1} < t_n$ . Consequently, the reliability function, considering the occurrence of  $n$  loads, can be formulated as follows

$$L(t_L | R_0 = r, N = n) = \int_0^{t_L} \int_{t_1}^{t_L} \dots \int_{t_{n-1}}^{t_L} \left[ \prod_{j=1}^n F_S(r g(t_j)) \right] \cdot \lambda^n \exp[-\lambda((n-1)t_L - t_n - t_{n-1} - \dots - t_2 + t_1)] dt_n \dots dt_1 \quad (23)$$

The reliability function presented in Eq. 23 can only be analytically evaluated if the time-dependent integral has a closed form. As the probability functions usually used to define the loading effect on a structure or structural component have complex formulations, the numerical value of the Eq. 23 can only be defined numerically, and solved based on the use of simulation techniques. This simulation is not easily executed taking in view that, for a given number of loads ( $n$ ), the reliability function is evaluated based on the random generation of the time of occurrence of each load. The simulation must be repeated for a given number of trials, until the reliability function presents a numerical convergence. This procedure must be executed for a set of values of number of random loads, since this variable is considered a random variable.

In order to evaluate the behavior of the reliability function presented in Eq. 23, the strength of the structure is assumed deterministic and a constant with magnitude  $r$ . The reliability can then be expressed as

$$L(t_L | R_0 = r, N = n) = \int_0^{t_L} \int_{t_1}^{t_L} \dots \int_{t_{n-1}}^{t_L} \left[ \prod_{j=1}^n F_S(r) \right] \cdot \lambda^n \exp[-\lambda((n-1)t_L - t_n - t_{n-1} - \dots - t_2 + t_1)] dt_n \dots dt_1 \quad (24)$$

or

$$L(t_L | R_0 = r, N = n) = [F_S(r)]^n \cdot \int_0^{t_L} \int_{t_1}^{t_L} \dots \int_{t_{n-1}}^{t_L} \lambda^n \exp[-\lambda((n-1)t_L - t_n - \dots - t_2 + t_1)] dt_n \dots dt_1 \quad (25)$$

Based on Eq. 25 a function  $A(n, \lambda, t_L)$  can be defined as the relation between the reliability index and the structural resistance

$$A(n, \lambda, t_L) = \frac{L(t_L / R_0 = r, N = n)}{[F_S(r)]^n} = \int_0^{t_L} \int_{t_1}^{t_L} \dots \int_{t_{n-1}}^{t_L} \lambda^n \exp[-\lambda((n-1)t_L - t_n - \dots - t_2 + t_1)] dt_n \dots dt_1 \quad (26)$$

The evaluation of the integral that defines  $A(n, \lambda, t_L)$  can be executed numerically, with the use of simulation techniques, for a given set of values of the variables  $\lambda$  and  $t_L$ , varying the number of loads ( $n$ ).

The Poisson probability function used to model the distribution of number of occurrence of load during the time interval  $(0, t_L)$  as a mean value equal to  $\lambda t_L$  and the standard deviation equal also to  $\lambda t_L$ . This mean value represents the expected number of load occurrences in the interval  $(0, t_L)$ .

The trends presented in Figure 2 represents the numerical variation of  $A(n, \lambda, t_L)$  as a function of the relation  $n/\lambda t_L$  i.e., (number of load occurrences divided by mean value of Poisson distribution), corresponding to the number of standard deviations, for a value of  $t_L$  equal to 10 units of time. The results presented in this figure were defined through the use of numerical simulation, in which a random set of time of occurrences were generated, allowing the evaluation of the function  $A(n, \lambda, t_L)$ .

Based on the results presented in Figure 2, the function  $A(n, \lambda, t_L)$  decreases with the increase in the relation  $n/\lambda t_L$ . The rate of decrease is reduced when the mean value of the Poisson distribution increases. For the case when this mean value is equal to 10, the rate of decrease can be considered very low, and the function  $A(n, \lambda, t_L)$  can be considered constant, and equal to 1.

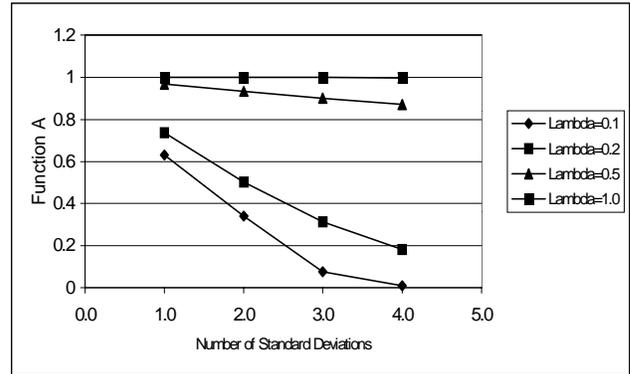


Figure 2. Numerical Variation of the Function  $A(n, \lambda, t_L)$  as a Function of the Relation  $(n/\lambda t_L)$

This result indicates that for high mean values of number of load occurrences, the joint distribution of the time of occurrence of loads can be considered uniform, since for high number of loads, the time of occurrence of these loads can be considered uniformly distributed in the interval  $(0, t_L)$ . Specifically for ship structures, the mean

value of number of load occurrences during the ship operational life is very high, so the above mentioned hypothesis can be used to model the distribution of the time of occurrence of loads. Ellingwood and Mori (1993) also used this hypothesis to model the joint probability distribution of the time of occurrence of the loads.

Considering that time of occurrence of loads are uniformly distributed, the joint probability density function of the time of occurrence of  $n$  loads in the interval  $(0, t_L)$  is expressed as

$$f_T(t_1, t_2, \dots, t_n) = \left( \frac{1}{t_L} \right)^n \quad (27)$$

According to Ellingwood and Mori (1993) the reliability function is expressed as

$$L(t_L | R_0 = r, N = n) = \int_0^{t_L} \dots \int_0^{t_L} \left[ \prod_{j=1}^n F_S(rg(t_j)) \right] \left( \frac{1}{t_L} \right)^n dt_n dt_{n-1} \dots dt_1 \quad (28)$$

or

$$L(t_L | R_0 = r, N = n) = \left[ \int_0^{t_L} F_S(rg(t)) \frac{1}{t_L} dt \right]^n \quad (29)$$

Equation 29 is used to define the reliability of the structure considering the action of  $n$  random loads in a given time interval  $(0, t_L)$ , and that the structural strength at the beginning of the operational life is deterministic.

Considering that the number of load events is random, defined according to a Poisson distribution, the reliability function in the interval  $(0, t_L)$  becomes

$$L(t_L | R_0 = r) = \sum_{n=0}^{\infty} \left[ \int_0^{t_L} F_S(rg(t)) \frac{1}{t_L} dt \right]^n \frac{(\lambda t_L)^n \exp(-\lambda t_L)}{n!} \quad (30)$$

According to Ellingwood and Mori (1993), Eq. 30 is expressed as

$$L(t_L | R_0 = r) = \exp \left\{ -\lambda t_L \left[ 1 - \frac{1}{t_L} \int_0^{t_L} F_S(rg(t)) dt \right] \right\} \quad (31)$$

As a final step for time-dependent reliability analysis, the conditioning on the initial strength  $R_0=r$  is removed in order to take into account the randomness in the structure initial strength by considering the probability density function associated with the structural resistance as follows

$$L(t_L) = \int_0^{\infty} \exp \left\{ -\lambda t_L \left[ 1 - \frac{1}{t_L} \int_0^{t_L} F_S(rg(t)) dt \right] \right\} f_{R_0}(r) dr \quad (32)$$

Equation 32 is used to define the reliability of the structure or structural component in the time interval  $(0, t_L)$ . This model can be used to analyze the reliability associated with the ultimate strength and buckling failure modes.

The time-dependent reliability analysis based on this methodology must be performed according to the procedure presented in Figure 3. The information necessary to perform the reliability analysis includes the structural loading, the structural strength and the time degradation characteristics represented by the corrosion effects.

The lifetime structural loading must be developed considering the operational conditions and the characteristics of a ship in the sea. The operational conditions are usually divided into different operation modes according to the combinations of ship speeds, ship headings, and wave heights. The ship characteristics include the length between perpendicular, the beam, the draft, and the displacement. As the reliability analysis requires the probabilistic characteristics of the random variables used in the reliability function, expressed in Eq. 32, the loading acting on the ship structure must be defined by a probability function ( $F_S(x)$ ), that represents the combination of the loading conditions faced by the structure during its operational life. This function can be expressed as a combination of short-term loading conditions probability functions, in accordance with the Lifetime Weighted Sea Method, proposed by Hughes (1988). In addition to this function, the rate of occurrence of loads ( $\lambda$ ) must be defined, in order to evaluate the time-dependent reliability. If necessary, a rate of occurrence of loads can be defined for each short-term load condition.

The ship structural characteristics allow the definition of the initial structural strength ( $R_0$ ), used for reliability analysis. By considering the structural configuration and the uncertainties associated with the dimensions of the structural elements (plates and stiffeners), a probability density function can be associated to the structural strength. This function is used to define the time-dependent reliability according to the expression presented in Eq. 32.

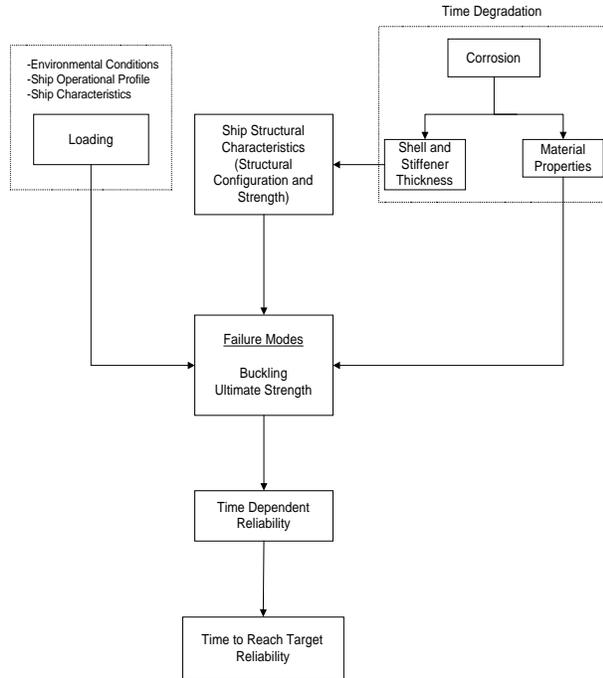


Figure 3. Time-Dependent Reliability Flowchart for First-Passage Failure Modes

The time degradation effects, represented by the corrosion process, can affect the structural strength or the material properties. With respect to the structural strength, the corrosion effect must be modeled according to a degradation function ( $g(t)$ ), that express the rate of structural strength degradation through the ship operational life as a function of the reduction of the structural scantlings dimensions due to corrosion. This function must be defined based on the statistical distribution of annual corrosion rates observed for ship structures. Besides this effect, the corrosion can also affect the material properties, such as ductility, due to the phenomena named Stress Corrosion Cracking (SCC), Jones (1992). So, the possible degradation in material properties due to the corrosion must also be modeled as a degradation function that expresses the rate of changes as a function of time.

Once the probabilistic characteristics of the loading and structural strength are defined, the time-dependent reliability of the ship structure can be defined, being developed a function similar to Eq. 32 for each of the failure modes analyzed in the study (ultimate strength and buckling). Based on these functions, the time-dependent reliability is defined for each of these failure modes. The solution of the time-dependent reliability equation can be executed numerically or analytically, depending on the

complexity of the equation developed for each failure mode.

As a summary, the following steps must be executed for time-dependent reliability analysis of first-passage failure modes:

- 1) Define the probability function associated with the ship long-term loading ( $F_S(x)$ );
- 2) Define probability density function of the initial structural strength of the ship structure ( $f_{RO}(x)$ );
- 3) Define the structural strength degradation function ( $g(t)$ ) and the material properties;
- 4) Define the time-dependent reliability equation, similar to Eq. 2-15, considering the possible material properties degradation due to corrosion action;
- 5) Evaluate the time-dependent reliability considering the ship operational life.

### 2.3.2. Failure Due to Cumulative Damage

The total fatigue life of a ship structural detail is computed through the sum of the number of load cycles necessary to develop the crack and the number of load cycles necessary to induce the crack propagation, from an initial size to a critical size, as shown in Figure 4.

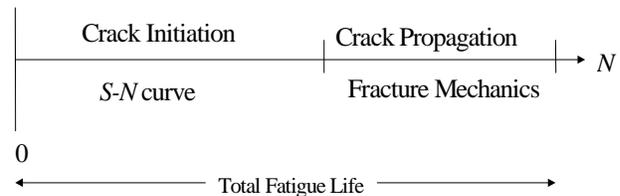


Figure 4. Comparison between the Characteristic  $S-N$  Curve and Fracture Mechanics Approach (Assakkaf and Ayyub 1999)

Two approaches are usually adopted in the evaluation of fatigue failure. The first one uses the  $S-N$  curve associated with Miner's rule as a base, considering the probabilistic distribution related to the  $S-N$  curve, Miner's damage parameter and the loading to estimate the fatigue reliability. The second approach focuses on the fracture behavior, where the fatigue failure of a mechanical or structural element under dynamic loading can be considered as the dominant time to a crack length grows up to a critical magnitude, which may be decided by serviceability requirement or fracture criterion.

Both approaches are discussed in the sequence of this section, being presented the fatigue reliability models

based on these approaches which were developed for ship structure design and analysis.

### 2.3.2.1. The $S-N$ Approach

Traditionally, the design of marine structures takes into account the fatigue analysis based on  $S-N$  curves and some models proposed to analyze the fatigue reliability of ships structures are based on  $S-N$  curves and Miner's rule.

According to Miner's rule, the total fatigue life under a variety of stress ranges is the weighted sum of the individual lives at constant stress range  $S$  as given by the  $S-N$  curves, with each being weighted according to a fractional exposure to that level of stress range (Fuchs and Stephens 1980). The mathematical expression of Miner's rule is

$$D = \sum_{i=1}^{n_s} \frac{n_i}{N_i} \quad (33)$$

where  $n_i$  = number of stress cycles in block  $i$ ,  $N_i$  = number of cycles to failure at constant stress range  $S_i$ , and  $n_s$  = number of stress blocks.

The fatigue behavior of different types of structural details is generally evaluated using constant-cycle fatigue tests, and the results are presented in terms of nominal applied stresses and the number of cycles that produce failure. The resulting  $S-N$  curves are expressed by the following relation

$$NS^b = A \quad (34)$$

where  $A$  = constant of  $S-N$  curve,  $N$  = number of cycles to fatigue failure,  $S$  = constant amplitude stress range at  $N$ , and  $b$  = slope of the  $S-N$  curve.

The books published by Fuchs and Stephens (1980), Suresh (1991) and Maddox (1991) present good reviews of the metal fatigue process, which can be used as the basis for the fatigue analysis of any metallic structure. According to these authors, the fatigue behavior of a structural detail is a function of a variety of factors, including: (1) the general configuration and local geometry of the member, (2) the material from which the members are made, and (3) the loading conditions to which the detail is subjected to.

The ship structure presents another important feature that influences the fatigue process which is the use of welding process for the assembly of the structural parts. According to many authors, such as, Morgan (1986); Mansour et al. (1995), Kihl and Sarkami (1996); Petenov and Thayambali (1998); Moan and Berge (1997), and Xu (1997), the weld can induce stress concentrations due to the weld geometry that influence crack growth behavior.

The choice of the appropriate stress representation is an important factor in reliability-based design and analysis for fatigue. Two different approaches can be used for fatigue design and analysis based on  $S-N$  curve: (1) the nominal stress approach, and (2) the hot spot stress approach.

The nominal stress approach is the simplest one between these two approaches. In this approach, the stress is represented by an average loading of the whole structural detail under study. The nominal stress is the maximum stress due to sectional forces or moments or the combination of the two at the location of possible cracking site in the detail. In this approach, neither the weld toe nor the properties of the weld material constitutive relations are taken into consideration. The  $S-N$  curve resulting from this analysis is unique to the structural detail for which it is established. Most design codes nowadays divide various structural details into different classes and provide standard  $S-N$  curve for each class, such as the American Welding Society (AWS 1990) and the British Standards, cited by Mansour et al (1995). For the fatigue analysis, a given structural detail must be classified according to a specific curve provided by the standards.

The hot spot stress is defined as the fatigue stress at the toe of the weld, where the stress concentration is the highest and where fatigue cracking is likely to initiate (Mansour et al 1995). The hot spot stress takes into account the local increase in stress due to the complex structural geometry of the welded joint. The advantage of the hot spot stress method is that only one universal  $S-N$  curve is required to define fatigue strength for all welds, if such curve exists. The disadvantage of this approach is the requirement for more elaborate stress analysis methods, such as the finite element method, to determine the hot pot stress.

Therefore, for welded structures, the weld geometry which produces stress concentrations must be considered as an additional local structural geometry effect which may be incorporated in nominal stress  $S-N$  curves or considered explicitly in hot spot  $S-N$  curves. Typically, for ship structures, the usual approach for fatigue analysis is based on the nominal stress.

In the fatigue analysis of ship structural details, the uncertainties associated with the following analysis variables may be considered: (1) Miner's fatigue damage ratio, (2) the fatigue life prediction related to  $S-N$  curve model, (3) the applied stress range, and the (4) theoretical stress analysis procedure (Wirsching 1984).

Fatigue reliability of ship structures, based on  $S-N$  approach, can be assessed using models proposed by

Wirsching (1984), Munse (1983), Mansour et al (1995), Ang et al (1999) and Assakkaf and Ayyub (1999).

The Ship Structure Committee funded the model developed by Munse et al (1983). The work is interesting not only for the recommended reliability-based fatigue design approach but also for the large amount of test data it contains for typical ship structural details

The design approach is based on calculating a “design” stress range  $S_{rd}$  for fatigue. This stress range is the maximum peak-to-trough stress range expected at the structural detail in analysis once under the most severe sea state during the entire life of the structure. The design stress  $S_{rd}$  must be less than or equal to the nominal permissible stress permitted once during the life of the structure by the basic design rules.

According to the Munse approach, the design stress range  $S_{rd}$  is found using the following equation

$$S_{rd} = S_N R_f \xi \quad (35)$$

where  $S_N$  = the mean value of the constant-amplitude stress range at the design life  $N_d$ ,  $R_f$  = a reliability factor, and  $\xi$  = a random load factor.

The mean value of the constant-amplitude loading stress range is found by entering the  $S-N$  curve of the structural detail of interest at the number of cycles  $N_d$  expected in the design life. The probabilistic nature of the design method is introduced by the factors  $R_f$  and  $\xi$  presented in Eq. 2-8.

The reliability factor  $R_f$  is meant to account for uncertainties in the fatigue data, workmanship, fabrication, use of the equivalent stress range concept, errors in the predicted load history and errors in the associated stress analysis (White et al 1995). The random load factor accounts for the probability of occurrence of the design stress range.

The lognormal format (Wirsching 1984) has been proposed as a convenient closed-form method for performing reliability assessments of existing designs or for developing probability-based design criteria. The fatigue damage, based on the Miner’s rule, is written as

$$D = \frac{n}{A} E(S^b) \quad (36)$$

where  $n$  = number of load cycles, and  $D$  = the damage accumulated in  $n$  load cycles. Instead of using the mean value of the random variable  $S^b$  ( $E(S^b)$ ), Wirsching (1984) proposed the use of an equivalent constant amplitude stress range,  $S_e$ , defined as

$$S_e = \sqrt[b]{E(S^b)} \quad (37)$$

and the damage can be written as

$$D = \frac{n}{A} S_e^b \quad (38)$$

In order to take into account the stress modeling error in the fatigue analysis, Wirsching proposed the use of a bias factor  $B$  to correct the equivalent stress, and the fatigue damage is expressed according to the following equation

$$D = \frac{n}{A} B^b S_e^b \quad (39)$$

The Miner’s rule states that fatigue failure occurs when the damage exceeds one, i.e.,  $D \geq 1$ . From fatigue experimental results, it was suggested that it is more appropriate to describe the fatigue failure as  $D \geq \Delta$ , in which  $\Delta$  is a random variable denoting damage at failure. The random variable is used to quantify the modeling error associated with Miner’s rule.

Because of the scatter in  $S-N$  data, Wirsching (1984) suggested that uncertainty in fatigue strength can be accounted for by considering  $A$  as a random variable with  $b$  taking as a constant.

The stress correction factor,  $B$ , is also considered a random variable by Wirsching (1984), and the uncertainty in  $B$  is assumed to stem from five sources: (1) fabrication and assembly operations, (2) sea state description, (3) wave load prediction, (4) nominal member loads, and (5) estimation of hot spot stress concentration factor.

Wirsching (1984) also recommended that a lognormal distribution should be used for the random variables  $A$ ,  $B$ , and  $\Delta$ , as a basis for fatigue reliability analysis and code development.

Based on these considerations, at failure, the damage ( $D$ ) is equal to  $\Delta$ , and the total number of load cycles ( $N$ ) until failure is

$$N = \frac{A\Delta}{B^b S_e^b} \quad (40)$$

The total number of load cycles until failure ( $N$ ) is a random variable. If  $A$ ,  $B$ ,  $\Delta$  are lognormally distributed random variables, then  $N$  will have an exact lognormal distribution. The mathematical properties of the lognormal distribution allow a closed-form solution for the probability of a fatigue failure prior to the end of the intended service life  $N_s$ . The fatigue failure probability ( $p_f$ ) is given by

$$p_f = P(N \leq N_s) \quad (41)$$

The probability of failure can be also defined in terms of the reliability index  $\beta$  and the standard normal distribution ( $\Phi(\cdot)$ ) as

$$p_f = \Phi(-\beta) \quad (42)$$

Based on Eq. 40, the reliability index can be expressed as

$$\beta = \frac{\ln\left(\frac{\tilde{N}}{N_S}\right)}{\sigma_{\ln N}} \quad (43)$$

where

$$\tilde{N} = \frac{\tilde{A}\tilde{\Delta}}{\tilde{B}^b S_e^b} \quad (44)$$

and

$$\sigma_{\ln N} = \sqrt{\ln\left[\left(1+C_{\Delta}^2\right)\left(1+C_A^2\right)\left(1+C_B^2\right)^{b^2}\right]} \quad (45)$$

in which  $C_{\Delta}$ ,  $C_A$ , and  $C_B$  denote the coefficient of variation (COV) for the variables  $\Delta$ ,  $A$ , and  $B$ , respectively. The tildes over the variables denote median values.

In evaluating fatigue reliability according to Wirsching's model, the equivalent stress range  $S_e$  needs to be estimated. The estimation can be based on the Weibull model, as presented in the previous section, or based on more sophisticated analysis, such as the spectral method proposed by Sikora et al (1983). Wirsching (1984) presented some expressions commonly used for the definition of the equivalent stress range.

Based on the lognormal format, Mansour et al (1995) developed a prototype fatigue design criteria structural code for cruisers and tankers.

The model developed by Assakkaf and Ayyub (1999) is based on the classical theory of structural reliability, being defined a functional relationship between the relevant load and resistance parameters as for fatigue analysis.

The reliability-based analysis for fatigue requires the definition of a performance function related to the Miner's rule. This function is expressed as (Assakkaf and Ayyub 1999)

$$g_{SN}(\underline{X}) = \frac{\Delta A}{k_S^b \left(\sum_{i=1}^n S_i^b\right)} - N_t \quad (46)$$

where  $g_{SN}$  = performance function,  $\underline{X}$  = vector of basic random variables, and the random variables are  $\Delta$  = fatigue damage ratio,  $A$  = constant of  $S$ - $N$  curve,  $k_S$  =

fatigue stress uncertainty factor, and  $\sum_{i=1}^n S_i^b$  = cumulative

dynamic stress range acting on the structure during a given number of load cycles  $n$ . The  $N_t$  is a deterministic value that corresponds to the number of load cycles expected during the structure operational life.

The limit surface or performance function of the limit state of interest can be defined as  $g_{SN} = 0$ . This is the boundary between the safe and unsafe regions in the design parameter space, and it also represents a state beyond which a structure can no longer fulfill the function for which it was designed. Therefore, the fatigue failure occurs when  $g_{SN}(\underline{X}) \leq 0$ .

As the joint probability function of the basic random variables is unknown, Assakkaf and Ayyub (1999) proposed the use of the advanced second moment (ASM) to solve the performance function presented in Eq. 46, defining the probability of fatigue failure, expressed in terms of reliability index.

The reliability index resulting from reliability assessment methods such as the advanced second moment (ASM) is compared with target reliability, and the structure is considered safe when the former is bigger.

The authors proposed the use of Eq. 46 to evaluate the reliability associated with a given ship structural detail. According to these authors, the spectral analysis is used to develop lifetime fatigue loads spectra by considering the operational and the characteristics of ship in sea, as proposed by Sikora et al (1983). With the proper identification of the structural detail resistance modulus, these loads spectra can be converted to stress range spectra. The stress range spectra are used to compute the equivalent stress range  $S_e$  as given by

$$S_e = \sqrt[2]{\sum_{i=1}^{n_b} f_i S_i^b} \quad (47)$$

where  $n_b$  = number of stress blocks in the stress histogram,  $f_i$  = fraction of cycles in the  $i^{\text{th}}$  block, and  $S_i$  = stress in the  $i^{\text{th}}$  block.

The performance function is written as

$$g_{SN}(\underline{X}) = \frac{\Delta A}{k_S^b S_e^b} - N_t \quad (48)$$

The reliability-based design and analysis for fatigue requires the probabilistic characteristics of the random variables in the performance function. It also requires specifying target reliability index  $\beta_0$  to be compared with a computed  $\beta$  resulting from reliability assessment methods such as ASM.

The reliability function for fatigue analysis suggested by Ang et al (1999) is based on the hypothesis that as fatigue is a process of cumulative damage, the conditional probability that failure will occur in the next loading cycle should be monotonically increasing with the life spent, i.e., the hazard function should be monotonically increasing. Ang et al (1999) used the Weibull probability distribution to express the fatigue reliability. The corresponding reliability function ( $L(t_L|N=n)$ ) for a given time interval ( $0, t_L$ ) is expressed by

$$L(t_L|N=n) = \exp\left[-\left(\frac{n}{E(N)}\Gamma\left(1+\frac{1}{k}\right)\right)^k\right] \quad (49)$$

where  $n$  = the number of load cycles in the time interval ( $0, t_L$ ),  $E(N)$  = the mean fatigue life, and  $k$  = shape parameter for the Weibull probability distribution.

The mean fatigue life is defined considering that the fatigue failure occurs when the mean damage, defined by Miner's rule, is equal to 1.0, and is expressed as

$$E(N) = \frac{E(A)}{E(S^b)} \quad (50)$$

and

$$E(S^b) = \int_0^{\infty} S^b f_S(s) ds \quad (51)$$

where  $E(A)$  = mean value of the  $S$ - $N$  curve parameter,  $E(S^b)$  = the mean value of the random variable  $S^b$ ,  $E(N)$  in the mean fatigue life value, and  $f_S(s)$  = the probability density function of the stress range acting on the structure.

The shape parameter  $k$  can be obtained as follows

$$k = COV(N)^{-1.08} \quad (52)$$

where  $COV(N)$  is the coefficient of variation of the fatigue life.

According to Ang et al (1999) the coefficient of variation is used to model the uncertainty of the random variables that influence the ship structure fatigue life. The main sources of uncertainty are: (1) uncertainty in strength, and (2) uncertainty in loading.

In establishing this coefficient, Ang et al (1999) take into account the following sources of uncertainty: (1) the scatter in fatigue data, related to the  $S$ - $N$  curve, (2) uncertainty due to the utilization of Miner's rule and errors in the fatigue model, (3) uncertainty in the stress range distribution and error in stress analysis and (4) uncertainty related to the effects of the quality of fabrication and workmanship.

According to Ang et al (1999), the total uncertainty ( $COV(N)$ ) in terms of fatigue life is given by

$$COV(N) = \sqrt{C_{mr}^2 + C_c^2 + C_{SN}^2 + b^2 C_B^2} \quad (53)$$

where  $C_{mr}$  = uncertainty ( $COV$ ) due to errors in fatigue model and use of Miner's rule,  $C_c$  = uncertainty ( $COV$ ) due to fabrication and workmanship,  $C_{SN}$  = uncertainty in  $S$ - $N$  curve,  $C_B$  = uncertainty in the stress range including error in stress analysis, and  $b$  = slope of mean  $S$ - $N$  regression line.

The method proposed by Ang et al (1999) considers the uncertainties in the variables that affect the fatigue failure of a structural detail through the use of their coefficient of variation. The reliability of the structural detail is expressed as a closed function, based on the hypothesis that the life of the structure is modeled by a Weibull probability function. The shape parameter of the distribution is calculated based on the coefficient of variation of the random variables. The great advantage of this method is the use of a closed equation to express the structural detail reliability, although the use of the Weibull distribution to model the dispersion in the fatigue life could be questionable.

### 2.3.2.2. Fracture Mechanics Approach

The fracture mechanics approach is based on crack growth data. For the structural detail under analysis the crack initiation phase is assumed to be negligible and the life can be predicted using the fracture mechanics method. The fracture mechanics approach is more detailed and it involves examining crack growth and determining the number of load cycles that are needed for small initial defects to grow into cracks large enough to cause fracture. The growth rate is proportional to the stress range. It is expressed in terms of a stress intensity factor  $K$ , which accounts for the magnitude of the stress, current crack size and geometry, and structure geometry. According to Fuchs and Stephens (1980) the basic equation that governs crack growth, named Paris Law, is given by:

$$\frac{da}{dN} = C \Delta K^m \quad (54)$$

where  $a$  = crack size,  $N$  = number of fatigue cycles,  $\Delta K$  = range of stress intensity factor, and  $C$  and  $m$  are crack propagation parameters that come from fracture mechanics. The range of the stress intensity factor is given by Fuchs and Stephens (1980) as:

$$\Delta K = S f(a) \sqrt{\pi a} \quad (55)$$

in which  $f(a)$  is a function of crack geometry and structure geometry and  $S$  is the stress range induced by the cyclic

loading. When the crack size  $a$  reaches some critical crack size  $a_{cr}$ , failure is assumed to have occurred. Although most laboratory testing is typically performed with constant amplitude stress ranges, Eq. 54 is always applied to variable stress range models that ignore sequence effects (Rolfe and Barsom 1987). Rearranging the variables in Eq. 54, the number of cycles for the crack grow from the initial size ( $a_i$ ) to a given crack size ( $a$ ) can be computed from:

$$N = \frac{I}{C(S)^m} \int_{a_i}^a \frac{da}{f(a)^m (\sqrt{\pi a})^m} \quad (56)$$

Eqs. 54 and 56 involve a variety of sources of uncertainty (Harris 1995). The crack propagation parameter  $C$  in both equations is treated as a random variable (Madsen et al 1991).

Considering the expression presented in Eq. 56, the fatigue damage related to one cycle of external loading can be calculated according to the following expression:

$$\Psi(a) = C(S)^m \quad (57)$$

where the function  $\Psi(a)$  represents the increase in crack size due to the loading cycle and is defined as:

$$\Psi(a) = \int_{a_i}^a \frac{da}{f(a)^m (\sqrt{\pi a})^m} \quad (58)$$

The failure criterion is taken as the exceedance of a maximum crack size admissible for the structure during  $N$  loading cycles, and expressed as:

$$a_f - a_n \leq 0 \quad (59)$$

where  $a_f$  = the maximum crack size admissible for the structure, and  $a_n$  = the crack size after  $N$  cycles of loading.

As the function  $\Psi(a)$  is monotonically increasing, the failure criterion can be written as:

$$\Psi(a_f) - \Psi(a_n) \leq 0 \quad (60)$$

Using the definitions presented in Eqs. 58 and 57 respectively for  $\Psi(a_f)$  and  $\Psi(a_n)$ , the failure criterion is written as a limit state function, that can be used for reliability analysis.

The limit state function for fatigue fracture analysis becomes:

$$Z_{\Delta K} = g_{\Delta K}(x) = \int_{a_i}^{a_f} \frac{da}{f(a)^m (\sqrt{\pi a})^m} - C \sum_{i=1}^n (S_i)^m \quad (61)$$

and the failure will occur when  $g_{\Delta K} \leq 0$ . The random variables in this equation are:  $a_i$  = initial crack size presented in the structure,  $a_f$  = the maximum crack size admissible for the structure,  $C$  = the Paris Law coefficient

and  $\sum_{i=1}^n (S_i)^m$  = cumulative dynamic stress range acting on the structure during a given time period.

Typically, the cumulative dynamic stress range is modeled as the product of the expected number of stress range during the time period studied and the equivalent mean stress range, defined based on the stress range density probability function.

Specifically for the case of structures subjected to a great variety of external loading, due to changes in environmental conditions, such as marine, offshore and aeronautical structures, the limit state function is modified to account for the influence of each loading condition, according to the method proposed by Hughes (1988), named "Lifetime Weighted Sea Method", and is expressed as:

$$Z_{\Delta K} = g_{\Delta K}(x) = \int_{a_i}^{a_f} \frac{da}{f(a)^m (\sqrt{\pi a})^m} - C \sum_{j=1}^k p_j \left[ \sum_{i=1}^n (S_i)^m \right]_j \quad (62)$$

where  $\left[ \sum_{i=1}^n (S_i)^m \right]_j$  = cumulative stress range acting on the

structure due to loading condition  $j$ , and  $p_j$  = probability of occurrence of this loading condition. So in this method, the long term cumulative stress range associated to the structure is composed by the combination of short term cumulative stress range related to each loading condition

acting on the structure. The term  $\sum_{j=1}^k p_j \left[ \sum_{i=1}^n (S_i)^m \right]_j$  can

be converted to an equivalent mean stress range  $\bar{S}_e$  according to the following equation

$$\bar{S}_e = \sqrt[m]{\sum_{j=1}^k p_j \left[ \sum_{i=1}^n (S_i)^m \right]_j} \quad (63)$$

The use of the first-order second moment reliability methods associated with the former limit state function allows the definition of the reliability index for a given structural detail. As used for the  $S-N$  approach, this index can be compared to a target reliability index, and the structure is considered safe if the former is bigger.

Usually, the crack growth curves for fatigue fracture analysis are developed for test specimens with a standard

geometry (ASTM 1999). Even for large structures, such as ship structures or offshore structures, the certification societies present some universal crack growth curves, that can be used for fatigue analysis (Kaminski and Krekel 1995, and Fricke and Muller-Schmerl 1998). In order to use those universal crack growth curves, the stress range used for reliability analysis must take into account the geometry of the structure, modeling the stress concentration due to the geometry of the weld line and structural detail. This approach corresponds to the hot spot stress range.

Once that the fracture mechanics approach is based on the stress range close to the crack, the residual stresses induced by the hull fabrication techniques influences the crack growth behavior, since they affect the stress field near the crack tip.

According to Souza and Ayyub (2000), the approach most frequently used to account for the effects of residual stresses on crack growth involves superposition of the respective intensity factors for the residual stresses and for the external loading induced stress.

As shown by the classical literature related to fatigue crack growth propagation analysis, such as the books written by Fuchs and Stephens (1980) and Rolfe and Barsom (1987), the fatigue crack growth depends on the stress intensity ratio ( $R$ ), which is defined as the relation between the minimum and maximum values expected for each stress cycle induced by the random loading. This factor takes into account the presence of a static constant load acting on the structure inducing a mean static stress that modifies the maximum and minimum stress intensity factor in one load cycle, although it does not influence the stress intensity range used in Paris Law, which is defined as the difference between the maximum and minimum value of the stress intensity range in a given load cycle. This residual stresses intensity factor is added to the external loading induced stress intensity factor, in order to define the stress intensity ratio ( $R$ ).

The limit state function for the fatigue crack growth analysis, presented in Eq. 62, must be correct to include the effect of the stress ratio on the crack growth rate. This correction is made applying the proposal done by Barsom, cited by Rolfe and Barsom (1987), where the Paris law is corrected by a factor  $(1-R)$ , as shown below:

$$Z_{\Delta K} = g_{\Delta K}(\underline{x}) = \int_{a_i}^{a_f} \frac{da}{(f(a))^m (\sqrt{\pi a})^m} - C \sum_{i=1}^k \frac{p_j}{\sqrt{1-R_j}} \left[ \sum_{i=1}^n (S_i)^m \right]_j \quad (64)$$

where  $R_j$ = stress intensity ratio related to the load condition  $j$ .

One advantage of fracture mechanics approach over  $S-N$  procedure, in addition to provide a more detailed description of the fatigue crack growth phenomenon, is the possibility of updating the fatigue failure probability based on the results of the non-destructive structural inspection executed through the structure operational life, as presented by Madsen et al (1991).

The great disadvantage of the fatigue fracture mechanics approach is the lack of data related to the probabilistic characteristics of the model random variables when compared to the database related to  $S-N$  model random variables.

### 3. APPLICATION OF PROBABILISTIC FRACTURE MECHANICS FOR LIFE PREDICTION OF SHIP STRUCTURES

This section presents an example of the application of the probabilistic fatigue fracture approach for the evaluation of structural reliability and life expectancy analysis of ship structures, including the effect of the residual stresses. The example is used to demonstrate a reliability analysis by evaluating the reliability index for a given structural detail, based on the statistical representation of the crack growth parameter,  $C$ , corresponding to the detail of interest. The performance function as defined in Eq. 64 is used in this example, where  $C$ ,  $a_i$ ,  $a_f$ , and  $\Delta S_e$  are random variables. The number of load cycles ( $N$ ) is considered to be a deterministic variable, and the reliability index is defined for a set of values of  $N$ , in order to evaluate its variation with cycles or time.

As the crack growth parameters are dependent on the material properties, the example considers a ship structure built with ABS-C steel which is a ferritic-pearlitic material usually used in merchant ships. Actually the ABS specifications for structural steels do not list the ABS-C steel, which was substituted by the material ABS-DS (Taggart 1980). The material mechanical properties are presented in Table 2.

The results of the fatigue fracture analysis are also compared to the results of fatigue analysis based on *S-N* approach, classically used for ship hull fatigue evaluation. The comparison is made using the reliability method proposed by Assakkaf and Ayyub (1999), using Eq. 48, once it is based on the analysis of a limit state function using the ASM method, similar to the procedure based on fracture mechanics approach.

Table 2. ABS-C Steel Mechanical Properties (Ramsamooj and Shugar 1998)

Mechanical Property	Magnitude
Elasticity Modulus (GPa)	207
Yield Strength (MPa)	269
Fracture Toughness (MPam <sup>0.5</sup> )	102
Charpy Energy (J)	78.6

### 3.1. Definition of Crack Growth Law

The procedure proposed by Gurney is used to define the mean  $\frac{da}{dN} - \Delta K$  curve equivalent to one *S-N* curve recommended for ship structural design, according to the methodology proposed by Souza and Ayyub (2000). The joint detail C used by Mansour et al (1995), corresponding to a welded plate with a full penetration weld, subject to a uniform load perpendicular or parallel to the weld line, is used in this study.

In order to define the equivalent  $\frac{da}{dN} - \Delta K$  curve, the initial crack is assumed to be an elliptical surface crack having a mean depth of 0.5 mm, as suggested by Kaminski and Krekel (1995) to study the crack growth process in the structure of a Floating Production Storage Off-loading (FPSO).

The upper limit for the crack dimension can be considered in two ways: (i) the failure occurs when the crack grows from a surface elliptical crack and penetrate the plate thickness, or (ii) the failure occurs when the crack grows until a given dimension that would cause the brittle fracture of the structure. The first criterion is based on serviceability analysis and considers that the structure is not suitable for service under the presence of a through thickness crack, due to the possibility of leakage. The second criterion is based on the linear fracture mechanics concepts, which state that the brittle fracture occurs in the presence of a given crack dimension that induces a stress intensity factor greater than the material critical stress intensity factor.

The materials used in hull structure of typical merchant ships or warships, such as the ABS-C used in this study, present high ductility, even in low temperature, in order to prevent the brittle fracture occurrence, Masubuchi (1980). So the ship structures are usually designed to withstand the presence of a small length through thickness crack in the hull plate, and the brittle fracture occurs when this crack continues to grow as a through thickness crack until the total crack length induces a stress intensity factor greater than the material resistance.

Although the presence of a through thickness crack, smaller than the critical crack length, cannot cause the brittle fracture of the hull structure, its presence allows the leakage of sea water inside the ship hull, which can cause the decrease in the ship operational performance or even contaminate a given liquid or dry cargo inside the hull.

In this study the fatigue failure is attained when the semi-elliptical crack becomes a through thickness crack, and therefore the fatigue criterion is dependent on the plate thickness. For the development of this example, the plate thickness is supposed equal to 6.35 mm.

Table 3 presents the parameters of the equivalent Paris law for the *S-N* curve used in this research.

### 3.2. Residual Stresses

The influence of residual stresses on the fatigue crack growth is assessed through the use of Equation 64. As the Paris law parameters are defined based on typical *S-N* curves defined for welded joints, the effects of the short-range residual stresses are already accounted in the crack growth law. However, the effect of the long-range residual stresses must be evaluated, since they cannot be simulated with the usual test specimens used in fatigue tests, whose dimensions are not large enough to allow the development of long-range residual stresses. According to Masubuchi (1980), the long-range residual stresses have a magnitude close to 20% of the material yield strength, at least for ferritic-pearlitic steels, as those used for hull fabrication.

To evaluate the effect of the long-range residual stresses on fatigue crack growth, the magnitude of the stress is considered constant through the plate thickness and the residual stress intensity factor is calculated using the geometry factor defined for the crack growth analysis, according to Gurney law. A parametric study is executed in this example to examine the effect of the residual stresses, considering two magnitudes: 10% and 20% of the material yield strength.

Table 3. Statistical Parameters of the Equivalent Paris Crack Growth Law Joint Detail C

Parameters of the Fatigue Curves		<i>S-N</i> Curve Joint Detail C
<i>S-N</i> Curve	<i>b</i>	3.5
	Mean of $A^{(1)}$	$4.23 \times 10^{13}$
	Coefficient of Variation of <i>A</i>	0.5
	Distribution of <i>A</i>	Lognormal
Equivalent $\frac{da}{dN} - \Delta K$ Curve	<i>m</i>	3.5
	Mean of $C^{(2)}$	$6.12 \times 10^{-15}$
	Coefficient of Variation of <i>C</i>	0.5
	Distribution of <i>C</i>	Lognormal
	Parameter $f(a)$	1.09

Notes: (1) Units of *A* according to stress in MPa  
(2) Units of *C* according to stress in N.mm<sup>-2</sup>

### 3.3. Stress Range

In order to evaluate the effect of the stress range, in combination with the residual stresses effects, on the crack growth, three mean stress range magnitude are selected, 125 MPa, 75 MPa, and 60 MPa, corresponding to 50%, 30% and 20% of the material yield strength, respectively.

### 3.4. Numerical Results

The probabilistic characteristics of the random variables that are used in this example are provided in Table 4. Reliability analyses are performed using the *S-N* and fracture mechanics limit states. The first-order second moment reliability method was used for this purpose. Summaries of the results are presented in Tables 5 through 7, and graphically presented in Figures 5 to 7.

The reliability index associated to the use of *S-N* curve is always higher than the reliability index associated to the use of the crack growth curve. Considering that the mean *S-N* curve is equivalent to the mean  $\frac{da}{dN} - \Delta K$  curve, according to the relation proposed by Gurney, the differences in the reliability indices are related to the random variables used in each of these approaches, since their respective statistical models are different. Furthermore, the limit state functions differ for these approaches, and this difference can influence on the results of the reliability analyses.

Figures 5 to 7 show the differences between the reliability indices based on the limit states. However, both approaches predicted about the same interval of number of cycles in which the reliability index is equal to

0.0, for all the cases analyzed. This result can be attributed to the initial hypothesis that the underlying mean curves for the two approaches are equivalent.

For fracture mechanics approach, the reliability index grows very rapidly for small numbers of load cycles indicating that until a specific number of load cycles the main concern of the structural design should be the occurrence of brittle fracture instead of fatigue failure. This critical number of cycles is dependent on the magnitude of the stress range acting on the structure, considering the same initial crack size. The *S-N* approach also presents an increase of the reliability index with the decrease of the number of load cycles, but the growth rate is smaller than the growth rate presented by the fracture mechanics approach.

The probabilistic analysis of brittle fracture failure can also be executed based on reliability methods. The limit state function that governs the brittle fracture can be expressed as

$$g_{K_I} = K_{IC} - K \quad (65)$$

where  $K_{IC}$  = the material fracture toughness and  $K$  = the stress intensity induced by external loading. This stress intensity factor is defined by an expression similar to Equation 4, but considering the maximum stress acting on the structure, instead of the stress range. Equation 65 can produce a reliability index associated with the brittle fracture process, considering as random variables the crack dimensions, the stress acting on the structure and the material fracture toughness. This analysis is not developed in this paper but can be explored in future studies.

Considering that the failure criteria for the structure is the presence of a through thickness crack, the life corresponding to a target reliability index can be defined for both fatigue analysis approaches. According to Mansour et al (1995), a failure criterion in this case that corresponds to a non-serious failure can be used, since it does not put at risk the crew life or the structural integrity of the ship, although the structure must be repaired in order to stop any leaks. The corresponding target reliability is 2.5. Table 8 presents the number of cycles for which the reliability index is equal to 2.5, for the joint detail C. The presence of the residual-stress field clearly reduces the life of the structure. Furthermore, the fatigue life predicted based on the fracture mechanics approach is 40% smaller than the life predicted by the *S-N* approach. This observation indicates that more research effort must be spent to define a target reliability index for ship structures based on fracture mechanics.

Table 4. Random Variables for Joint Detail C

Random Variable		Mean	Coefficient of Variation	Distribution Type
Stress Range	$\Delta S_e$ (MPa)	125.	0.10	Lognormal
		75.	0.10	
		60.	0.10	
S-N Model	$b$	3.5	-	Constant
	$A$	$4.23 \times 10^{13}$	0.40	Lognormal
	$k_s$	1.0	0.10	Normal
	$\Delta$	1.0	0.48	Lognormal
Fatigue Fracture Model	$m$	3.5	-	Constant
	$C$	$6.12 \times 10^{-15}$	0.50	Lognormal
	$a_i$ (mm)	0.5	1.00	Exponential
	$a_f$ (mm)	6.35	0.10	Normal

Table 5. Fatigue Reliability Results for Joint Detail C Using a Mean Stress Range 125 MPa

Fatigue Analysis Model		RELIABILITY INDEX						
		Number of Load Cycles						
		$5 \times 10^4$	$10^5$	$5 \times 10^5$	$10^6$	$5 \times 10^6$	$10^7$	$10^8$
S-N Model		4.24	3.38	1.39	0.54	-1.41	-2.24	-4.92
Fatigue Fracture Model	$\sigma_{res} = 0.00$ MPa	3.41	2.71	0.86	0.14	-1.14	-1.59	-3.54
	$\sigma_{res} = 26.9$ MPa	3.32	2.61	0.76	0.06	-1.21	-1.65	-3.63
	$\sigma_{res} = 53.8$ MPa	3.25	2.51	0.67	-0.02	-1.26	-1.70	-3.48

Table 6. Fatigue Reliability Results for Joint Detail C Using a Mean Stress Range 75 MPa

Fatigue Analysis Model		RELIABILITY INDEX						
		Number of Load Cycles						
		$5 \times 10^4$	$10^5$	$5 \times 10^5$	$10^6$	$5 \times 10^6$	$10^7$	$10^8$
S-N Model		6.48	5.61	3.60	2.74	0.76	-0.90	-2.85
Fatigue Fracture Model	$\sigma_{res} = 0.00$ MPa	29.06	3.83	2.89	2.10	0.31	-0.31	-1.89
	$\sigma_{res} = 26.9$ MPa	23.68	3.82	2.72	1.92	0.17	-0.43	-1.98
	$\sigma_{res} = 53.8$ MPa	18.98	3.82	2.59	1.79	0.06	-0.53	-2.04

Table 7. Fatigue Reliability Results for Joint Detail C Using a Mean Stress Range 60 MPa.

Fatigue Analysis Model		RELIABILITY INDEX						
		Number of Load Cycles						
		$5 \times 10^4$	$10^5$	$5 \times 10^5$	$10^6$	$5 \times 10^6$	$10^7$	$10^8$
S-N Model		7.46	6.59	4.57	3.71	1.72	0.87	-1.92
Fatigue Fracture Model	$\sigma_{res} = 0.00$ MPa	45.14	31.41	3.59	2.97	1.13	0.40	-1.43
	$\sigma_{res} = 26.9$ MPa	42.60	25.38	3.45	2.77	0.92	0.22	-1.54
	$\sigma_{res} = 53.8$ MPa	40.31	20.38	3.33	2.62	0.77	0.10	-1.62

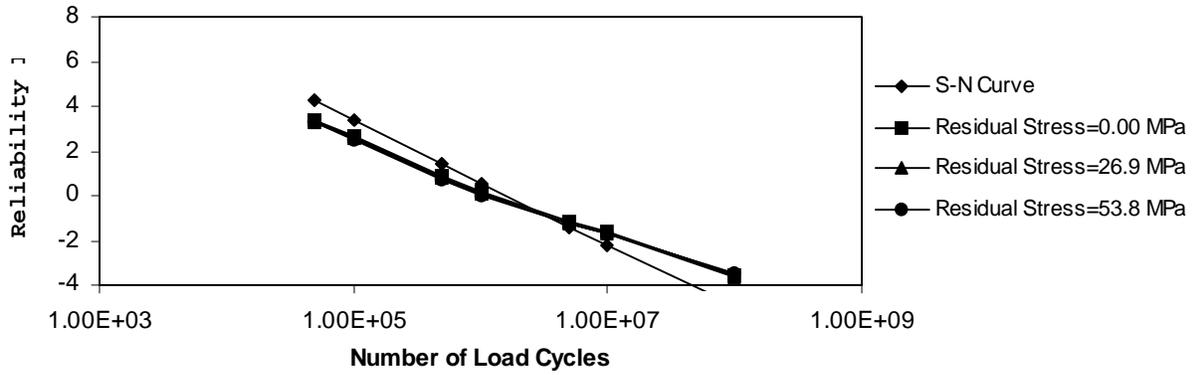


Figure 5. Reliability Index as a Function of Load Cycles for Joint Detail C Using a Mean Stress Range 125 MPa

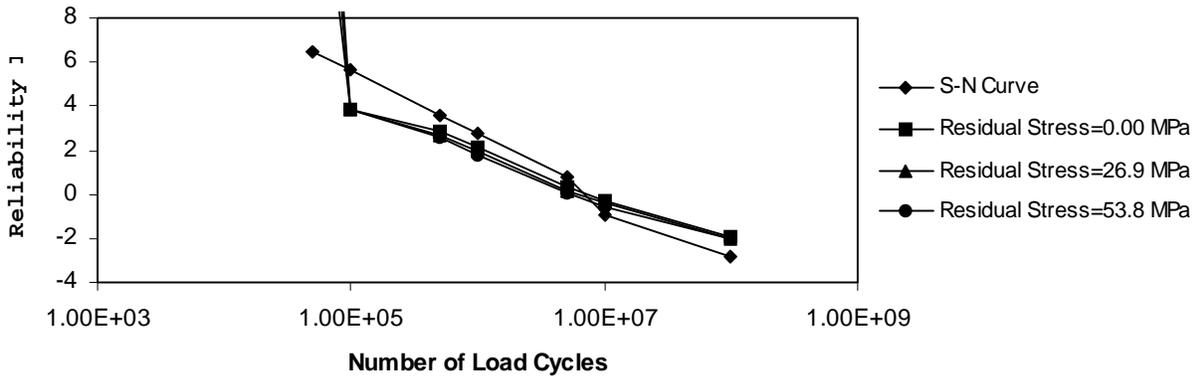


Figure 6. Reliability Index as a Function of Load Cycles for Joint Detail C Using a Mean Stress Range 75 MPa

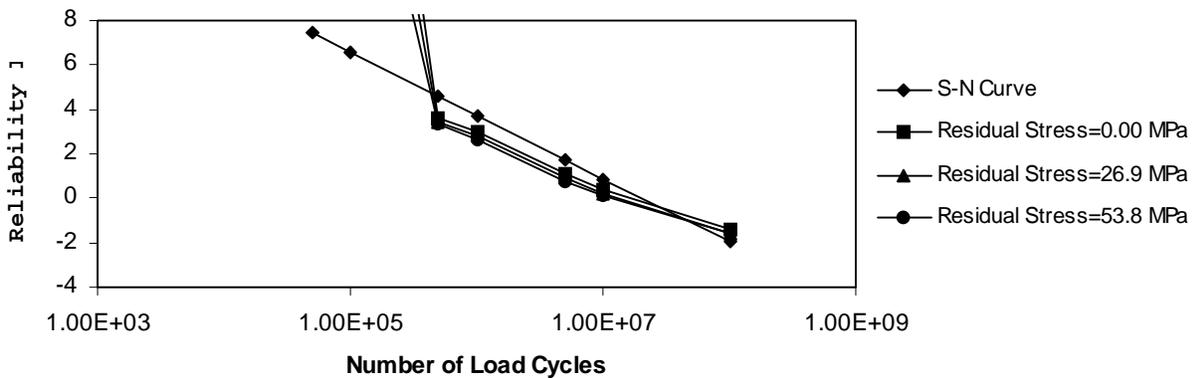


Figure 7. Reliability Index as a Function of Load Cycles for Joint Detail C Using a Mean Stress Range 60 MPa

Table 8. Fatigue Life of Joint Detail C for a Target Reliability Index 2.5 without Residual Stress

Mean Stress Range (MPa)	Structural Life <i>S-N</i> Curve	Structural Life Fracture Mechanics
125	$2.00 \times 10^5$	$1.20 \times 10^5$
75	$1.20 \times 10^6$	$7.00 \times 10^5$
60	$2.60 \times 10^6$	$1.50 \times 10^6$

The influence of the long-term residual stresses on the fatigue crack growth is significant, once the variable *C* influence the minimization procedure used by the ASM algorithm. This procedure, in order to determine the shortest distance to the limit state function, evaluates directional cosines for all random variables. Table 9 presents the influence factors, corresponding to the square of the directional cosines defined in the minimization procedure, considering  $10^5$  load cycles and a mean stress range of 125 MPa for the purpose of illustration. These results show that although the influence factor for  $a_i$  is large for joint detail C, the influence factor for *C* is the second higher among the random variables, and has influence on the minimization procedure. So the reliability index is influenced by the presence of the residual stress field.

Table 9. Influence Factor for Joint Detail C for Mean Stress Range 125 MPa and  $10^5$  Loading Cycles

Random Variable	Influence Factor Joint Detail C
<i>C</i>	0.222
$a_i$	0.653
$a_f$	$3.75 \times 10^{-3}$
$\Delta S_e$	0.122

An increase in the residual stress magnitude cause a decrease in the reliability index. The difference in the reliability index defined for null residual stresses and for maximum residual stresses is about 10%. Therefore, for fatigue fracture analysis, the long-term residual stresses field has an influence on the crack growth rate, and must be considered in reliability analysis.

The effect of the residual stresses over the crack growth rate can be more significant for structures that were repaired. The welding process used during maintenance procedures can induce long-term residual stresses of high magnitude, since the displacement

restraint in a built structure is higher than the restrictions during the initial structural fabrication.

As presented in section 1.2, the effects of corrosion can affect the ship structural resistance. As for fatigue analysis, the corrosion affects the material properties, increasing the crack propagation rate due to the phenomena named Corrosion fatigue, Jones (1992). The association of corrosion with the action of cyclic stress tends to increase the rate of crack propagation. The degradation effects of corrosion in the material properties is usually modeled with the use of a new coefficient *C* for the crack growth curve, which is related to an environment different from air, Rolfe and Barsom (1987). This new curve is assessed based on experiments executed with tests specimens subjected to the environment of interest, for the case of ships, salt water. This coefficient is usually higher than the coefficient defined with tests in air. The exponent of the crack growth curve can be considered as a constant. Finally, the effect of corrosion is also dependent on the area of the structure that is being analyzed. Usually the structural parts close to the draft line present a higher corrosion rate than the values observed for the structure totally submerged or the structure in the air.

In order to study the effect of corrosion over the fatigue reliability index defined with the probabilistic fracture mechanics approach, a parametric study is presented considering two values for the constant *C* of the crack growth curve, both of them higher than the value presented in Table 4. The study is executed considering values 15% and 30% higher than the original value, representing an increase in the crack growth rate.

The effects of corrosion on the reliability index are presented in Figure 8. This chart presents the reliability index associated with joint detail C, considering an external induced stress range of 60 MPa, with null residual stress. The reliability index is reduced as a function of the increase in the crack growth rate. For an increase of 30% in the crack growth rate, the reliability index presents a decrease of almost 10%.

Regarding fatigue analysis, as observed for the residual stress effects, the corrosion effect is detrimental for the structural safety. The only way to avoid the corrosion effect over the crack growth is to keep a cathodic protection system on the ship hull. Painting does not avoid the occurrence of corrosion fatigue, since once the crack becomes a surface crack, the paint film in the area and does not offer protection against the environmental effects.

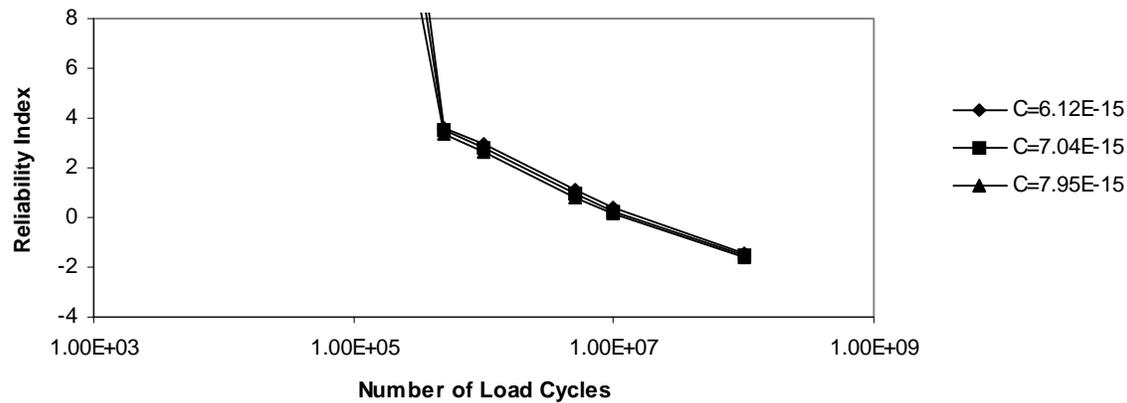


Figure 8. Effects of Corrosion on the Reliability Index for Joint Detail C Using a Mean Stress Range of 60 MPa and Null Residual Stresses

#### 4. CONCLUSIONS

This paper has discussed the development of a reliability-based for life expectancy analysis of ship structures. The methodology considers the time dependency of the loads acting on the ship structure and also the through life degradation of the structure due to environmental effects.

The methodology is developed considering different approaches for first-passage failure modes and for cumulative failure modes. The reliability function for the first-passage failure modes is developed based on the model proposed by Ellingwood and Mori (1993). This function allows the evaluation of the structure reliability in a closed form, without use of numerical methods to calculate the reliability index. This method also allows the consideration of ship structural degradation due to environmental effects.

The reliability analysis for the cumulative failure mode, specifically the fatigue failure, is developed based on the probabilistic linear fracture mechanics model. This model is suitable for the fatigue analysis of ship structures once it assumes the existence of fabrication process induced defects in the structure and considers that the fatigue failure will be dependent on the growth rate of these defects. The fatigue analysis is not based on a closed-form reliability function, such as the one defined for the first-passage failure modes. The reliability index must be evaluated with the use of numerical methods, such as the Advanced Second moment, used in this paper.

Probabilistic analysis of fatigue crack growth in ship hull requires the utilization of reliability methods for

assessing fatigue life by considering the crack propagation process and the uncertainties associated with it. The limit state function that governs fatigue crack growth failure is developed based on Paris law by considering as random variables the coefficient of the  $\frac{da}{dN} - \Delta K$  curve, the initial and maximum admissible crack size, and the stress induced by external loading. The residual stresses induced by welding are modeled as a deterministic loading acting on the structure that can accelerate the crack growth rate.

The fatigue analysis of a typical ship structural joint is developed, comparing the reliability index related to fracture mechanics approach with the reliability index defined with the use of  $S-N$  approach. The reliability index defined by the former approach is similar to those related to the fracture mechanics approach. For the joint analyzed, the reliability index defined by  $S-N$  approach is slightly higher than the reliability index calculated with fracture mechanics approach. This is an indicative that the uncertainties of the random variables in fracture mechanics approach are higher than in the  $S-N$  approach, and must be better characterized, mainly the material properties and initial crack size.

The fracture mechanics approach is suitable for the study of the influence of the residual stresses on the fatigue failure of a ship structure. The difference in the reliability index defined for null residual stresses and for a residual stress field with magnitude equal to 20% of the material yield strength is about 10%.

The effects of corrosion on the reliability index are analyzed considering an increase in the crack growth rate.

For an increase of 30% in that rate, there is a decrease of 10% in the reliability index.

The alliance between the effects of corrosion and residual stresses field can reduce the predicted ship structural life as for fatigue analysis. Both detrimental effects must be considered during the ship design.

## 5. ACKNOWLEDGEMENTS

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