Residual Strength of Damaged Ship Hull

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ABSTRACT

Hull-girder residual strength after grounding was investigated for four double hull tankers, three bulk carriers and one single hull VLCC. It is noted that the loss of section modulus (to the deck and to the bottom) and the loss of ultimate strength (sagging and hogging conditions) are approximately proportional to the transverse damage extent in bottom. Such relationships, when expressed as dimensionless form, do not seem to depend on the ship's length, but are different for different types of ships. The ship's speed that results in a given bottom damage extent in a grounding accident is predicted using a simplified analytical method. The influential effects of ship's speed on the residual strength of hull-girder are shown. It is revealed that when damaged to a same percent of the ship's breadth, a double hull tanker and a bulk carrier have comparable hull-girder residual strength, and are better than a single hull tanker. If sailing at a specific speed and running aground, a single hull tanker will have a residual hull-girder strength that is comparable to a double hull tanker of a similar size which loses outer skin only. Oil tankers, both single hull and double hull, seem to have more reserve in residual strength than bulk carriers.

Presentation

1. INTRODUCTION

A ship's structure has been designed to sustain all the loads expected to arise in its seagoing environment. There are static components due to weight and buoyancy in calm water, dynamic components caused by wave-induced motions of the water around the ship, impact loads such as green water, slamming, collision and grounding, and specialized operational loads such as ice loads and thermal loads. From the viewpoint of normal operation, the objective in structural design has been to maintain structural integrity of hull-girder and details.

Protection against accidental flooding has been another essential issue of the design of watercraft. The most effective way to minimize the adverse effects of flooding is by use of internal subdivision by means of watertight bulkheads and by some horizontal subdivision. National and international standards related to damage as a result of an accident have focused on requirements for watertight bulkheads and subdivision. Structural strength in a collision, grounding or internal accident, such as an explosion, has been attracted very limited attention.

In 1995, ABS published "Guide for assessing hull-girder residual strength" (ABS 1995). It provides guidelines and assumptions for facilitating an assessment of structural redundancy and hull-girder residual strength, and may be easily employed at an early design stage. After a ship sustains damage in the prescribed most unfavorable condition, a minimum residual strength of hull-girder is to be maintained with regard to preventing, or at least substantially reducing, the risk of a major oil spill or loss of ship due to a post-accident collapse or disintegration of the hull during tow or rescue operation. The ABS Guide for hull-girder residual strength is among the few criteria published by classification societies or international organizations that regulate the design of ship structure.

The residual strength of hull-girder, particularly, the residual strength of a grounded tanker, has emerged as an important issue in early design stage, since hull-girder failure will generally lead to pollution of environment. Unfortunately, assessment of the post-accident strength of a ship's hull is limited. Therefore,

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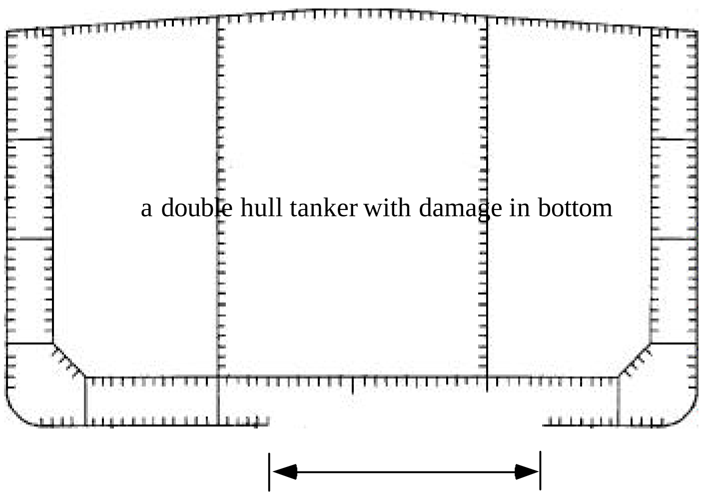

Figure 1 A double hull tanker with damage in bottom

discussion of the influence of damage extent on the hull-girder residual strength is limited, and analyses on the impact of ship's speed on the loss of hull-girder strength are rare.

This paper presents some results on an investigation of hull-girder residual strength of commercial ships. Typical double hull tankers, bulk carriers and a single hull tanker were analyzed for grounding accident scenarios. The general relations between section modulus and transverse damage extent, between ultimate strength and transverse damage extent, between loss of section modulus and ship's speed, and between loss of ultimate strength and ship's speed were obtained. The advantages of having an inner bottom are discussed from the viewpoint of hull-girder residual strength.

2. BENDING CAPACITY AND RESIDUAL STRENGTH OF HULL-GIRDER

The hull-girder analysis assumes that hull-girder bending satisfies simple beam theory. The midship section modulus is the means for the evaluation of the bending strength of the ship's primary longitudinal members for vertical bending moment. The calculation of midship section modulus is an important step in basic ship design.

In 1991, IACS unified the requirements for longitudinal hull-girder (Nitta et al. 1992). All vessels classed for unrestricted service are to comply with the requirements of section modulus and hull-girder moment of inertia, and shearing strength. The required hull-girder section modulus for 0.4L amidships SM is obtained by dividing the total longitudinal bending moment by the nominal permissible bending stress. The total longitudinal bending moment is the sum of the maximum still water bending moment and the wave induced bending moment. In addition, a ship should have its section modulus larger than an established minimum value. The total bending moment determines most of ship hulls, while the ships with minimum required section modulus occupy about 20% of all ships sailing unlimited service areas.

The ultimate strength of the hull-girder determines the bending moment that will break the back of the ship by causing extensive yielding and buckling. The bending moment corresponding to the initial buckling of compressive panels or the initial yielding of tensile components in a ship's hull is, usually, not the true maximum hull-girder bending capacity. Individual plates and longitudinals may experience elastic buckling, plastic buckling, post buckling, yielding, and/or fracture in the process of approaching hull-girder ultimate strength.

Figure 1 shows a transverse section of a double hull tanker which has damage in its bottom due to a grounding accident. The hull-girder residual strength of a damaged ship can be either a reduced section modulus or a reduced ultimate strength. This paper investigates hull-girder residual strength in terms of residual section modulus to the bottom, residual section modulus to the deck, residual ultimate strength under sagging conditions and residual ultimate strength under hogging conditions.

The improved knowledge of the behavior of structural members and the hull-girder itself has led to the development of various approximate methods predicting the collapse load of ship's hulls. Closed-form approximate formulae can be obtained based on simplified analytical models, but no formulae have been developed yet which are unequivocally acceptable. Detailed non-linear finite element analysis, which accounts for buckling, yielding and post-buckling behaviors of plate and stiffened panel, may be a reliable approach for determining ultimate strength. Since they require enormous modeling efforts and computing time, FEM analyses have limited applications to particular ship's hulls. Simplified methods generally take into account most of the influential factors and are much easier to use to model initial deflections and residual stresses, which may be a difficult task for an FEM simulation. The modeling easiness and quick calculation time have lent simplified analytical approaches to extensive usage.

All approximate approaches rely on an idealization of a transverse section into elements. The stress-strain curve for each individual element is determined, in advance, through theoretical formulations or through finite element analysis. For a given vertical curvature, stresses of all elements are integrated over the whole section to provide the applied bending moment. The complete moment-curvature response is thus built up by applying this technique incrementally (e.g., Smith et al. 1987).

Another approach is the so-called idealized structural unit method, which uses a coarse mesh idealization. ALPS/ISUM (Paik et al. 1996), one of the programs based on this idea, includes failure modes of local buckling, panel buckling, overall buckling, yielding, ultimate tensile rupture, and ductile fracture. An element may fail in one of these modes initially and progress subsequently to another mode in the progressive collapse process. The program reduces the modeling effort and computing time required of a conventional finite element analysis using large elements, and thus makes it suitable for the ultimate strength analysis of this study.

3. GROUNDING DAMAGE AND THE PREDICTION

A grounding accident results in permanent set-in, or rupture, or loss of bottom shell. Bottom longitudinals that support the bottom shell may be heavily deformed, twisted, broken or even lost. Damaged structures are unable to carry longitudinal stress, and should therefore be excluded from the calculation of the bending capacity.

New tankers have a height of double bottom of more than 2 meters or one fifteenth of ship's breadth, whichever is the lesser. The depth of double bottom, required by MARPOL, has been specified in accordance with an acceptable probability-based risk of "non-failure" of the inner skin due to grounding. It is expected that the inner bottom of tankers is the final barrier separating oil from seawater, and that it will maintain its full capacity of withstanding the hull-girder bending in and after a grounding accident. Therefore, a grounding accident is modeled as loss of the bottom shell and the attached bottom longitudinal within a damage zone. The inner bottom, though may be deformed, is assumed to carry longitudinal stress and is retained in the calculation of hull-girder bending strength. The vertical damage extent, in this paper, is less than the height of double bottom (Figure 1). For bulk carriers and container carriers, the same assumption will be used.

Some numerical simulations of the grounding process, both small and full scale, have been reported recently. FEM has been recognized as a reliable tool for analyzing an entire collision or grounding process. Because of its high costs and long modeling and computation time, FEM simulation is still limited to studies of particular designs under specific accident scenarios.

Simplified analytical approaches have emerged as powerful calculation tools. They are based on theoretical analysis so that they are suitable for a wide range of designs and accident scenarios. They require relatively short computing time and are more attractive than FEM simulations.

The Joint MIT-industry Project on Tanker Safety has developed various models for ship structural members, which are then assembled into a computer program DAMAGE. The general theory is applicable for determining the vertical and horizontal resistance force of hull plating interacting with wedge and cone shaped rocks, based on a given specific ship's speed, geometrical parameters of the rock and the relative location of the rock.

Wang et al. (1997) developed a simple analytical method for predicting the strength of ship bottom in a raking accident. This method considers main energy absorbing mechanics, i.e., plasticity and friction, and accounts for primary failure modes of major structural members, i.e., the tearing and concertina type tearing of the bottom shell, membrane stretching of floors, and bulging of bottom plating behind floors. It separates the internal mechanics, which deals with the energy dissipation in structures, from the external mechanics, which deals with ship motions of a raking process. This scheme of uncoupling internal mechanics and external mechanics simplifies the calculations, and makes comparisons of different designs much easier. This method has the advantage of very small calculation effort, especially when compared to FEM simulations. The simplicity of calculations makes the method well suitable for a comparative study.

Table 1 Principal dimensions of four double hull tankers, three bulk carriers and one single hull tanker

Vessel	LBP (m)	B (m)	D (m)	DWT (ton)
DHT 1	320.00	58.00	31.00	307,000
DHT 2	258.16	46.20	25.30	125,000
DHT 3	234.00	42.00	21.00	105,000
DHT 4	172.00	32.20	19.10	46,500
BC 1	277.0	45.00	24.10	169,930
BC 2	217.0	32.20	19.15	75,200
BC 3	180.6	30.95	16.40	48,000
SHT 1	315.0	58.0	28.7	260,000

4. CALCULATIONS OF TYPICAL COMMERCIAL SHIPS

Table 1 lists principal dimensions of four double hull tankers, three bulk carriers and one single hull tanker, all of which are typical designs of typical sizes. The transverse damage extent ranges from 5% (a minor grounding damage) to 70% (an extreme grounding damage) ship's breadth. ALPS/ISUM was used to determine the ultimate strength under intact condition and damaged conditions. Wang et al.'s (1997) simplified method was used to calculate the ship's speed for a given damage extent.

4.1 Double hull tankers

The four double hull tankers used in analysis were built recently and classed with ABS. Vessels DHT1, DHT2 and DHT3 carry ABS SafeHull notation. Vessel DHT3 has heavier scantlings than SafeHull requirements to satisfy special requests from the owner. The four vessels are of typical sizes, ranging from 46,500 tons to 307,000 tons in deadweight, or 172 meters to 320 meters in ship's length. This tanker fleet, although limited in number, is representative of most double hull tankers in service.

In order to easily illustrate analyses result trends, transverse damage extent has been expressed as percent of the ship's breadth. A minimum of six damage cases for each vessel were investigated, covering the range of small damage to very large damage. For each damage case, section modulus to the bottom SM_btm, section modulus to the deck SM_dk, ultimate strength under sagging conditions Ult. Str. (sag), and ultimate strength under hogging conditions Ult. Str. (hog) were calculated.

Figure 2a plots the residual section modulus to the bottom as a function of the transverse damage extent. Values of SM_btm are normalized with respect to their values at intact condition. The loss of bottom, or the transverse damage extent, is normalized with respect to the ship's breadth. It is revealed that there exists, for a specific vessel, a linear function correlating the section modulus to the bottom with the loss of bottom, which is as the follows,

$$\textit{Residual strength} = 1.0 - C1 * b \qquad (1)$$

Where, b is the transverse damage extent as percent of the ship's breadth, and C1 is a coefficient, which is dependent on the geometry of the transverse section and the design of the ship's hull.

Furthermore, Figure 2a suggests that the ship's length has no obvious influence on the value of C1. The points for the four vessels are very close to a single linear relationship of SM_btm to percent of bottom loss, although the four vessels represent a wide range of sizes of double hull tankers. A common value of C1 would therefore be adequate for the residual section modulus to the bottom for the four double hull tankers. A value of 0.439 provides the best correlation.

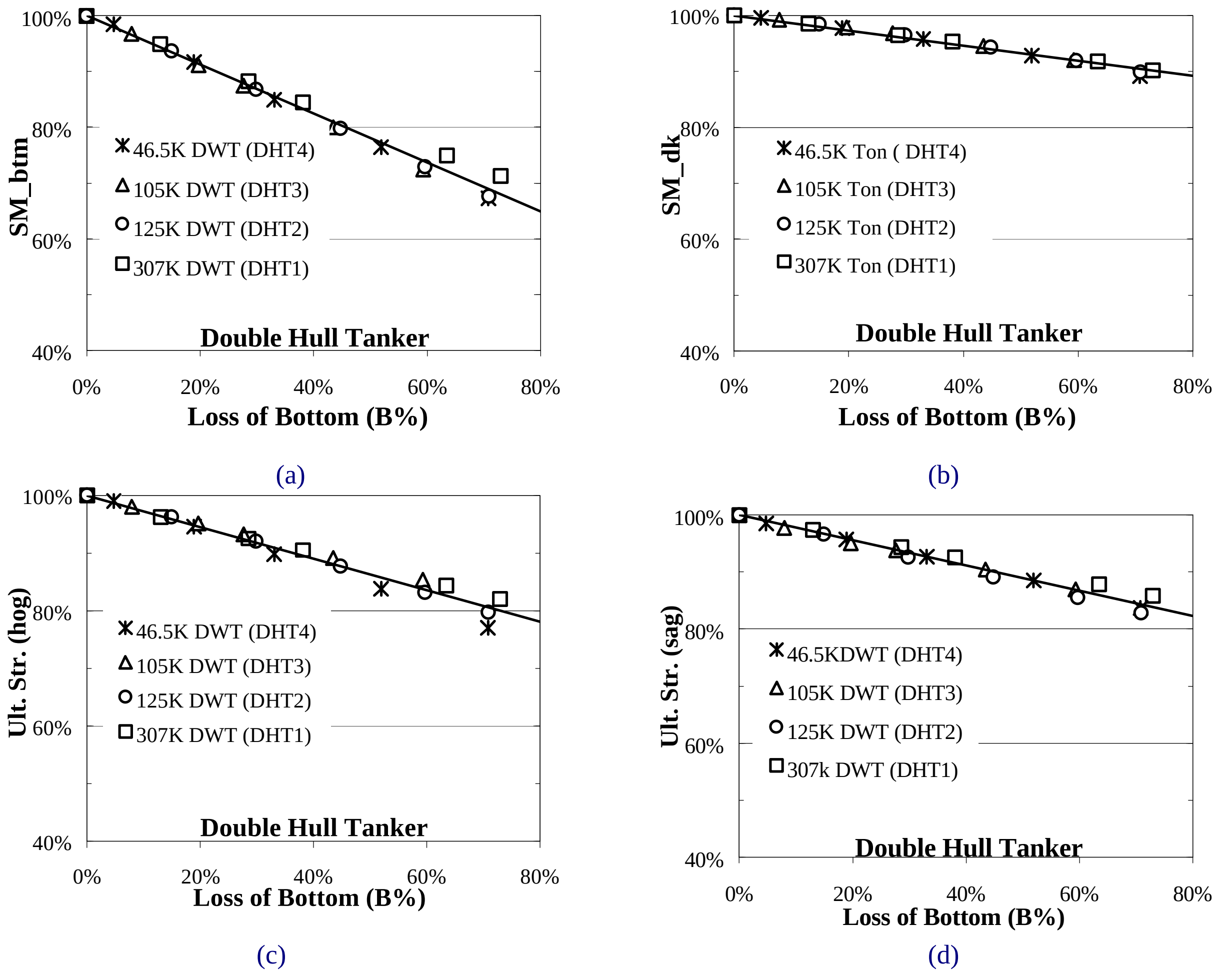

Figure 2   Double hull tankers: hull-girder residual strength versus transverse damage extent

In addition, a second series of damage cases were analyzed in which one bottom girder was damaged too. Calculation results suggest that the influence of one bottom girder can be adequately accounted for by adding another item to the end of Equation 1. Then Equation 1 becomes the following expression:

$$Residual\ strength = 1.0 - C1 * b - C2 \qquad (1a)$$

Where, b and C1 are the same as Equation 1, and C2 is a coefficient, which depends on the height of double bottom, the ship’s depth, etc. The value of C1 is approximately the same as that in Equation 1. The value of C2 is in the range of 1.40~2.00%.

The four tankers were sampled to represent a wide range of typical tanker designs. One can conclude that all typical double hull tankers have the same relation as Equations 1 and 1a, and the values of C1 and C2 obtained herewith can be applied to all double hull tankers.

Figure 2b plots the residual section modulus to the deck as a function of the transverse damage extent. Values of SM_dk are normalized with respect to their values at the intact condition. Again, a linear correlation between SM_dk with percent of loss of bottom is obvious. Equations 1 and 1a are also applicable for all double hull tankers, but with a different combination of C1 and C2 values.

Figure 2c shows calculations of residual ultimate strength under hogging conditions. All these calculation points are very close to a linear line, although some small deviations exist. Equations 1 and 1a can be used for these four tankers, and also for all other double hull tankers in service. Ultimate strength is the limit-state of the hull-girder. When approaching this limit-state, structural components have experienced buckling,

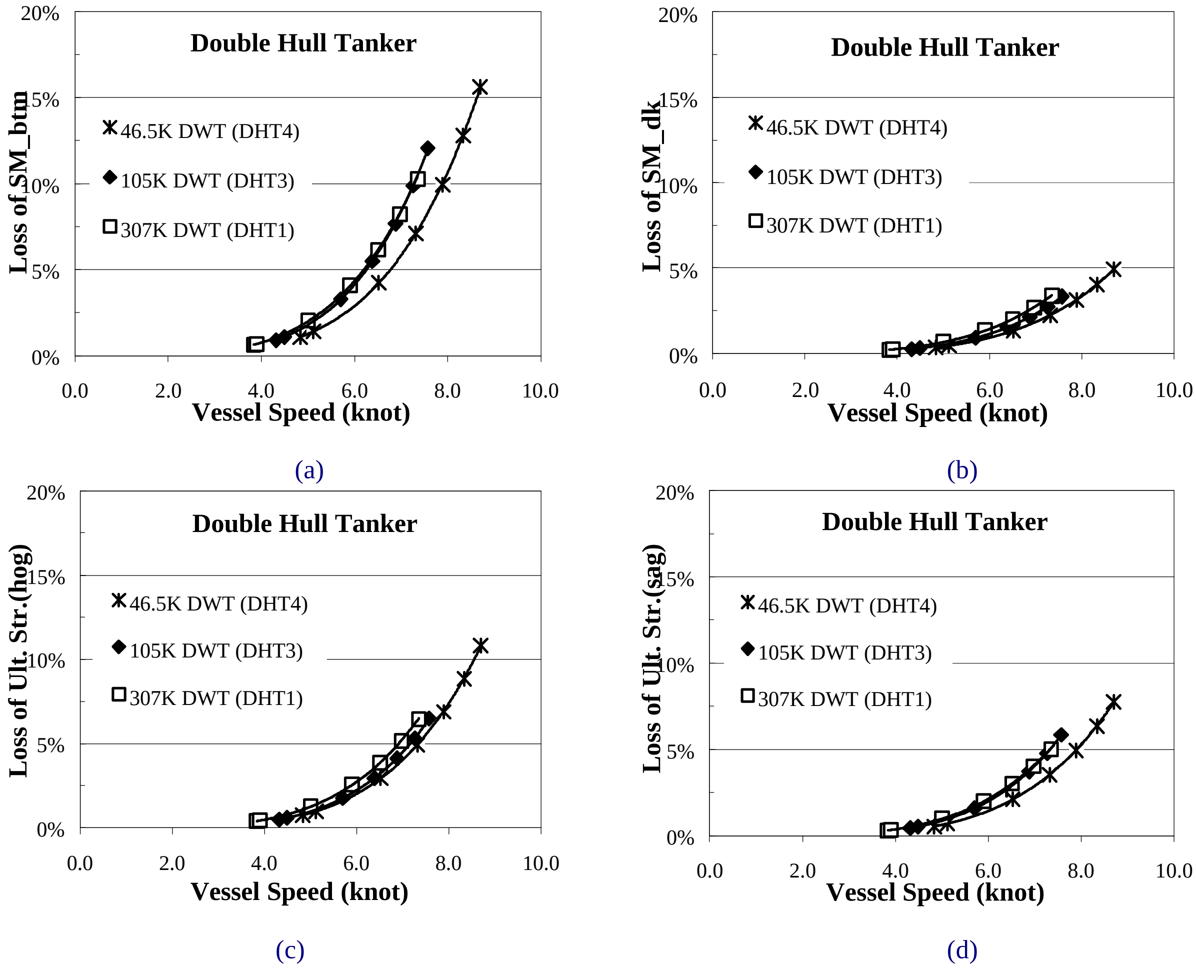

Figure 3   Double hull tankers: loss of hull-girder strength versus ship's speed

yielding and even fracture where non-linearity plays a major role. Interestingly, as suggested by Figure 2c, this limit-state linearly correlates with the transverse damage extent.

Figure 2d plots residual ultimate strength under sagging conditions. Again, there exists an approximate linear correlation between residual ultimate strength and the transverse damage extent, and Equations 1 and 1a are applicable for all double hull tankers.

Table 2 summaries C1 values for double hull tankers. Generally, a 10.0% loss of bottom results in a 4.4% loss in section modulus to the bottom, a 1.4% loss in section modulus to the deck, a 2.7% loss of ultimate strength under hogging conditions and a 2.2% loss in ultimate strength under sagging conditions.

The value of section modulus to the bottom is the most sensitive to the transverse damage extent in the bottom, while the value of section modulus to the deck is the least sensitive. The residual ultimate strength for sagging and hogging conditions are very close to each other, and their curves fall between those for section modulus to the deck and section modulus to the bottom.

For a fully laden ship that is damaged to 0.6 times the ship's length in a raking accident, the ship's speed can be predicted for a specific transverse damage extent using the simplified method of Wang et al. (1997). Combining the prediction of bottom damage with Figures 2a to 2d leads to Figures 3a to 3d, which are the relations of the loss of hull-girder strength with the ship's speed. In the calculation, forces of the tearing damage and the concertina tearing type damage are averaged, the full depth of the floor (removing man-hole height) is assumed to effective in resisting the penetration of a rock, and bottom girders are assumed to be intact.

Figures 3a to 3d clearly reveal the influential effects of ship's speed on the loss of hull-girder strength,

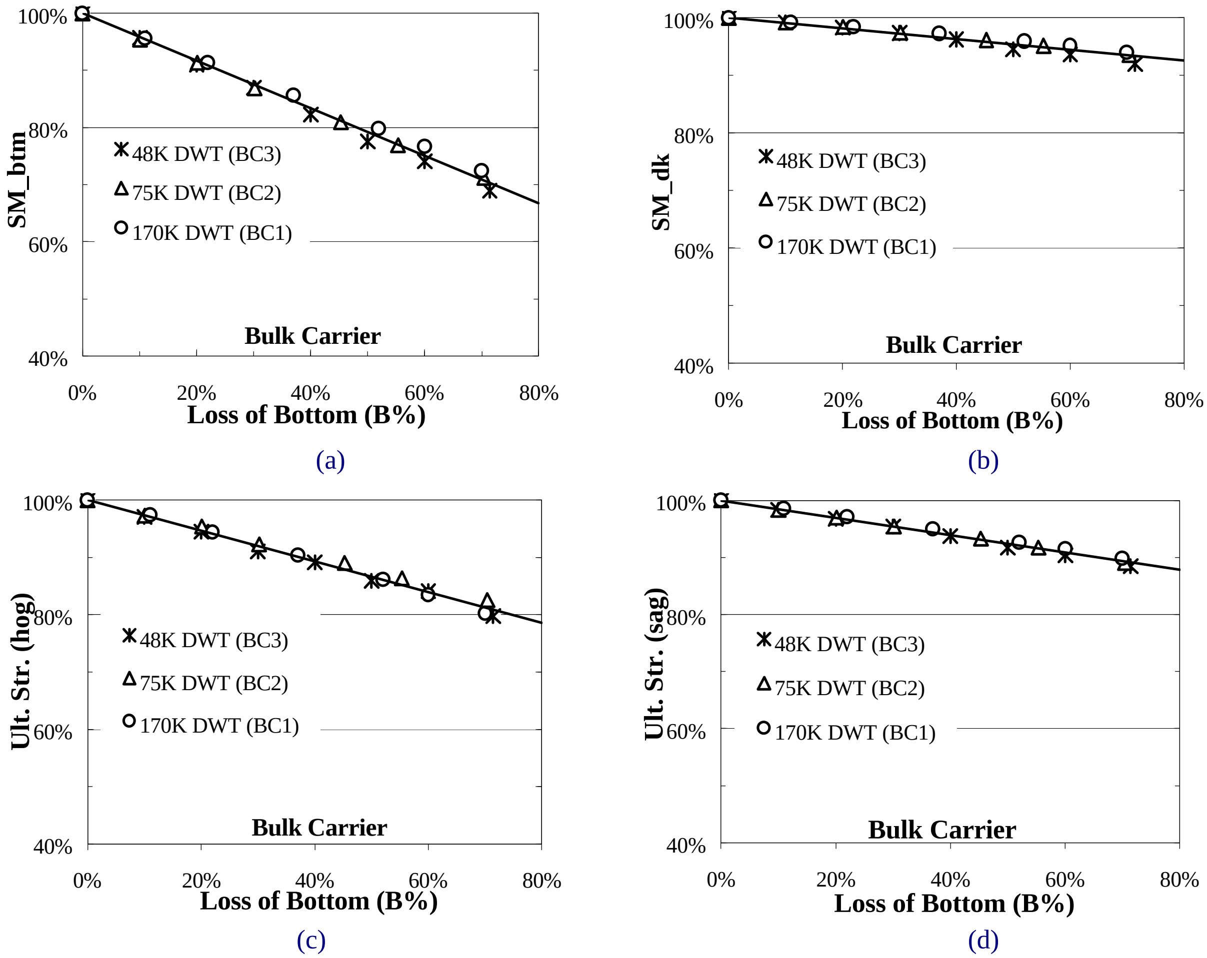

Figure 4 Bulk carriers: hull-girder residual strength versus transverse damage

in terms of either section modulus or ultimate strength. Generally, the hull-girder strength decreases in the order of V^{α}, where V is the ship's speed, α is a value larger than 4.0 and C is a coefficient:

$$\textit{Loss of hull-girder strength} = C * V^{a} \qquad (2)$$

All four tankers exhibit a similar trend, but a simple equation with unique values for C and α can not be used to represent all four tankers. Since α is a value larger than 4.0, the loss of hull-girder strength is very sensitive to the ship's speed, so that unified values of C and α for all double hull tankers, though still possible, may not be meaningful.

Generally, when a tanker sails very slowly (say, less than 5 knots) and runs aground, its lose of hull-girder strength is small. When it sails faster, the loss of the hull-girder strength becomes very drastic and even catastrophic.

It should be noted that the influence on the motion of the ship caused by grounding reaction is not taken into account in the calculation. Therefore, these figures provide conservative estimations. It can be expected that the loss of hull-girder strength will be smaller than that shown in these figures, if the influence that the ship is raised during the raking process is included. However, the general trend shown herewith should not change.

4.2 Bulk carriers

The three bulk carriers that were studied in this paper are typical, including one handy size, one Panamax size and one cape size. Again it is assumed that the inner bottom will keep intact in a grounding accident and

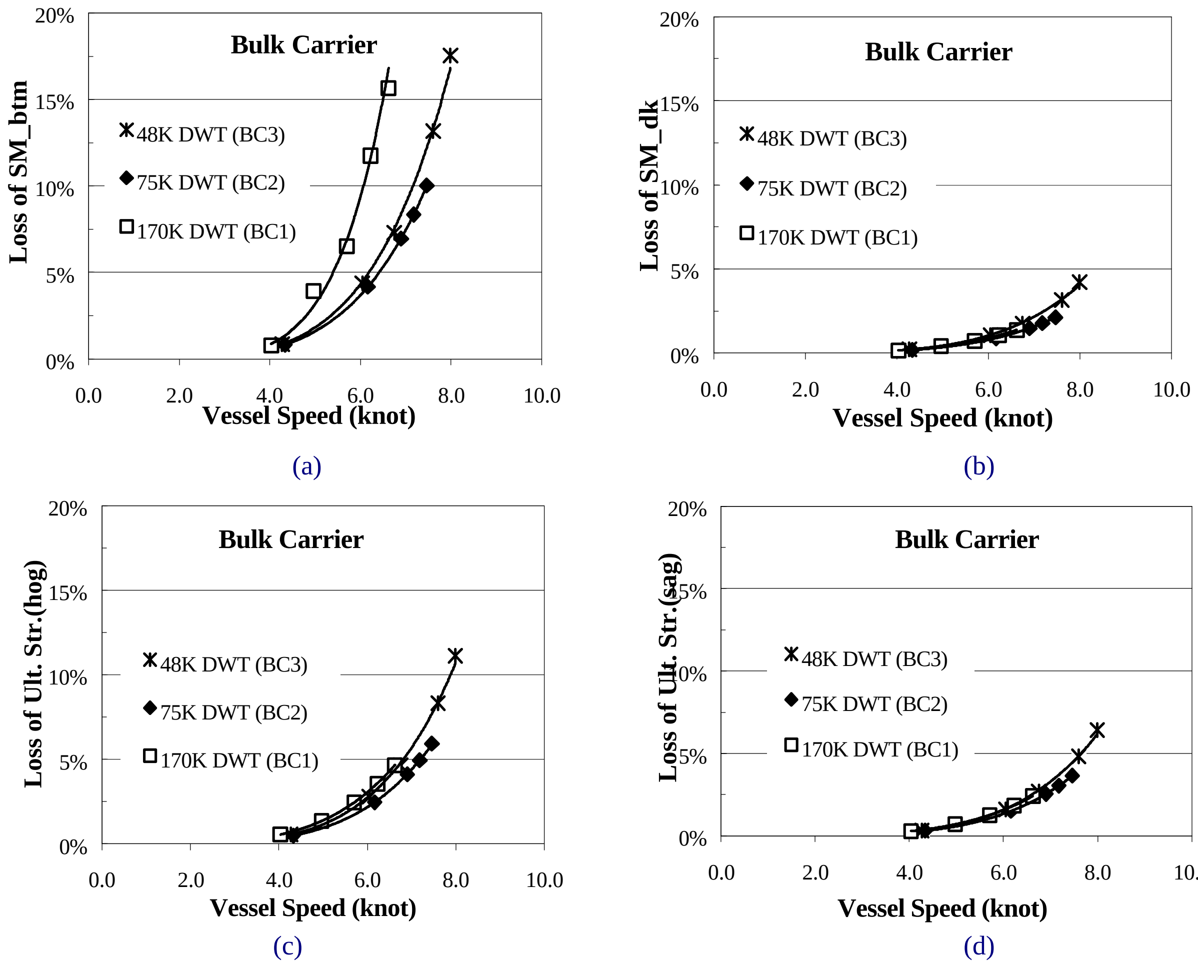

Figure 5 Bulk carriers: loss of hull-girder strength versus ship's speed

only the bottom shell and the attached bottom longitudinal are damaged. The calculations for the hull-girder residual strength are in Figures 4a to 4d, and the calculations of the loss of hull-girder strength as function of ship's speed are in Figures 5a to 5d.

Similarly, Equations 1, 1a and 2 are applicable here, while the values of C1 and C2 are different. The general conclusions for double hull tankers are applicable to bulk carriers also.

For bulk carriers, a 10.0% loss of bottom results in a 4.2% loss in section modulus to the bottom, a 0.9% loss in section modulus to the deck, a 2.7% loss of ultimate strength under hogging conditions and a 1.5% loss in ultimate strength under sagging conditions.

Bulk carriers, when sailing slowly and running aground, will lose a small portion of their hull-girder strength, and when the speed is faster, the strength loss can be extremely large.

4.3 Single hull VLCC

One typical single hull VLCC was analyzed. This single hull VLCC may be viewed as a representative of all single hull tankers. The designs of different sizes of single hull tanker are generally very similar, so it may be possible to generalize conclusions for this specific vessel to all single hull tankers.

Calculations of this tanker are in Figure 6a to 6d, and Figures 7a to 7d, and the values of C1 are in Table 2. Damage in floors is assumed to be 2 meters from the bottom, so that the vertical penetration is comparable to that of a double hull tanker of similar size. For comparison, the corresponding values of the 260k DWT double hull tanker (DHT1) are plotted in the same figures.

For this single hull VLCC, a 10.0% loss of bottom results in a 6.7% loss in section modulus to the bottom, a 1.7% loss in section modulus to the deck, a

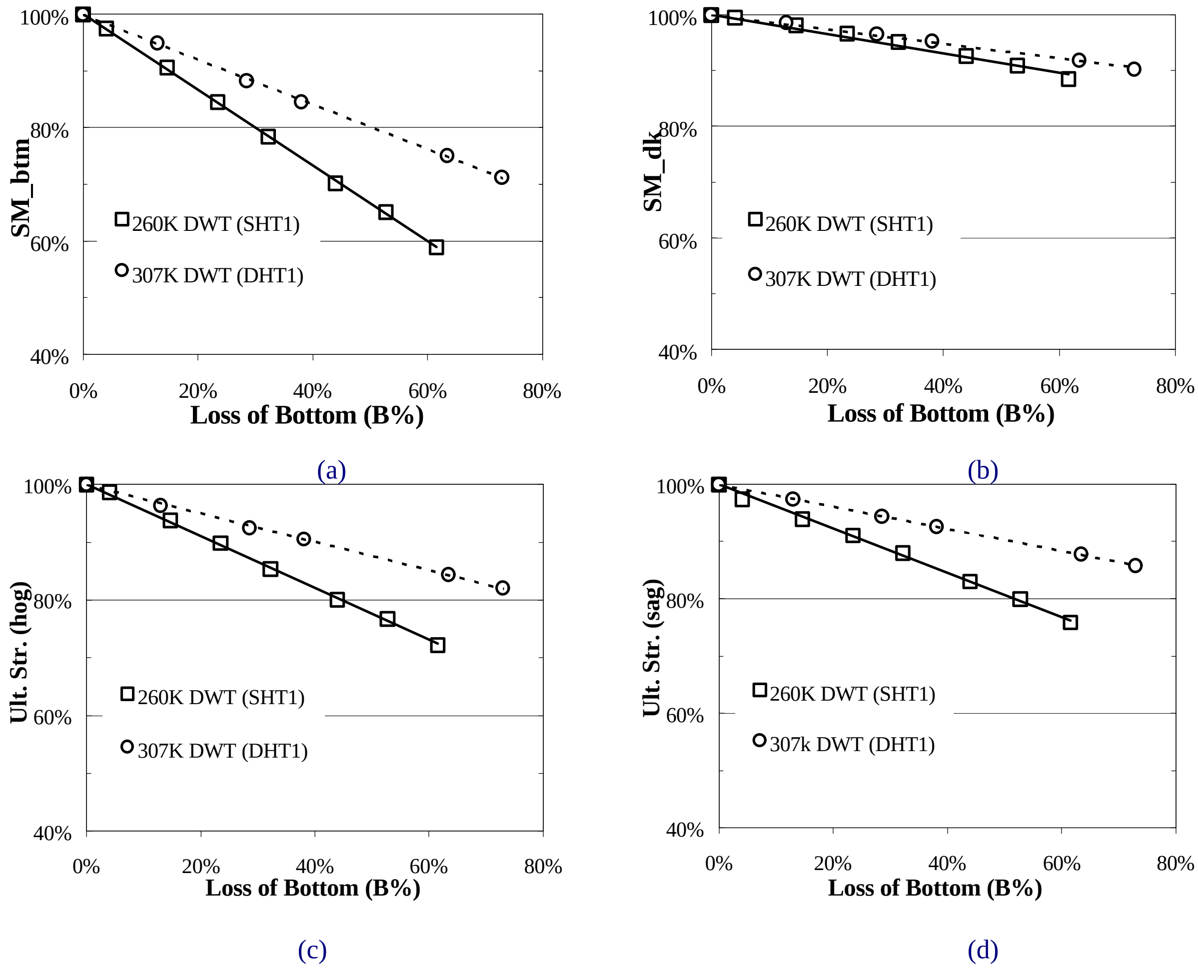

Figure 6 A single hull VLCC and a double hull VLCC:
hull-girder residual strength versus transverse damage extent

4.5% loss of ultimate strength under hogging conditions and a 3.9% loss in ultimate strength under sagging conditions.

4.4 Discussions

If the bottom skin is damaged to a specific percent of ship's breadth, double hull tankers and bulk carriers have comparable residual section modulus to bottom, which is much larger than that of single hull tankers (Figures 2a, 4a and 6a). A similar comparison is noted for the residual ultimate strength of hogging condition when bottom is under compression (Figures 2c, 4c and 6c). Clearly, fitting of inner bottom, which is characteristic in double hull tankers and bulk carriers, provides much more reserve strength capacity at the bottom flange of a ship's hull, and therefore, more reserve residual strength after a grounding accident.

In the analysis it is assumed that the vertical penetration in grounding is less than the height of the double bottom and inner bottom does not have damage. It should be noted that if the inner bottom loses its capacity to withstand longitudinal stress as well, the loss of hull-girder strength of double hull tankers and bulk carriers will become prominent and may be as large as that of single hull tankers under severe circumstances. Fortunately, existing probability analysis shows that the chances of such severe grounding accidents are few. One can conclude, generally, that double bottom design is better than single bottom design with respect to hull-girder residual strength.

If the bottom skin is damaged to a specific percent of ship's breadth, bulk carriers have the highest

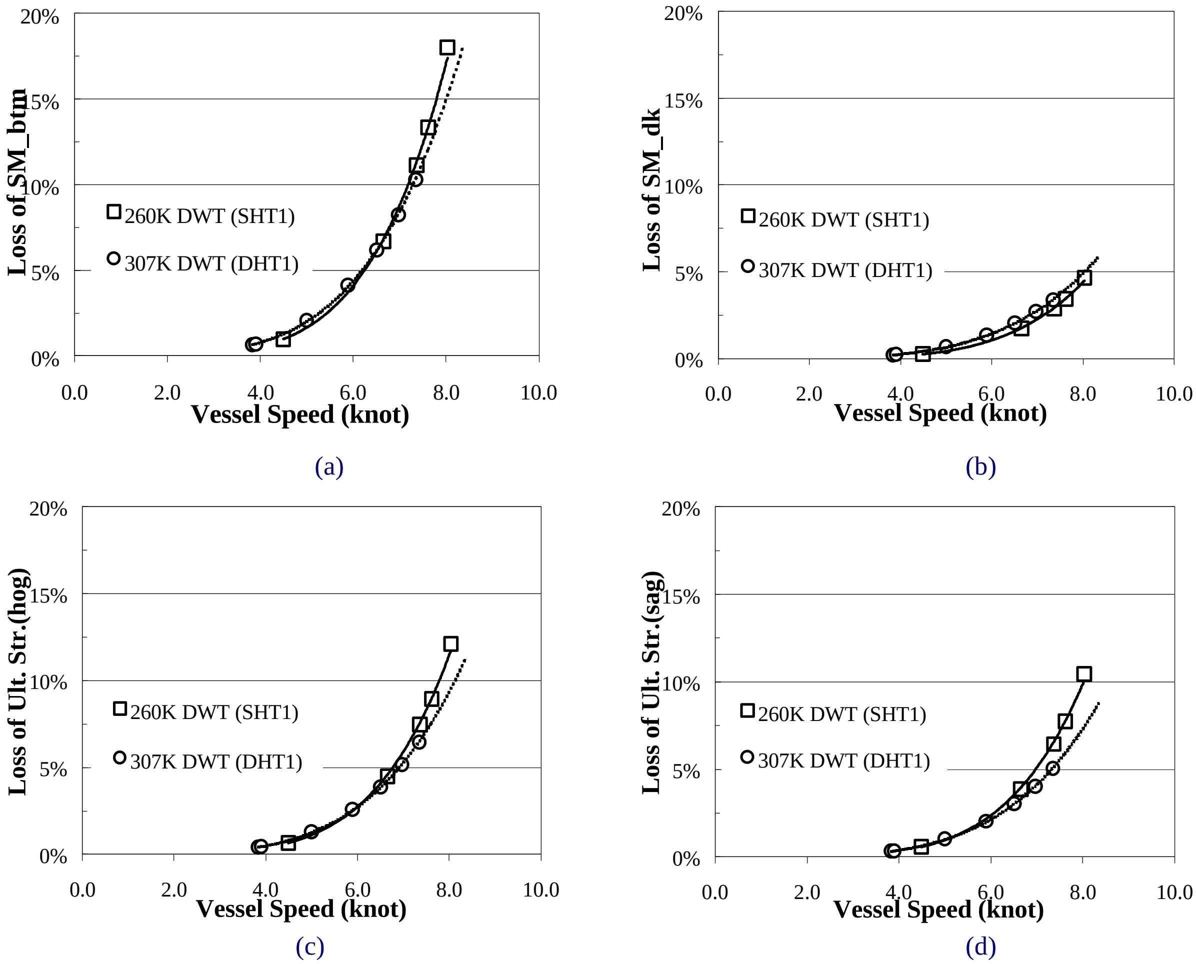

Figure 7 A single hull VLCC and a double hull VLCC:
loss of hull-girder strength versus ship's speed

value of residual section modulus to the deck, followed by double hull tankers, and the single hull tanker has the smallest (Figures 2b, 4b, 6b). A similar comparison may be noted for the residual ultimate strength under sagging conditions, when the deck is under compression (Figures 2d, 4d and 6d). The difference of having or having no inner bottom is reflected again in the hull-girder strength to the top flange.

When comparing Figures 3a to 3d with Figures 5a to 5d, and 7a to 7d, it seems that a damaged bulk carrier loses much more hull-girder strength than an oil tanker. The fact that relative less material is placed in the bottom skin of bulk carriers may lead to this trend. However, this conclusion may be considered by some as too simplistic and conservative. Bulk carriers usually have more bottom girders to support heavy density cargoes. Because of the narrow spacing of bottom girders, in a grounding accident bulk carriers may see more damaged bottom girders. In the present calculation, the contributions of bottom girders to energy absorption were not considered, resulting in lower prediction of ship's speed for a given transverse damage extent. If the bottom girder(s) is included, the predicted loss of hull-girder strength will be smaller and may be close to that of tankers. Future investigations are needed and the comparison of bulk carriers and tankers should be based on more realistic damage scenarios.

As revealed by Figures 7a to 7d, when a single hull tanker sails at a specific speed and runs aground, its loss of hull-girder strength is comparable to that of a double hull tanker of a similar size which loses its outer skin and has no damage to its inner bottom. Generally, the bottom shell of a single hull tanker is thicker than that of a double hull tanker of a similar size, so that the

Table 2 Values of C1 in Equation 1

Value of C1	SM_btm	SM_dk	Ult. Str. (Hog)	Ult. Str. (Sag)
Double hull tankers	0.439	0.135	0.274	0.221
Bulk carriers	0.416	0.092	0.267	0.152
Single hull VLCC	0.667	0.172	0.448	0.386

extent of damage that a single hull tanker sustains in a particular grounding accident is smaller. As a result, although the residual strength for a given damage extent is different for single and double hull tankers, the loss of hull-girder strength when expressed as function of the grounding energy, or the ship's speed, becomes comparable.

Single hull tankers have no reserve strength capacity once their bottom is punctured, while double hull tankers have the inner bottom to act as a reserve both in absorbing grounding energy and in residual hull-girder strength. Having a double bottom is also advantageous from the viewpoint of hull-girder residual strength.

5. CONCLUSIONS

Hull-girder residual strength after grounding was investigated for four double hull tankers, three bulk carriers and one single hull VLCC. It was noted that the loss of section modulus and the loss of ultimate strength are approximately proportional to the transverse damage extent in the bottom. Such relationships, when expressed in dimensionless form, do not seem to depend on the ship's length, but are different for different types of ships.

For double hull tankers, a 10.0% loss of bottom results in a 4.4% loss in section modulus to the bottom, a 1.4% loss in section modulus to the deck, a 2.7% loss of ultimate strength under hogging conditions and a 2.2% loss in ultimate strength under sagging conditions. For bulk carriers, a 10.0% loss of bottom results in a 4.2% loss in section modulus to the bottom, a 0.9% loss in section modulus to the deck, a 2.7% loss of ultimate strength under hogging conditions and a 1.5% loss in ultimate strength under sagging conditions. For single hull tankers, a 10.0% loss of bottom results in a 6.7% loss in section modulus to the bottom, a 1.7% loss in section modulus to the deck, a 4.5% loss of ultimate strength under hogging conditions and a 3.9% loss in ultimate strength under sagging conditions.

The ship's speed that results in a given bottom damage extent in a grounding accident was predicted using a simplified analytical method. The relationships between the loss of hull-girder strength and the ship's speed were obtained. The influential effects of ship's speed on the residual strength of the hull-girder are shown.

If the bottom skin is damaged to a specific percent of ship's breadth, double hull tankers and bulk carriers have comparable residual strength, which is much larger than single hull tankers. However, when a single hull tanker sails at a specific speed and runs aground, its loss of hull-girder strength is comparable to that of a double hull tanker of a similar size which loses its outer skin and has no damage to the inner bottom. In double hull tankers, the inner bottom serves as a reserve both in absorbing grounding energy and in hull-girder residual strength. The double hull tanker is therefore better than the single hull tanker in so far as the residual strength of a damaged hull is concerned.

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