EVALUATION OF SL-7 SCRATCH-GAUGE DATA

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1981
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This report is one of a group of Ship Structure Committee Reports which describe the SL-7 Instrumentation Program. This program, a jointly funded undertaking of Sea-Land Service, Inc., the American Bureau of Shipping and the Ship Structure Committee, represents an excellent example of cooperation between private industry, classification authority and government. The goal of the program is to advance understanding of the performance of ships' hull structures and the effectiveness of the analytical and experimental methods used in their design. While the experiments and analyses of the program are keyed to the SL-7 Containership and a considerable body of the data developed relates specifically to that ship, the conclusions of the program will be completely general, and thus applicable to any surface ship structure.

The program includes measurement of hull stresses, accelerations and environmental and operating data on the S.S. Sea-Land McLean, development and installation of a microwave radar wavemeter for measuring the seaway encountered by the vessel, a wave tank model study and a theoretical hydrodynamic analysis which relate to the wave induced loads, a structural model study and a finite element structural analysis which relate to the structural response, and installation of long-term stress recorders on each of the eight vessels of the class. In addition, work is underway to develop the initial correlations of the results of the several program elements.

Results of each of the program elements are being made available through the National Technical Information Service, each identified by an SL-7 number and an AD- number. A list of all SL-7 reports available to date is included in the back of this report.

This report documents the evaluation of the long-term stress recorders.

Clyde T. Lusk, Jr.
Rear Admiral, U.S. Coast Guard
Chairman, Ship Structure Committee
16. **Abstract**

This report assesses the value and application potential of the SL-7 scratch-gauge data base. The principal advantage of the mechanical extreme-stress recorders is large quantities of inexpensive data. Their drawbacks are that contributions from different load sources cannot be separated; contributions from torsional, lateral and vertical bending cannot be separated; and there is no reasonable way to distinguish between hogging and sagging response. However, when the scratch-gauge data and electrical strain-gauge data from a second operational season aboard the McLEAN were correlated, a comparison between the form of the curves showed good agreement.

Several statistical models were found to describe the data well enough to be used as a basis for statistical inference beyond the range of measured values. The Type-I Extreme-Value distribution, the Weibull distribution, and a four-parameter distribution proposed by M. K. Ochi satisfactorily represent the data in most cases.
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#### Approximate Conversions to Metric Measures

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Note: 1 °C = 1.8 °F. For other exact conversions and more detailed tables, see NBS Spec. Publ. 286, "Units of Weights and Measures," 1975; U.S. Catalogue No. G12-10-286.
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1. INTRODUCTION

1.1 Background and Objectives

Previous Ship Structure Committee projects attempting to establish load criteria on a probabilistic basis indicated that lifetime extreme loads could not yet be predicted with confidence. In order to acquire this confidence, mechanical extreme-stress gauges were installed on eight SL-7 ships: the SEA-LAND McLEAN, GALLOWAY, COMMERCE, EXCHANGE, TRADE, FINANCE, MARKET, and RESOURCE. The scratch gauge, unattended and continuously running, records the maximum to minimum stress excursion in a four-hour period. Two scratch gauges were installed in the McLEAN on October 7, 1972, and the seven other SL-7 container ships were installed with a single gauge as they were delivered to their owners. In the first five years of operation, over twenty "ship-years" of data were recorded on paper tape in the form of 36000 + records. The data are presented in histogram form by Fain and Booth in SSC-286 (1) and represent a substantial measurement and reduction effort.

This project assesses the value and application potential of the data base. Specifically, the three-fold objective of the present project is to evaluate SL-7 scratch-gauge data as a basis for extreme-load prediction, to determine correlations with SL-7 electrical strain-gauge data, and to recommend when and how many scratch gauges can be recovered for placement aboard other ships.

Although the data presented as histograms in SSC-286 form the basis of the present study, remeasurement and data reduction of some original scratch records were necessary to carry out many of the analyses. It is not within the scope of this investigation, however, to remeasure, reduce and reprocess the data in bulk.

A scratch-gauge instrumentation program is not without precedent. The Naval Construction Research Establishment (NCRE) of Dunfermline, Fife, United Kingdom, (now AMTE) outfitted over sixty British warships of various classes with simple maximum reading mechanical strain gauges. The approaches used in that particular program are detailed by Yuille (2) and Smith (3). Other full-scale measurements that produced the same type of data, although not necessarily with a scratch gauge, are reported by Jasper (4) and Ward (5).

† Numbers in parenthesis designate references at end of paper. Ship Structure Committee Reports will be denoted by "SSC-###".
1.2 Organization of Report

This report consists of three parts. Part I presents background information and studies the scratch-gauge instrumentation project from a scientific perspective. Within the framework of the generalized experimental process, the scratch-gauge project is examined on a physical basis, from instrumentation to data reduction. New methods used to remeasure and reduce subsets of the original data base are described. Part I consists of Chapters 1 and 2.

Part II is concerned with the data-analysis work. It should have particular interest to the statistician. The studies carried out in this part bring to bear various tools of statistical science. The emphasis in several chapters is more on analysis than application, in contrast to Part III. This second part is comprised of Chapters 3 through 7. Chapter 3 attempts to put the SL-7 electrical instrumentation data and the scratch data on an equivalent basis. This is the first step in evaluating the scratch data as an alternative data-gathering method for ship lifetime load prediction. Correlations are performed on a statistical and deterministic basis. Chapters 4 and 5 present a study of various statistical models which may provide the means to extrapolate to longer periods of time; particular attention is given to the Type-I extreme-value distribution in Chapter 4. Chapter 6 investigates the use of the scratch data for long-term prediction as suggested by Hoffman, et al, in SSC-234 (6), using the correlation information derived in Chapter 3. Chapter 7 studies several miscellaneous topics of interest.

Part III provides an evaluation of the program and is presented from the perspective of a naval architect, emphasizing the value of the data to methods currently in use for the rational determination of ship structural load criteria. To assess the value of the scratch data and program, it is necessary to critically assess the methods for ship lifetime extreme-load prediction. This is done in Chapter 8, which examines the utility of many of the "traditional" procedures within the context of the rapidly evolving ship motion and load simulation techniques currently in development. Chapter 9 categorically evaluates the potential applications of the data. Chapter 10 presents the conclusions of the study and makes recommendations as to disposition of the gauges and further possibilities for data reduction and analysis.
2. INSTRUMENTATION AND PROCEDURE

2.1 Measurement Phenomena

Evaluation of the scratch data must rely heavily on the tools of statistical analysis. To ensure that any conclusions drawn from such analyses are valid, however, a preliminary validation of the data is necessary. This preliminary validation is conducted within the conceptual framework diagrammed in Figure 2-1, after Bury (7).

The primary phenomenon underlying the measurement-generating process (M.G.P.) is the environmental loading on the ship hull girder. The generated data of interest are strains in the instrumented structural components. The measurement process includes both the conversion of strain into a scratch mark on the paper tapes, and the conversion of scratch marks into stress records. These perceived data are organized into histograms which form the information bank presented in SSC-286.

Perceived data and generated data are usually not identical, the difference is normally attributed to experimental error. There are essentially two kinds of error: systematic and random. An example of systematic error would be a consistent nonlinear response of the scratch gauge at higher strains. A random error may be introduced, for example, by the process of measuring scratch lengths, transcribing results, etc.

2.2 Measurement Process - Instrumentation and Data Collection

The details of the instrumentation are presented in SSC-286. For completeness, however, this section presents a brief description of the hardware and data collection procedures involved. The maximum reading strain gauge, recorder and clock units, as shown in Figure 2-2, were obtained from Elcomatic Limited of Glasgow, Scotland. Figure 2-3 shows the placement position of the unit in the starboard tunnel of each SL-7 and in the port tunnel of the McLEAN. Figure 2-4 provides a more detailed illustration of the component layout.

The scratch-gauge consists of a simple extensometer with mechanical amplification of approximately 100:1 at the stylus. The stylus moves against pressure-sensitive paper causing positive or negative deflections. The paper is advanced about 0.13 inch every four hours. Every sixth interval (24 hours) the paper advances 0.4 inches. This produces a data tape as shown in Figure 2-5. Each vertical marking represents the maximum peak to maximum trough stress which has occurred during the four-hour sampling period.
FIGURE 2-1. General Process of an Engineering Investigation
GAUGE ACTION: As shown in the sectional diagram below, the lever system is actuated by distortion of the structure under test and requires no external power supply. The instrument is bolted in position, bearing against the test surface on two sets of hardened conical studs. Any change in separation of bearing points is magnified by the lever system which drives the recording pen across the stationary reel of carbon-backed paper. Time related maximum strain records are obtained by forward movement of recording paper programmed by a precision battery-rewound clock and powered by a small motor also battery powered.

| DETAILS |
|-----------------|-----------------|-----------------|
| Prime Function  | Fully automatic recording of maximum strain. |
| Duration of Continuous Unattended Operation | Three months depending on programme. |
| Magnification Factor | Nominal 100 – subject to precise calibration by a dial gauge reading to 0.001". |
| Resolution | A strain change of 0.001 will produce a 1" pen deflection. |
| Chart Loading | Cassette. |
| Linearity | Substantially linear over strain range of 0.0025. |
| Temperature Effects | Uniform temperature changes of gauge and steel test structure produce no discernible pen movement. |
| Vibration | Tested by dynamic strains of double amplitude 0.0008 at frequencies 25 to 200 cycles per minute – no significant inaccuracy. |

**FIGURE 2-2. N.C.R.E. - Maximum Reading Strain Gauge Recorder**

**FIGURE 2-3. Ship Gauge Location**
(from ref. 1)

**FIGURE 2-4. Component Layout**
(from ref. 1)
Teledyne Engineering Services has measured each data marking to the nearest 0.02 inches and tabulated the results for each vessel over the entire data-gathering period. Prior to installation, the scratch gauges were calibrated so a relationship between force and deflection was established. This was transformed to stress vs. line length so that a stress value for each data interval could be calculated from:

\[
\sigma_{\text{psi}} = (\text{length of scratch line in inches}) \times \text{(scale factor)}
\]

The scale factors are contained in SSC-286. Histograms that represent peak-to-trough stress levels versus the number of occurrences have been prepared by Teledyne Engineering Services. They are arranged in order of data years; one histogram is provided for each gauge for each year. With each year, summary plots of all Atlantic and Pacific data were prepared, as was a grand total plot of all data collected within the year. Additionally, a five-year Atlantic summary, a five-year Pacific summary, and a summary of all data collected in the five-year period were also prepared. Thus, a total of 63 histograms represents the information base for the present project.

2.3 Measurement-Generating Process

As noted in SSC-286, it is important to keep in mind several characteristics of the system when interpreting the scratch-gauge data:

- The record indicates the combined wave-induced and first- (or higher) mode vibratory stresses and there is no way to separate them.
- The maximum-peak and maximum-trough stresses indicated on the record may not have occurred as part of the same cycle; i.e., they may have occurred at different times during the four-hour interval.
- Slow "static" changes in the average stress caused by thermal effects, ballast changes, etc., will contribute to the total length of the scratched line.

These effects are illustrated in Figure 2-6. Consequently, one scratched line can represent as many as five different load sources. These loads include:

- Still-water bending due to weight and buoyancy
- Ship's own wave train
- Wave-induced bending
- Dynamic loads, including slamming, whipping and springing
- Thermal effects
Although the scratch-gauge mark represents strain from these loads at different times, there are several notable examples where a severe transient load has produced a distinct, single excursion well above the portion of the mark that represents wave-induced bending. This is depicted in Figure 2-7. As remarked upon in SSC-286, specific events such as loading or drydocking can be identified. Also, on a "smooth" and sunny day at sea, the thermally induced strains can be followed on the paper tape.

2.4 Experimental Errors

Any area that might be a potential source of experimental error was identified and investigated prior to the statistical analysis of the data.

One possible source of random error is the procedure to measure each scratch. Obviously, there are limitations to the accuracy and consistency obtainable with the human eye and hand. The histograms in SSC-286 are based on measurements with an accuracy of 0.02 inches. Such a distance represents about 630 psi on the average— an amount which can move an observation into the next higher or lower stress "bin" or category in the histogram. To evaluate this aspect and to facilitate remeasurement of original data when required, a new measurement and reduction process was developed and is described in Section 2.5.
Nonlinear, biased or unaccounted-for effects in the scratch-gauge data were considered. Review of the calibration curves for each gauge indicated that they were linear within the entire range of gauge movements. Another potential source of error would be in the amplification or reduction of actual strains due to the dynamic characteristics of instruments. An answer to this concern as well as several others can be gained from discussions and author's reply that ensued after Yuille presented his paper to RINA, "Longitudinal Strengths of Ships," in 1963 (2). Part of this paper reported on an extensive scratch-gauge program conducted by B.S.R.A.* The gauges used are very similar to the instruments used in the present project. In reply to discussors who raised questions concerning the design of the scratch-gauge itself, Yuille made several points which are pertinent to the present investigation. These points are summarized below:

- With regard to concerns that the gauge possesses dynamic response characteristics that either amplify or damp the actual strains, Yuille stated that a prototype gauge was mounted on a large steel specimen in a Losenhausen fatigue-testing machine, and was found to accurately (within 5%) record strain fluctuating with a range of frequencies that far exceeded the expected higher modal ship response associated with slamming.

* British Ship Research Associates, Wallsend, Northumberland
One discusser, Mr. T. Clarkson was concerned with the effect of local bending using a ten-inch gauge length. He quoted a paper presented to the N.E.C.I.E.S.† ("Measurements and Predictions of the Influence of Deckhouses on the Strengths of Ships," by A. J. Jackson and P. W. Ayling) which indicated that appreciable local bending stresses may exist even for a gauge length of 100 inches. Dr. Yuille noted that the use of a longer gauge length would not eliminate the effects of local bending, although it might increase the accuracy of the measurement. However, by placing the instrument on the web near the neutral axis of a longitudinal girder under the maindeck, Yuille felt that strains other than those of interest were reduced to a minimum.

With regard to temperature effects, Yuille indicated that the gauge, whose "important" parts were made of steel, would extend or contract just as the longitudinal girder upon which it is mounted.

Other possible sources of error are:

- When relating a scratch length to its particular weather condition such as Beaufort Number recorded in the log, it is probable that the scratch mark represents the worst conditions that existed during a four-hour period. This, however, may not correspond to the sea condition at the time a log entry was made.
- The inaccuracies and biases associated with observed wave heights, periods, etc., are obvious. Any of the analyses using observed data must be viewed with caution.
- Grouping of observations (as in histograms) decreases the accuracy of estimated parameters in some of the statistical analyses.
- All instruments (particularly scratch gauge) truncate measurements below some threshold level of sensitivity.

Two distinguishing characteristics of the scratch-gauge data which are important enough to be reemphasized are:

- The scratch-gauge data are not strain response resulting purely from longitudinal vertical bending; they represent components of horizontal and torsional bending as well. There is no way by which to separate the response modes.
- The scratch-gauge data represent the strain response resulting from all sources of loading. There is no explicit technique by which to separate the combination. Furthermore, there is no technique to distinguish contributions from hogging and sagging.

† North East Coast Institute of Engineers and Shipbuilders
2.5 Analysis of the Data: Procedure

The series of scratch marks visible on a paper tape contain three types of information: relative, absolute, and sequential. Relative information, in this case, pertains to the length of each scratch, irrespective of its absolute position on the paper tape. In precise terms, this scratch value represents the maximum positive peak to maximum negative peak stress excursion, symbolized as "p-to-p". The more frequently used expression is maximum peak-to-trough excursion, symbolized as "p-to-t", and this terminology will be used throughout the report, recognizing that the "trough" does not necessarily occur with the "peak" recorded by the scratch mark. It is the peak-to-trough type of information most often produced in full-scale instrumentation programs. Assuming certain conditions are met, the Rayleigh distribution is conveniently employed in the analysis of this type of data. In addition, it is one of the simpler information "elements" obtained from analyses of data. Thus, this relative information has immediate appeal in data studies.

The nature of the absolute type of information is typified by terms such as maximum stress, minimum stress, and mean stress. It requires more knowledge about the conditions under which the measurements are taken, as well as the maintenance of an accurate reference point.

The third type of information is sequential and is related to the relative order of the scratches and to knowledge of the date and time of each mark.

The histograms comprising the information base represent only one of these three types of information: relative. There is no way to extract any of the other types of information. Thus, in order to fully exploit the information potential of the data, a number of marks were remeasured using a digitizing tablet and keeping track of the locations of the marks on the record tape as well as the times and dates. The remeasured data are from the voyages 1-37, McLEAN's starboard gauge.

The digitizing tablet is shown in Figure 2-8. It is accurate to 0.005 inch. Using this device, a data file for each voyage was created. Software was developed to read the data file, which is comprised of an X-coordinate, y-coordinate, and "flag" number representing the location of the cursor on the tablet when one of the four buttons are pressed. The data are converted into stresses. One output is a sequence of values corresponding to the order of the scratch marks: providing information as to the p-to-t, maximum, minimum, and mean stress with respect to the centerline on the paper tape. Another output provides histogram type information, which is also stored in a data file. These data files can be manipulated to form combined data...
FIGURE 2-8. Digitizing Tablet and Four Button Cursor Used for Remeasurement

sets or to combine adjacent scratches. Additionally, the software was developed so that a Beaufort Number is assigned to a scratch measurement, by terminal input, to allow for automatic breakdown of the data by weather. The details of this procedure are presented in Appendix A. Further description of data processing techniques required by certain analyses is presented with a discussion of those analyses throughout this report.

2.6 Port vs. Starboard Gauge Data

Of the eight SL-7 ships instrumented with scratch gauges, the McLEAN is the only ship with a port and starboard gauge. Thus, the majority of the eight-ship data base is composed of starboard-gauge data only. The implications of this fact are considered in this section.

Visual comparison of the port and starboard scratch records for the same time periods indicate that when the ship is encountering waves from the port side, the starboard scratch mark is larger than its corresponding port scratch mark. The reverse is also true.

A comparison of the zeroth, first and second moments and maximum value of the starboard to the port data for each of the first five years was conducted. Table 2-1 shows the results of these calculations. Comparing the port gauge statistics to the starboard shows similar values in most cases; although year 5, for example, represents a significant discrepancy. Also shown are the representative histograms for Data Years 1 and 5. Further consideration of this aspect is given in Chapter 6.
<table>
<thead>
<tr>
<th>DATA YEAR</th>
<th>MOM 0</th>
<th>MOM 1</th>
<th>MOM 2</th>
<th>MAX</th>
<th>MOM 0</th>
<th>MOM 1</th>
<th>MOM 2</th>
<th>MAX</th>
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<tr>
<td>1</td>
<td>4.63</td>
<td>21.02</td>
<td>42.68</td>
<td>37.95</td>
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<td>12.66</td>
<td>25.70</td>
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<td>8.30</td>
<td>16.69</td>
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<td>2.80</td>
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<tr>
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<td>3.14</td>
<td>8.87</td>
<td>18.73</td>
<td>18.13</td>
<td>3.20</td>
<td>11.29</td>
<td>22.17</td>
<td>19.14</td>
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<tr>
<td>5</td>
<td>2.55</td>
<td>10.47</td>
<td>16.98</td>
<td>21.88</td>
<td>3.10</td>
<td>13.51</td>
<td>23.11</td>
<td>26.80</td>
</tr>
</tbody>
</table>

MOM 0 - 0th moment of sample about origin
MOM 1 - 1st moment of sample about origin
MOM 2 - 2nd moment of sample about origin
MAX - maximum value of sample

TABLE 2-1. Comparisons of Scratch Gauge Data McLEAN Port vs. Starboard Gauges (Max Peak-to-Trough Stress-KPSI)

If it is assumed that the ship will experience seas uniformly from all directions, then the accumulation of "under-response" scratches due to asymmetric loading will be offset by "over-response" scratches. Carrying this one step further, as the total data sample becomes larger, the sample average can be assumed to approximate that data sample average which would have been acquired if the single scratch gauge had been mounted on the ship centerline.

2.7 Remarks

The data collection and reduction process upon which five years of scratch data is based is clearly susceptible to some "experimental" error. Each scratch mark represents a complex response to combined loads; and there is no technique to simplify the response or separate the load effects. The principal benefit of mechanical extreme stress recorders is large quantities of inexpensive data. The limitations of the data have been pointed out. Within the scope of these limitations, the measurements generated by the gauges appear valid and the data-reduction process introduces no significant error.
3. CORRELATION WITH STRAIN-GAUGE DATA

It was concluded in SSC-234, that ship stress data could be extrapolated to obtain long-term trends by either of two mathematical models; one based on rms values, and the other using the extreme value of stress amplitude per record. In Chapter 6, the utility of this conclusion will be reevaluated in light of the present data. It is first necessary to correlate the scratch-gauge data and the relevant electrical strain-gauge data, so that both may be applied on an equivalent basis. The majority of electrical strain-gauge analysis in the SL-7 program to date has been based on Longitudinal Vertical Bending Stress (LVBS)*. The scratch-gauge data, however, differs from the LVBS data as a result of the following factors:

- **Location** - The scratch gauges are mounted on the fourth longitudinal stringer down from the deck in the vicinity of frame 184%. LVBS is the average of signals from port and starboard Longitudinal Strain Gauges mounted on the underside of the main deck, frame 186%.
- **Combined Stress Components** - Whereas the LVBS data represent only midship vertical bending, the scratch-gauge data represent contributions from vertical, lateral, and torsional bending.
- **Sampling Time** - The strain-gauge data represent four 20-minute samples per four-hour watch; the scratch-gauge data reflect a four-hour sample.
- **Sampling Type** - The bulk of the strain-gauge data has been reduced so that wave bending and transient higher modal bursts are presented as separate responses. The combined maximum p-to-t excursion is not presented. The scratch data, on the other hand, represents combined sources of loading, as listed in Chapter 2.
- **Data Reduction** - Random errors are introduced in the data reduction process for both sets of data. Additionally, there may be systematic error introduced due to calibration inaccuracy.

Several approaches will be used to correlate scratch and strain-gauge measurements:

**STATISTICAL**

- Linear regression/statistical correlation

---

*LVBS is an electrical combination of longitudinal strain gauges in the port and starboard tunnels mounted on the main deck underside.*
DETERMINISTIC

- Hull structural analysis
- Comparison of calibration data

These three approaches are presented in the subsequent sections.

3.1 Linear Regression

Various data subsets were subjected to linear regression analysis. The electrical strain-gauge data is assigned as the independent variable \((x)\) and the scratch-gauge data is the dependent variable \((y)\). If it is assumed they are linearly related, this relationship is represented as:

\[ y = ax + b \]

In terms of analytical geometry, "a" would represent the slope of a line; "b" would be the y-axis intercept.

Booth (8) performed such an analysis with LVBS and a scratch-data subset of voyages 1-5, and 29, average of port and starboard. In addition to this, the present investigation analyzed several other subsets of data. The results are presented in Table 3-1.

<table>
<thead>
<tr>
<th>VOYAGES</th>
<th>DATA SETS</th>
<th>SCRATCH</th>
<th>STRAIN DATA</th>
<th>(a)</th>
<th>(b)</th>
<th>(r)</th>
<th>(N)</th>
<th>REMARKS</th>
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</thead>
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<tr>
<td>1-9, 29</td>
<td>PORT/STBD AVG</td>
<td>LVBS Stress</td>
<td>0.79</td>
<td>-267</td>
<td>0.91</td>
<td>238</td>
<td>TES (ref 8)</td>
<td></td>
</tr>
<tr>
<td>32</td>
<td>STBD</td>
<td>LVBS Stress</td>
<td>0.87</td>
<td>+886</td>
<td>0.81</td>
<td>61</td>
<td></td>
<td></td>
</tr>
<tr>
<td>60 + 61</td>
<td>STBD</td>
<td>MAX P-to-P (max. of four 20 min. samples)</td>
<td>0.64</td>
<td>-382</td>
<td>0.93</td>
<td>98</td>
<td>scratch zeroes excluded</td>
<td></td>
</tr>
<tr>
<td>60 + 61</td>
<td>STBD</td>
<td>MAX P-to-P (avg of four 20 min. samples)</td>
<td>.76</td>
<td>-710</td>
<td>0.92</td>
<td>98</td>
<td>scratch zeroes excluded</td>
<td></td>
</tr>
</tbody>
</table>

\(a, b\) - coefficients in linear least-squares curve fit

\(r\) - correlation coefficient

\(N\) - sample size

TABLE 3-1. Statistical Correlations - Scratch Gauge Data vs.
Electrical Strain Gauge Data - McLEAN
The results of reference 8 represent the largest data subset, as well as the average of port and starboard scratch readings. The linear relationship was \( y = 0.79x - 267 \). For the present investigation, scratch data from voyages 32E*, 32W, 60W, 61E, and 61W were remeasured for correlation. Voyage 32 scratch data were correlated to LVB Maximum wave-induced p-to-t data. The results show the effect of not including higher modal transient stresses; \( y = 0.87x + 884 \).

The processing of the electrical strain-gauge data presented in the McLEAN's Third Operational Season report (9) included a special reduction which gave the maximum peak-to-trough LVBS excursions per 20-minute sample. This information represents exactly the type of information provided by the scratch marks, i.e. the maximum positive excursion does not necessarily follow the maximum negative excursion; it combines wave bending and transient loads, etc. For each four-hour watch corresponding to a scratch mark, there were four 20-minute strain-gauge samples. The largest of the four values was used for the correlation. The results of this particular analysis \( y = 0.64x - 382 \) seem to reflect the effect of using only the starboard gauge data. Contributions from lateral and torsional bending are thought to be the primary cause for the difference between this correlation and that from reference 8.

### 3.2 Hull Structural Analysis

The most direct approach to determine the scratch/electrical strain-gauge correlation is through straightforward structural analysis of the hull girder. Booth (8) carried out such an analysis for vertical bending. He showed the relation to be

\[
y = 0.77x
\]

in which

- \( x \) = LVB stress
- \( y \) = average P/S scratch stress

These calculations are reproduced in Figure 3-1. The SSC reports on structural analysis of the McLEAN (10-12) were studied in an effort to pinpoint any peculiarities in stress flow in the region of interest; none were identified.

* E = eastward bound, W = westward bound
3.3 Comparison of Calibration Data

This approach involves the comparison of changes in stress for various sensors, including scratch gauges, as loading conditions were systematically varied during the static structural calibration of McLEAN's instrumentation on 9-10 April 1973 in Rotterdam, Holland (13). Table 3-2 shows the magnitude of stress changes from one loading condition to the next for the following sensors: LVB, LSTS, LSTP, port and starboard scratch gauge. It was hoped that this comparison would provide definitive relationships between the scratch gauges and all the other sensors. However, the scratch marks resulting from the induced strains of the calibration experiment were very short and exact changes were difficult to discern. Although the trends between the LST and scratch gauges show good agreement, it is not possible to derive an accurate numerical relationship.

<table>
<thead>
<tr>
<th>LOADING CONDITIONS</th>
<th>PORT</th>
<th>STBD</th>
<th>LVB</th>
<th>AVERAGE SCRATCH</th>
<th>AVERAGE LSTP + LSTS</th>
<th>RATIO SCRATCH STRAIN</th>
</tr>
</thead>
<tbody>
<tr>
<td>From To</td>
<td>Temp °F</td>
<td>Scratch</td>
<td>LSTP</td>
<td>Temp °F</td>
<td>Scratch</td>
<td>LSTS</td>
</tr>
<tr>
<td>1 3</td>
<td>51-59</td>
<td>-814</td>
<td>-1369</td>
<td>52-64</td>
<td>-1445</td>
<td>-1756</td>
</tr>
<tr>
<td>3 4</td>
<td>49</td>
<td>+798</td>
<td>+1281</td>
<td>64-63</td>
<td>+1950</td>
<td>+270</td>
</tr>
<tr>
<td>4 5</td>
<td>49-45</td>
<td>+658</td>
<td>+870</td>
<td>63-52</td>
<td>0</td>
<td>+991</td>
</tr>
<tr>
<td>5 6</td>
<td>45-43</td>
<td>+798</td>
<td>+504</td>
<td>52-46</td>
<td>+650</td>
<td>+675</td>
</tr>
<tr>
<td>6 7</td>
<td>43-60</td>
<td>+798</td>
<td>+504</td>
<td>66</td>
<td>+1300</td>
<td>+2072</td>
</tr>
</tbody>
</table>

LSTP - Longitudinal Stress - Top - Port
LVB - Longitudinal Vertical Bending

TABLE 3-2. Correlations from Static Structural Calibration of Ship Response Instrumentation System - Rotterdam, Holland, 9-10 April 1973
3.4 Remarks

It was shown in Section 2.6 that the port and starboard scratch gauges from the McLEAN produce two data samples with different statistics. This is largely a consequence of a non-uniform distribution of ship-wave relative headings over the sampling period and possibly non-uniform temperature effects. It seems reasonable to assume that over the long term ships generally would experience a uniform distribution of headings, although in some cases a circuitous trade route may be characterized by a consistently one sided ship-wave relative heading. Intuitively, such bias may be introduced when a ship makes "one-way" passages, returning by some other route.

The SI-7's make easterly and westerly transoceanic passages on the same general trade routes. Some time is spent in coastal passages which are typically one-way; however, they represent a small portion of the data sample. Thus, over the long run, a large sample from a starboard gauge only should provide a fair approximation of the vertical bending strains. It is emphasized that, in the short term, a "starboard only" data sample will provide an approximation of vertical bending, since there may be a significant contribution of asymmetric lateral and torsional bending.

In view of the uncertainties associated with the regression analysis, it is recommended that the relationship derived from the hull girder structural analysis be used to relate scratch-gauge stress to LVB stress:

\[
\text{SCRATCH} = 0.77 \text{ LVBS}
\]
4. TYPE I EXTREME VALUE MODEL

4.1 Statistical Models

The aim of the statistical analysis described in this and the following chapter is to construct a statistical model that describes the scratch data base, or subsets and derivations thereof. The purpose of constructing models is to derive objective conclusions about the underlying phenomena (ship response) and to ascertain the degree of uncertainty associated with such conclusions. In this manner, we can systematically evaluate the scratch-gauge data as a basis for extreme load prediction as well as the adequacy of the present data base.

It is suggested that Appendix B, "Extreme-Value Statistics", be reviewed for a better understanding of the following analysis. The Type-I Extreme-Value model is particularly appropriate to the scratch data, and its use with respect to the data is the principal topic of this chapter.

As indicated in Appendix B, the Type-I Extreme-Value model is applicable to initial distributions that are unbounded in the direction of the extreme value and where the initial probability density function decreases at least as rapidly as the exponential function. It follows that the maximum extreme value from a normal, log-normal, gamma, or Weibull distribution is modeled by a Type-I asymptotic distribution. If we assume that the random process of strain excursions in a four-hour period of ship operation can be modeled by one of the above distributions, then the Type-I model may approximate the probability distribution associated with the maximum peak-to-peak strain in a four-hour period. As a sample of four-hour periods becomes larger, then the Type-I asymptotic distribution of extreme values approaches the exact distribution of extremes. Aside from the condition that the initial distribution must be an exponential type, there are two other conditions which are generally applicable to any extreme-value distributions. First, the initial distribution from which the extremes have been drawn and its parameters must remain constant. Secondly, the observed extremes should be extremes of independent data. A complicated situation can be replaced by a comparatively simple asymptotic model if the actual system conditions are compatible with the assumptions of the model.

The method used to estimate the parameters of the extremal distributions are contained in Appendix B. It will be assume a priori that the variates underlying the extreme value records are independent and identically distributed (i.i.d.). The validity of this assumption and the postulated distribution can be judged by a test of fit. Gumbel (14) suggested that the $\chi^2$ and Kolmogorov-Smirnof tests are not appropriate to test extremal distribution fits to observed data.
Probability plotting, however, furnishes a quick and simple method by which to examine the postulate. Additionally, procedures do exist to derive the upper and lower bounds for specified confidence limits. Although the method is essentially subjective, it provides an excellent test of fit for extremal distributions. To test the postulate that a Type-I Extreme-Value model \( G_0(y) \) is appropriate, extremal probability paper will be used.

For large samples, if the plot of data is markedly nonlinear, then there is reason to suspect the postulated distribution \( G_0(y) \). For small samples, the deviations of the sample points from a straight line will usually be more pronounced, even where \( G_0(y) \) is true. There is no definite rule to tell when, for a given sample size, the deviations are large enough to reject the hypothesis \( G_0(y) \). It should also be noted at this point, that like other tests, probability plotting cannot be used to establish the validity of the postulate.

In that the evaluation of the data plot on probability paper is a subjective test, each reader may have different conclusions. The following information is provided to guide the evaluation of such plots. Several possible types of nonlinear plots are shown in Figure 4-1. Figure 4-1a shows a mixture of two distinct populations. Figure 4-1b indicates that the sample may have been censored at both ends. The convex curve shown in Figure 4-1c may suggest that the actual distribution is more skewed to the right than the postulated model. The concave plot of Figure 4-1d may indicate a more negatively skewed underlying distribution.

**FIGURE 4-1. Several Examples of Nonlinear Plots (7)**
4.2 Data Analysis

The initial analysis looks at the Type-I Extreme Value postulate for the two largest data samples - Summary Atlantic Grand Total and Summary Pacific Grand Total, presented in Figures 4.2e and 4.3e, respectively. Additionally, data samples of Progressive Yearly Accumulations are given showing the changing character of the cumulative distribution as the data sample grows larger by yearly increments. The Progressive Yearly Accumulations are presented as Figures 4.2a - 4.2d (Atlantic) and 4.3a - 4.3d (Pacific).

In general, the fact the data plots are not markedly non-linear would indicate that the Type-I Extreme-Value model may represent the data. Strictly speaking, the results do not warrant rejection of the hypothesis that the data is modeled by a Type-I Extreme-Value distribution.

A common characteristic of the data plot is a mild "s" shape. This is a result of the fact that the postulated p.d.f. is not as peaked as the frequency histograms representing the data. This can be seen in Figure 4-4 which shows the Type-I p.d.f. and the Summary Grand Total Atlantic histogram from which its parameters were estimated.

If we can assume at this point a Type-I model is appropriate, we then have a means by which to extrapolate to greater periods of time and make long-term predictions. The first analysis that may provide some indications of the adequacy of our data base, in terms of sample size or time, is to compare the long-term predictions made by the Yearly Accumulations for the same probability or return period. As a basis for such comparisons, we will predict the stress for a return period of 12319 for the Atlantic and 23692 four-hour watches for the Pacific. These values are conveniently chosen to be the actual number of records for which measurable strains were experienced in the first five data years.

Table 4-1 shows the stress predictions from the above analysis. Figures 4-5 and 4-6 also illustrate this analysis. It was hoped that, with each additional year's increment of data, the predicted extreme would converge. As can be seen, no definite trend is apparent. It was also hoped that the predicted extreme value from the postulated model would be a good estimation of the five-year extreme that actually occurred. The actual extreme is certainly within the 95% confidence bounds, although not exactly as predicted.

The second technique is very similar to the first, except that both parameters of the Type-I Extreme Value distribution are examined as the data sample is incrementally increased. It was anticipated that analysis of the location parameter and the scale parameter would provide greater insight into the changes. Table 4-2 shows the parameters. As can be seen, no identifiable trend is apparent.
Figure 4-2b. Data Years 1 and 2 – Atlantic – Type 1 Extreme Value Fit

Figure 4-2a. Data Year 1 – Atlantic – Type 1 Extreme Value

100,000  100,000
  10,000  5,000
  2,000  1,000
  0,200  100
    0

100,000  100,000
  10,000  5,000
  2,000  1,000
  0,200  100
    0
Figure 4-2c. Data Years 1, 2 and 3 - Atlantic - Type 1 Extreme-Value Fit

Figure 4-2d. Data Years 1, 2, and 4 - Atlantic - Type 1 Extreme-Value Fit
FIGURE 4-2e. Summary Grand Total - Atlantic - Type 1 Extreme Value Fit
Figure 4-3a. Data Year 1 - Pacific - Type 1 Extreme-Value Fit

Figure 4-3b. Data Years 1 and 2 - Pacific - Type 1 Extreme-Value Fit
Figure 4-3c. Data Years 1, 2 and 3 - Pacific - Type 1 Extreme-Value Fit

Figure 4-3d. Data Years 1, 2, 3 and 4 - Pacific - Type 1 Extreme-Value Fit
FIGURE 4-3e. Summary Grand Total - Pacific - Type 1 Extreme Value Fit

-SUMMARY GRAND TOTAL-
PACIFIC

Number of observations: 23683
Type 1 Extreme Value Fit
FIGURE 4-4. Comparison of Scratch-Gauge Data to Type-I Extreme-Value Probability Density Function for Summary Grand Total Pacific

ATLANTIC DATA
estimated for 12319 four-hour watches

<table>
<thead>
<tr>
<th>DATA SET (YEARS)</th>
<th>MUST LIKELY VALUE</th>
<th>LOWER 2.5% CONTROL VALUE</th>
<th>UPPER 2.5% CONTROL VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>32.24</td>
<td>28.40</td>
<td>43.07</td>
</tr>
<tr>
<td>1+2</td>
<td>30.04</td>
<td>26.43</td>
<td>40.22</td>
</tr>
<tr>
<td>1+2+3</td>
<td>28.06</td>
<td>24.68</td>
<td>37.57</td>
</tr>
<tr>
<td>1+2+3+4</td>
<td>27.67</td>
<td>24.35</td>
<td>37.06</td>
</tr>
<tr>
<td>All 5</td>
<td>28.43</td>
<td>25.00</td>
<td>38.09</td>
</tr>
</tbody>
</table>

PACIFIC DATA
estimated for 23692 four-hour watches

<table>
<thead>
<tr>
<th>DATA SET (YEARS)</th>
<th>MUST LIKELY VALUE</th>
<th>LOWER 2.5% CONTROL VALUE</th>
<th>UPPER 2.5% CONTROL VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>37.32</td>
<td>32.54</td>
<td>50.76</td>
</tr>
<tr>
<td>1+2</td>
<td>34.22</td>
<td>29.86</td>
<td>46.49</td>
</tr>
<tr>
<td>1+2+3</td>
<td>33.95</td>
<td>29.63</td>
<td>46.12</td>
</tr>
<tr>
<td>1+2+3+4</td>
<td>33.83</td>
<td>29.08</td>
<td>45.98</td>
</tr>
<tr>
<td>All 5</td>
<td>34.42</td>
<td>30.026</td>
<td>46.77</td>
</tr>
</tbody>
</table>

TABLE 4-1. Long-Term Estimates Using Type-I Extreme-Value Distribution for Yearly Accumulations of Scratch Data
Figure 4-5. Atlantic Statistical Models Yearly Accumulations - 1st Five Years - Type 1 Extreme-Value Fit

Figure 4-6. Pacific Statistical Models Yearly Accumulations - 1st Five Years - Type 1 Extreme-Value Fit
The preceding analysis suggests that there may be insufficient data. It may also indicate that the data may not conform to the conditions necessary for the application of the Type-I Extreme-Value model. A systematic evaluation of the adequacy of the data base will be presented in Chapter 9, and will be based only in part on the preceding analysis. However, it is worthwhile to look now at the possible nonconformance of the data base to the conditions required by extreme-value models.

Recall that the first essential condition is, in Gumbel's words, "that the initial distribution from which the extremes have been drawn, and its parameters, remain constant, from one sample to the next, or that changes which have occurred, or will occur, may be determined and eliminated." (14) The second condition is that the observed extremes should be extremes of samples of independent data. Regarding the second condition, it should be noted that extreme-value methods have been shown to be very robust against dependence. However, the use of four-hour samples may be severely straining the limits of robustness. In the following analysis, a portion of the data base has been re-measured by digital tablet in order to evaluate the impact of nonconformity to the above conditions. The original scratch marks of the first 37 voyages of the McLEAN (Starboard Gauge) were associated with the actual data and time. The logbook of the McLEAN provided information as to visually estimated wave conditions and Beaufort Number. Histograms of four-hour extremes were then developed for each Beaufort Number to be used in the analysis described in the following section.

### TABLE 4-2. Parameters of Type-I Extreme-Value Distribution for Yearly Accumulations of Scratch Data

<table>
<thead>
<tr>
<th>DATA SET (YEARS)</th>
<th>U</th>
<th>α</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.8878</td>
<td>0.2735</td>
</tr>
<tr>
<td>1+2</td>
<td>2.7782</td>
<td>0.2996</td>
</tr>
<tr>
<td>1+2+3</td>
<td>2.7880</td>
<td>0.3022</td>
</tr>
<tr>
<td>1+2+3+4</td>
<td>2.7409</td>
<td>0.3029</td>
</tr>
<tr>
<td>All 5</td>
<td>2.7497</td>
<td>0.2975</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>DATA SET (YEARS)</th>
<th>U</th>
<th>α</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.5843</td>
<td>0.3396</td>
</tr>
<tr>
<td>1+2</td>
<td>2.1629</td>
<td>0.3613</td>
</tr>
<tr>
<td>1+2+3</td>
<td>1.9984</td>
<td>0.3865</td>
</tr>
<tr>
<td>1+2+3+4</td>
<td>1.9643</td>
<td>0.3997</td>
</tr>
<tr>
<td>All 5</td>
<td>1.9603</td>
<td>0.3805</td>
</tr>
</tbody>
</table>
4.3 Examination of the Identically Distributed Condition

Consider the 12000+ records which comprise the Summary Atlantic Grand Total. The assumption that the underlying distributions from which each extreme value was taken are identically distributed seems intuitively suspect.

For example, during one four-hour period, if the ship remained on a constant course, at a constant speed, at a constant draft and ballast condition, and experienced an unchanging moderate sea condition, it would be generally accepted that the initial p.d.f. underlying the scratch mark recorded during that period would be Rayleigh distributed. On the other hand, the underlying distribution for the ship in severe seas, experiencing high transient loading from flare shock or slamming, along with ballast shifts, and course and speed changes would probably be poorly modeled by the Rayleigh distribution.

Intuitively, the underlying distribution would be more identical if they were grouped according to sea severity. This was done by Beaufort Number, and the samples were then plotted on extremal probability paper. The results are presented in Figures 4-7a to 4-7j. Some of the sample sizes are small, and random deviations are to be expected. Nevertheless, in general, the plots appear to be quite linear. Classifying the data by weather conditions allows for the prediction of lifetime extreme values using the concept of conditional probability of weather.

4.4 Examination of the Independence Condition

The remeasurement of the data provided a sequential list of stresses in the relative order in which they occurred. From this data set, three subsets of data were derived; 8-Hour-Maxima, 16-Hour-Maxima, and 24-Hour-Maxima. To derive the 24-Hour-Maxima sample, for example, six adjacent scratch marks were measured, the largest of which would be used. These data sets were plotted on extremal probability paper. The results are presented in Figures 4-8a to 4-8c. Figure 4-8d shows the three together.

It was hoped that the Type-I line fits would be parallel in these plots, from which it could be deduced that the independence condition was fulfilled. This is not the case as can be seen from Figure 4-8d. However, the obvious divergence of lines may be a result of factors other than non-independence, e.g. non-fulfillment of the identically distributed initial distribution condition or inadequate sample size.
Figure 4-7a. Beaufort Number 2 Stress Occurrences - Type I Extreme Value Fit

Figure 4-7b. Beaufort Number 3 Stress Occurrences - Type I Extreme Value Fit
Figure 4-7c. Beaufort Number 4 Stress Occurrences - Type 1 Extreme-Value Fit

Figure 4-7d. Beaufort Number 5 Stress Occurrences - Type 1 Extreme-Value Fit
Figure 4-7e. Beaufort Number 6 Stress Occurrences - Type 1 Extreme-Value Fit

Figure 4-7f. Beaufort Number 7 Stress Occurrences - Type 1 Extreme-Value Fit
Figure 4-7g. Beaufort Number 8 Stress Occurrences - Type 1 Extreme-Value Fit

Figure 4-7h. Beaufort Number 9 Stress Occurrences - Type 1 Extreme-Value Fit
Figure 4-71. Beaufort Number 10 Stress Occurrences - Type 1 Extreme Value Fit

Figure 4-72. Beaufort Number 11 Stress Occurrences - Type 1 Extreme Value Fit
Figure 4-8a. Extreme Value 8-Hour Intervals - Type 1 Extreme-Value Fit

Figure 4-8b. Extreme Value 16-Hour Intervals - Type 1 Extreme-Value Fit
Figure 4-8c. Extreme Value 24-Hour Intervals - Type 1 Extreme-Value Fit

Figure 4-8d. Composite of Type-I Extreme-Value Model Fits to 8, 16 and 24 Hour Maxima of Voyages 1 - 37, McLean, Starboard Scratch Gauge
Two other sets of extreme values were plotted. The maximum extreme value for each voyage for all ships were derived from the working lists used to develop the original histograms. Additionally, the maximum extreme values for each year (all ships) by ocean was taken from the histograms. Figures 4-9 and 4-10 show the Atlantic and Pacific voyage maxima plotted. These plots of Voyage Maxima are visually excellent fits. The Yearly Maxima, on the other hand, were not plotted due to the very small sample size. The concept of yearly maxima has great appeal in that it is the standard period of observation of meteorological and related phenomena. However, five data points are simply inadequate.

4.5 Remarks

Numerous subsets of data have been analyzed. Each set can be used in some capacity to predict extreme loads. In order to give the strongest possible theoretical justification for inferences beyond the range of measured values, the data sets which best fulfill the necessary conditions of the extreme value theory were selected. The data subsets which are thought to be the most appropriate are the ones classified by Beaufort Number and also the maximum value per voyage (Voyage Maxima). In each of these cases, the underlying phenomenon which generates the extreme value should be described by identically distributed probability functions. Using either of these two data sets, lifetime extreme values can be predicted according to the following two procedures:

(a) Using the single maximum value from each voyage, fit the Type-I E.V. Model to the data (graphically or numerically) and extrapolate to a "return period" equal to the lifetime number of voyages.

(b) Using the four-hour maxima broken down by sea severity, fit the Type-I E.V. Model to the data associated with each of the more severe sea conditions. Estimate the number of four-hour watches associated with each of the sea conditions and extrapolate to a return period equal to the respective number of watches. Of these, choose the highest value. The worst sea condition may not necessarily yield the highest value due to its low number of encounters.

Each of these methods are evaluated below. The Beaufort Number subsets of data were used from Voyages 1-37 of the McLEAN. Beaufort Numbers 8-11 were considered. The number of watches associated with each Beaufort Number were determined from the logbook for these voyages. The extremal probability plots (Figures 4-7g - 4-7j) were used and the Type-I line on these plots were used to determine the
VOYAGE EXTREME
ATLANTIC
Number of observations: 331
Type 1 Extreme Value Fit

FIGURE 4-9. Voyage Extreme Atlantic – Type 1 Extreme Value Fit
FIGURE 4-10. Voyage Extreme Pacific - Type 1 Extreme Value Fit

VOYAGE EXTREME
PACIFIC
Number of observations: 746
Type 1 Extreme Value Fit
The most likely p-to-t value associated with a return period equal to the number of watches for each Beaufort Number with the following results:

<table>
<thead>
<tr>
<th>Beaufort Number</th>
<th>Most Likely p-to-t Stress (kpsi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>28</td>
</tr>
<tr>
<td>9</td>
<td>28</td>
</tr>
<tr>
<td>10</td>
<td>24</td>
</tr>
<tr>
<td>11</td>
<td>22</td>
</tr>
</tbody>
</table>

Thus, the largest expected value is 28 kpsi. These values are essentially derived from data of 35 voyages (1-37 less 30 and 31). Entering the extremal plot for Atlantic Voyage Maxima (Figure 4-9) with a return period of 35 voyages, the most likely value is 27.9 kpsi. The fact that these two values are very close suggests that the two methods are based on a proper theoretical foundation.
This chapter presents an evaluation of Atlantic and Pacific Five Year Summary Totals using various statistical models, with the exception of the Type-I extreme value model. The Type-III extreme value model is evaluated using the Voyage Maxima for the Pacific.

5.1 Four-Parameter Expression Proposed by M. K. Ochi

This new expression has been proposed by Dr. M. K. Ochi, and the best description of the motivation for its development is presented by Ochi and Whalen (41). The main points are summarized below:

- The log-normal probability law describes the data well, except at the higher values.
- The Weibull distribution describes the data well, except at the lower values, where it is a poor representation.
- The data for the higher values, above \( F(x) > 0.99 \) maybe extremely unreliable.
- A probability function that precisely represents the data over the entire range for the cumulative distribution up to about 0.99 would be most accurate for extrapolation to extreme values.
- In general, the cumulative distribution function \( F(x) \) is expressed as:

\[
F(x) = 1 - e^{-q(x)}
\]

where, \( q(x) \) = monotonically increasing real function.

- The new expression is to represent \( q(x) \) as:

\[
q(x) = a x^m e^{-px^k}
\]

- The probable extreme value \( \bar{y}_n \) in \( n \) observations, for large \( n \), is:

\[
\bar{y}_n = q^{-1}(\ln n)
\]

where \( q^{-1} \) is the inverse function of \( q(x) \).

The constants involved in the function \( q(x) \) are determined numerically by using a non-linear least squares fitting method. Figure 5-1 shows the Atlantic and Pacific Data plotted on log-log graph paper and the estimated four-parameter distributions that fit the data. Evaluation of the distribution for the Atlantic Data at 5, 10, 20, and 30 years of life based on 12319 watches per five years, is 36, 38.2, 41.0, and 42 kpsi, respectively.
5.2 Log-Normal Distribution

The data are plotted on probability-log paper to evaluate the postulate that the data was modeled by the log-normal distribution. The plots shown in Figure 5-2 provide the basis to reject this hypothesis.

5.3 Weibull Distribution (2-parameter)

The data are plotted on Weibull probability paper. As can be seen from Figure 5-3, the plot is fairly linear, and there is no cause to reject the Weibull model.
Figure 5-2. Atlantic and Pacific Summary Data Plotted on Probability Paper

5.4 Type-III Extreme Value Distribution

A glance at the data plots in Chapter 4 will show that most of the data sets do not suggest an upper stress limit. In fact, many of them seem to fall off in the "wrong" direction. However, the Voyage Maxima Pacific, Figure 4-10, appears to be amenable to a Type-III Extreme Value Fit:

\[ H(y_n) = \exp \left\{ \left( \frac{b-y_n}{b-v} \right)^k \right\} \]

in which \( b \) is the upper bound, \( k \) is a shape parameter, and \( v \) is the value of \( y_n \) at \( H(y_n) = 0.368 \). The parameters were estimated using a
graphical procedure (see Appendix A). The Type-III fit is shown in Figure 5-4. The graphical procedure requires the selection of arbitrary points and by choosing a different set of points, the parameters can be manipulated.

A numerical procedure is also given in Appendix A. However, convergence is usually difficult to obtain using this method.

FIGURE 5-3. Weibull Fit
Number of Observations: 746
Type 3 Extreme Value Fit

b = 45  v = 2.7497  k = 5.5

FIGURE 5-4. Type-III Fit of Voyage Maxima Pacific - All Voyages
6. SCRATCH GAUGES: AN ALTERNATIVE TO ELECTRICAL STRAIN GAUGES

It was concluded by Hoffman, et. al. (6) that "ship stress data can be analyzed and extrapolated by either of two mathematical models, one using rms values and the other extreme values of regularly recorded stress records". The value of the scratch-gauge data partly depends on its utility with respect to the above conclusion, and this chapter will evaluate the data using methods discussed in Reference 6.

6.1 Comparison of Extrapolations Based on Electrical and Mechanical Strain Measurements

The data subset for this analysis represents those voyages classified as the "second operational season" of the S.S. SEA-LAND McLEAN in North Atlantic Service. Specifically, this includes voyages 25-29 and 32-37. (Voyage numbers skipped from 29 to 32). Cumulative distributions of the following samples were compared:

- (Electrical) Wave-Induced Maximum p-to-t LVBS
- Scratch-Gauge Port
- Scratch-Gauge Starboard

These data samples are compared graphically in Figure 6-1. The format is the familiar long-term trend plot relating stress to probability of exceedance. The port scratch data is plotted with several representative starboard gauge points included. Port and starboard gauge data fall together except for a few of the highest value data points. The scratch data were corrected for location so that it corresponds to the LVB stress, in accordance with the recommendation of Section 3.4. The plot of the scratch data represents four-hour maxima whereas the plot of the LVB data represents 20-minute maxima. Thus, there are twelve LVB records for one scratch record for an equivalent period of time. Additionally, the LVB curve represents wave-induced maximum amplitudes only, whereas the scratch curve represents a variety of loading sources. The figure shows that the LVBS and scratch curves are separated by approximately log 12 at very low probability levels. As pointed out in SSC-234, as the probability of exceedance approaches zero, the separation in our case would be exactly log 12. This would be true, of course, if the scratch represented only wave-induced double-amplitude vertical bending stress. The lines drawn through the data points are visual fits to the data. The graph suggests that extrapolated lines to higher stress values would be roughly parallel, as would be expected. The dotted line (Curve 3) in Figure 6-1 represents a theoretical calculation of the cumulative distribution for all stress cycles. The calculations for this curve have been carried out in connection with the SSC project entitled "Fatigue Considerations in View of Measured Load Spectra", SR-1254.
FIGURE 6-1. Long-Term Trend on Longitudinal Vertical Bending Stress for McLEAN based on Second Operational Season, Voyages 25-37
It was shown in SSC-234 that the curve of "all stresses" could be analytically related to the curve of "extremes" for the lower exceedance probabilities. This capability suggested that the analytically derived long-term distributions, e.g. Curve 3, could be checked and validated using experimentally acquired record extremes e.g. scratch data, rather than a complete count of all excursions, e.g. electrical strain-gauge records.

Figure 6-1 certainly lends some support to the above method and illustrates the approximate relationships whereby scratch-gauge data can be operated on to produce a curve similar in form to that acquired by the electrical strain-gauge instrumentation.

The same data are also presented in Figure 6-2. The data are plotted on extreme probability paper and Type-I extreme-value distributions are fit to the data. Ideally, for an equivalent stress value, the return period for the LVB data should be twelve times greater than the scratch data. This discrepancy results from scratch-to-LVB transformation error, stress contributions other than wave-induced bending, and possible bias from Type-I distribution parameter estimation.

Table 6-1 was constructed using the parameters estimated for the Type-I extreme-value distribution. It compares projected lifetime extreme peak-to-trough longitudinal vertical bending stresses from scratch data and LVB data. The port and starboard gauges provide very similar projections. The LVB stress data provide a lesser estimate, as expected, being 88% of the average scratch projections.

<table>
<thead>
<tr>
<th>DATA SAMPLE</th>
<th>SAMPLE SIZE</th>
<th>MOST LIKELY ESTIMATE IN SHIP'S LIFE* KPS1</th>
<th>LOWER 2.5% CONTROL POINT KPS1</th>
<th>UPPER 2.5% CONTROL POINT KPS1</th>
</tr>
</thead>
<tbody>
<tr>
<td>LVB Stress Maximum P-to-T Wave Induced</td>
<td>2081</td>
<td>56.21</td>
<td>50.80</td>
<td>71.44</td>
</tr>
<tr>
<td>SCRATCH Starboard Corrected to LVB Location</td>
<td>523</td>
<td>63.51</td>
<td>56.14</td>
<td>84.30</td>
</tr>
<tr>
<td>SCRATCH PORT Corrected to LVB Location</td>
<td>673</td>
<td>64.03</td>
<td>56.34</td>
<td>85.13</td>
</tr>
</tbody>
</table>

* Ship's Lifetime 10000 4 hr. watches or 360000 20 min. samples. The most likely value is the most probable value.
† Upper and lower control points represent the limits of a 95% confidence interval. Thus, there is 95% assurance that the value will be between these control points.

TABLE 6-1. Comparison of Long-Term Estimated Stress Values From Scratch and Electrical Strain Gauge by Fitted Type-I Extreme Value Distribution – McLEAN – Second Season
FIGURE 6-2. McLEAN Second Operational Season - Type-I Extreme Value Fit
6.2 Remarks

It has been shown that the scratch data can be applied in the manner suggested in SSC-234, comparing well in shape to the electrical strain-gauge extreme data and theoretically derived curve. It cannot be relied upon, however, as an accurate validation tool for wave-induced bending data. The principal reason is that the extrapolation of the curve is strongly affected by the last few points representing the highest strains. The highest several strains as obtained from scratch data probably consist largely of strain response from flare shock, bottom slamming or green water. Thus extrapolation using the scratch data is suitable for validation support with theoretically predicted values which represent combined loading and response. (Of course the designer of a SL-7 type ship would be interested in combined loading. Its value to the designer is discussed in Chapter 9.) With respect to the use of the data base to validate theoretically predicted wave-induced loads only, it is not appropriate.
7. MISCELLANEOUS APPLICATIONS

7.1 Mean Stress

One of the most difficult pieces of information to acquire in the SL-7 instrumentation program is the still-water bending stress. In order to define the still-water stress conditions at the beginning of each voyage, the complete loading condition, including all liquid tanks, stores and cargo, must be known in detail. This information is difficult to obtain in most cases. Nevertheless, mean-stress information is particularly necessary to any investigator attempting to determine the actual combined loading on ship structure. It was hoped that the scratch-gauge records could provide the capability to determine this source of loading for a three-month period (length of one paper tape) on the average. This section describes an evaluation of this application.

In the SL-7 stress records, information on the initial mean stress was lost when the instrumentation was reset to zero at the beginning of each voyage, or when required during the voyage due to instrument saturation. In this analysis, the times at which the instrumentation was zeroed were extracted from the electrical stress records. The point on the scratch-gauge paper tape (McLEAN-starboard) corresponding to the "zero" time was located. At this point, the scratch-mark measurements were taken with the digitizing tablet, using the middle horizontal line of the paper tape as the common reference point between tapes in the folders. This analysis was attempted for voyages 25-38. In some cases, the scratch record at "zero" time showed no dynamic stress. In most cases, however, the ship was in a seaway. Thus, it was necessary to estimate the mean stress. A hogging/sagging ratio of 1:1.2 was used to estimate the mean stress from the scratch measurement. This ratio resulted from analysis of SL-7 electrical sensor data performed by Dalzell et al (16). The results of this effort are summarized in Table 7-1. The last column on the right represents the stress variations at the location of the starboard scratch gauge from that which existed at the beginning of 32 W. This voyage was chosen because the LVB stress has been calculated based on loading conditions (15). It has been estimated that the initial mean LVB stress for 32 W was 6481 psi; corrected to scratch gauge location 4990 psi.

It is quite obvious, however, that the variations in mean stress as shown in Table 7-1 are large and do not appear credible. Further analysis of the paper tapes and the procedure used indicates several difficulties associated with using the scratch paper tape as a continuous long-term record of mean stress.

It appears that the scratch paper tape itself slowly migrates on its spool. Also, it is difficult to be sure, in many cases, that the paper-tape segments in the record folders are from the same roll. It is also difficult to identify which scratch corresponded to which time.
TABLE 7-1. Mean Stress As Derived From Scratch-Gauge Paper Tape - McLEAN Starboard Gauge

The tapes were annotated with time and date at various points. But, there are occasionally discrepancies in the sequencing of scratch marks; sometimes an extra scratch between two annotations, sometimes too few.

In short, there are numerous difficulties associated with using the scratch data for the long-term tracking of mean stress. Despite these problems, it is believed that the scratch-gauge records can provide long-term mean stress data if considerable effort is expended to account for the difficulties just discussed. Additionally, reduction of the scratch-gauge tapes for mean-stress analysis could be enhanced through the following actions:

- Annotate the tape at the time the ship finishes loading and time the ship gets underway.
- Do not cut the paper tape for placement in folders. Leave it as a roll.
- Clearly annotate the end of one tape and the beginning of another when the roll is changed.
- Take care not to disturb the instrument stylus arm when changing paper.

<table>
<thead>
<tr>
<th>McLEAN Voyage</th>
<th>Date</th>
<th>Time GMT</th>
<th>Absolute Maximum (psi)</th>
<th>Absolute Minimum (psi)</th>
<th>Mean Stress Measured (psi)</th>
<th>Mean Stress Corrected (psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>25E</td>
<td>9/23/73</td>
<td>0400</td>
<td>-456</td>
<td>-1732</td>
<td>-1221</td>
<td>+1016</td>
</tr>
<tr>
<td>25W</td>
<td>10/1/73</td>
<td>1200</td>
<td>-279</td>
<td>-538</td>
<td>-2775</td>
<td>+596</td>
</tr>
<tr>
<td>26E</td>
<td>10/9/73</td>
<td>0400</td>
<td>-164</td>
<td>+596</td>
<td>-1641</td>
<td>+745</td>
</tr>
<tr>
<td>26W</td>
<td>10/17/73</td>
<td>0800</td>
<td>-732</td>
<td>-2006</td>
<td>-1492</td>
<td>+3872</td>
</tr>
<tr>
<td>27E</td>
<td>10/22/73</td>
<td>2400</td>
<td>-184</td>
<td>+589</td>
<td>-1868</td>
<td>+4085</td>
</tr>
<tr>
<td>28E</td>
<td>11/29/73</td>
<td>1600</td>
<td>3256</td>
<td>549</td>
<td>+1635</td>
<td>+3832</td>
</tr>
<tr>
<td>28W</td>
<td>12/6/73</td>
<td>1600</td>
<td>-32</td>
<td>+1615</td>
<td>+1615</td>
<td>+3832</td>
</tr>
<tr>
<td>29E</td>
<td>12/11/73</td>
<td>2000</td>
<td>2800</td>
<td>1206</td>
<td>+1847</td>
<td>+4085</td>
</tr>
<tr>
<td>29W</td>
<td>12/18/73</td>
<td>2000</td>
<td>4839</td>
<td>-2836</td>
<td>+233</td>
<td>+2670</td>
</tr>
<tr>
<td>32E</td>
<td>12/30/73</td>
<td>1200</td>
<td>0</td>
<td>692</td>
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<td></td>
</tr>
<tr>
<td>32W</td>
<td>1/8/74</td>
<td>1500</td>
<td>2949</td>
<td>-5663</td>
<td>-2737</td>
<td>0</td>
</tr>
<tr>
<td>33E</td>
<td>1/17/74</td>
<td>1600</td>
<td>1200</td>
<td>-5663</td>
<td>-2918</td>
<td>-680</td>
</tr>
<tr>
<td>33W</td>
<td>1/23/74</td>
<td>NOT ZERED</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>34E</td>
<td>1/29/74</td>
<td>2000</td>
<td>1827</td>
<td>877</td>
<td>+1280</td>
<td>+3517</td>
</tr>
<tr>
<td>34W</td>
<td>2/5/74</td>
<td>2000</td>
<td>0</td>
<td>+686</td>
<td>+2923</td>
<td></td>
</tr>
<tr>
<td>35E</td>
<td>2/12/74</td>
<td>1600</td>
<td>-232</td>
<td>-3471</td>
<td>-2370</td>
<td>-133</td>
</tr>
<tr>
<td>35W</td>
<td>2/20/74</td>
<td>1600</td>
<td>1121</td>
<td>-2393</td>
<td>-987</td>
<td>+1250</td>
</tr>
<tr>
<td>36E</td>
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<td>2000</td>
<td>690</td>
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<td>-107</td>
<td>+2130</td>
</tr>
<tr>
<td>37E</td>
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<td>4255</td>
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<td>-772</td>
<td>+1500</td>
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<td>37W</td>
<td>3/21/74</td>
<td>0800</td>
<td>-3151</td>
<td>-4905</td>
<td>-4200</td>
<td>-1963</td>
</tr>
</tbody>
</table>
7.2 Short-Term Statistics

Most long-term prediction methods are based on probability density functions describing short-term steady-state ship response. It is generally accepted that short-term records of wave and ship response can be satisfactorily described using the Rayleigh probability distribution. This is because it can be assumed that a 15 - 30 minute record of the phenomena will yield a statistical distribution of maxima and minima that conform to the following conditions:

- Random process is a steady-state Gaussian (normal) process with zero mean.
- Process has a narrow-band spectrum.
- Maxima are statistically independent.
- Linearity must hold between input and output processes when the input process satisfies the previously stated conditions.

Many investigators have found that these assumptions are valid in light of the fact that the Rayleigh distribution does indeed satisfactorily represent the observed short-term distribution of wave heights and related phenomenon, i.e., ship motions and bending moment responses. A brief review of the conditions to be considered in short-term analysis is presented in support of subsequent evaluations.

Non-narrow Band

If one of the required conditions is not satisfied, then the Rayleigh distribution may no longer be applicable for predicting statistical properties of the maxima. For example, if the condition regarding the narrow-band property of a spectrum is removed, then maxima are predicted by using a probability function that carries a parameter representing the bandwidth of a spectrum. The spectra of many random phenomena observed in practice cover a certain range of frequencies. Often they have several maxima and minima during one cycle as determined from zero crossings. This contrasts to a single peak and trough for a narrow-band random process. This can have a significant impact on the prediction of extreme values.

A bandwidth parameter (\( \varepsilon \)) of the spectrum was defined by Cartwright and Longuet-Higgins (17),

\[
\varepsilon = \sqrt{1 - \frac{m_2^2}{m_0 m_4}}
\]

where,

- \( m_0 = \int S(\omega) d\omega \) = zeroth moment of spectrum \( S(\omega) \)
- \( m_2 = \int \omega^2 S(\omega) d\omega \) = second moment
- \( m_4 = \int \omega^4 S(\omega) d\omega \) = fourth moment
- \( S(\omega) \) = ordinate of power density spectrum of the random process
- \( \omega \) = frequency
Figure 7-1 illustrates the variation in probability density function of a random variable $\eta$ as a function of bandwidth parameter $\epsilon$. With the change in distributions, the characteristics of extreme values will also vary. Ochi (18) provides a formula to determine extreme values as a function of $c$ in terms of either number of observations or time.

**Statistical Dependence**

If the condition concerning statistical independence of maximum values is removed, spectral analysis of the Gaussian process can still be used. Ochi (19) discusses the impact of statistical dependence of maxima on extreme values of a random process.

**Non-Gaussian Process**

For a non-Gaussian random process, a mathematical relationship between the spectrum of a random process and the probability function for the maxima has yet to be developed. Statistical prediction of maxima, in this case, must be achieved through random sampling of the maxima, since the probability function of the maxima is no longer Rayleigh.

Figure B-2 illustrates the comparison between a Rayleigh, Weibull, and a Generalized Gamma Distribution. How these distributions impact on extreme values is illustrated in Figure B-3. As can be seen, a significant variation exists in the extreme values predicted. This emphasizes the importance of correct assumptions when statistically evaluating random processes.

Using the remeasured data from voyages 1 - 37 of the McLEAN, classified by Beaufort Numbers, histograms have been derived and are presented in Figure 7-2. This type of presentation is typically used to illustrate the suitability of a normal distribution to describe rms response values conditional on weather. In SSC-234, such a presentation was used to show that the extreme p-to-t stress values per 20-minute

*B-2 is Figure 2 in Appendix B*
FIGURE 7-2. Histogram of Peak-to-Trough Stress vs. Beaufort Number for Voyages 1-37, McLEAN, Starboard Scratch Gauge.

record for weather groups was better approximated with Longuet-Higgins curves. The scratch-gauge data can obviously be used to carry out similar demonstrations; it must be kept in mind, however, that combined dynamic loads are represented, in contrast to wave-induced response only.

Another analysis was conducted to evaluate the postulate that the most severe strain response per voyage occurred in the most severe sea condition. Several McLEAN voyages were chosen arbitrarily—Voyages 4, 9, 26, 27, 29, 34, and 36. The procedure was to visually select the largest scratch marks on the original record and determine the times to which they correspond. By referring to the logbook, the conditions at the time of the mark were noted. The log data for the entire voyage were then examined to determine if a larger or more severe sea condition occurred than those associated with the selected scratch marks. The results are shown in Table 7-2. In two cases, the maximum strain occurred in the maximum sea. In other cases, larger seas occurred but did not generate the extreme strains for that voyage.

Another application of the scratch data to short-term statistical analysis is to validate the theoretically predicted extreme values for short-term steady-state conditions. A proposed procedure is to postulate the p.d.f. associated with the conditions occurring during the four-hour period, then evaluate the exact extremal distribution associated with the initial p.d.f. A comparison to the actual extreme value as provided by the scratch measurement to the predicted value would help assess the validity of the postulate.
<table>
<thead>
<tr>
<th>Voyage</th>
<th>Remarks</th>
<th>Stress</th>
<th>Index No.</th>
<th>Speed Knots</th>
<th>Beaufort S.S. No.</th>
<th>Rel. Wave Direction</th>
<th>Average Wave Ht.</th>
<th>SWELL</th>
<th>Ht.</th>
<th>Direction</th>
</tr>
</thead>
<tbody>
<tr>
<td>- 4 -</td>
<td></td>
<td>17,206</td>
<td>325</td>
<td>25</td>
<td>9</td>
<td>8P WSW</td>
<td>10-12</td>
<td>10-12</td>
<td>208</td>
<td>W</td>
</tr>
<tr>
<td>1,2,3</td>
<td>pitching</td>
<td>14,036</td>
<td>314</td>
<td>29</td>
<td>8</td>
<td>0 W</td>
<td>5</td>
<td>10-12</td>
<td>158</td>
<td>W</td>
</tr>
<tr>
<td></td>
<td>wave -</td>
<td>14,036</td>
<td>366</td>
<td>23</td>
<td>10</td>
<td>315 WNW</td>
<td>15</td>
<td>15</td>
<td>32.15</td>
<td>W</td>
</tr>
<tr>
<td></td>
<td>spray over bridge</td>
<td>14,036</td>
<td>11/12 - 0000 - not in log book</td>
<td>one case of heavier seas (10) with ship rolling heavy</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>- some Beaufort No. but wave ht. less (2-3 ft.)</td>
<td></td>
<td>9</td>
<td>7656</td>
<td>54</td>
<td>25</td>
<td>8</td>
<td>145S W</td>
<td>5</td>
<td>10-12</td>
<td>145S NE</td>
</tr>
<tr>
<td></td>
<td></td>
<td>7656</td>
<td>55</td>
<td>25</td>
<td>8</td>
<td>145S W</td>
<td>5</td>
<td>10-12</td>
<td>145S NE</td>
<td></td>
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<tr>
<td></td>
<td></td>
<td>9570</td>
<td>26</td>
<td>35</td>
<td>10</td>
<td>130P NW</td>
<td>15</td>
<td>15</td>
<td>107.5P NW</td>
<td></td>
</tr>
<tr>
<td></td>
<td>- worst weather of voyage</td>
<td></td>
<td>8932</td>
<td>25</td>
<td>30</td>
<td>8</td>
<td>152P NW</td>
<td>10</td>
<td>15</td>
<td>122P NW</td>
</tr>
<tr>
<td>- some higher Beaufort No. 6 - wave ht. 5 ft.</td>
<td></td>
<td>27</td>
<td>4466</td>
<td>7</td>
<td>32.8</td>
<td>4</td>
<td>146S NW</td>
<td>3</td>
<td>5</td>
<td>144P NE</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4466</td>
<td>19</td>
<td>32.7</td>
<td>2</td>
<td>34P NE</td>
<td>2</td>
<td>3-5</td>
<td>109P W</td>
<td></td>
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<tr>
<td>- worst weather of voyage</td>
<td></td>
<td>34</td>
<td>19,788</td>
<td>16</td>
<td>30.5</td>
<td>6</td>
<td>49.5S SWW</td>
<td>5</td>
<td>8</td>
<td>275 W</td>
</tr>
<tr>
<td></td>
<td></td>
<td>19,140</td>
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<td>31.4</td>
<td>6</td>
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<td>5</td>
<td>8</td>
<td>138P SW</td>
<td></td>
</tr>
<tr>
<td>- two cases of Beaufort No. 8 - wave ht. 10-15 ft.</td>
<td></td>
<td>36</td>
<td>24,606</td>
<td>24</td>
<td>8.55</td>
<td>8-10</td>
<td>10-11</td>
<td>NW</td>
<td>20</td>
<td>15</td>
</tr>
<tr>
<td></td>
<td></td>
<td>19,140</td>
<td>15</td>
<td>32.16</td>
<td>7-8</td>
<td>W</td>
<td>10</td>
<td>10-10</td>
<td>W</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>18,502</td>
<td>22</td>
<td>32.36</td>
<td>8</td>
<td>W</td>
<td>12</td>
<td>12-15</td>
<td>W</td>
<td></td>
</tr>
<tr>
<td>- 2 higher Beaufort No. 12 &amp; 11-12 - wave ht. 20 ft.</td>
<td></td>
<td>29</td>
<td>32,219</td>
<td>9</td>
<td>12.0</td>
<td>12</td>
<td>0 NW</td>
<td>35</td>
<td>35</td>
<td>375 NW</td>
</tr>
<tr>
<td></td>
<td></td>
<td>23,606</td>
<td>8</td>
<td>12.0</td>
<td>12</td>
<td>375 NW</td>
<td>35</td>
<td>35</td>
<td>375 NW</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>32,130</td>
<td>12</td>
<td>10.0</td>
<td>12</td>
<td>125S NW</td>
<td>50</td>
<td>50</td>
<td>125S NW</td>
<td></td>
</tr>
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<td>- worst weather of voyage</td>
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<td></td>
<td></td>
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</tbody>
</table>

**TABLE 7-2.** Evaluation of the Postulate that the Most Severe Weather is Always Associated with the Maximum Strains

When this technique was attempted with several of the larger extreme values recorded by the scratch gauge for which there was log data, it became obvious that there was not sufficient data to determine the broadness parameter of the initial distribution f(x). Additionally, the ship-response events associated with the larger extreme scratch marks were probably nonlinear, not zero-mean, not Gaussian, and not narrow banded. Thus the scratch marks are applicable in a direct way, only to the validation of short-term theoretical predictions of extreme values resulting from a complex, nonlinear combined load system. This type of validation would probably be better served by statistical analysis of the time histories of the response, as provided by electrical gauges.
One concern associated with the scratch-gauge data base is the effect of corrosion, if any, on the amount of strain detected by the scratch gauge. Equivalent bending moments should produce equivalent strains if there is no degradation of the structural material over the years. If, on the other hand, the structural member is degraded by corrosion, then it may experience greater strains with equivalent bending moments. Inherent to this approach is the determination of import still-water bending moments. The procedure consists of the following steps:

Step 1: From a "data-year" paper tape, for an import period, when it has been possible to calculate the bending moment $M_1$, the scratch mark is pinpointed at point $p_1$. At some later time, not greater than three months, another import bending moment is calculated, $M_2$, and is located at point $p_2$. Thus, the difference in bending moment $M_2 - M_1$ is reflected in a strain $p_2 - p_1$. In the computation of bending moments, it is important to carefully account for the fuel, stores, and water as well as the cargo.

Step 2: For each consecutive data year after year 1, the same process is carried out. A sequence of change in BENDING MOMENT/DEFLECTION can be noticed by this method. If over the years the structure is degrading from corrosion, the ratio of $\Delta M/\Delta \text{DEFL.}$ will decrease. The trend should show the change in the structural strength over the years.

This method depends on the ability to determine still-water bending moments very precisely. Additionally, it requires measurements of an accuracy difficult to obtain from the paper records. In view of these considerations, and the difficulties associated with the mean-stress analysis discussed in Section 7.1, this procedure did not prove to be viable.

The most straightforward assessment of the potential effects of corrosion is from visual inspection of the hull girder. Booth (20) indicates that there is no corrosion in the areas of the scratch instruments. Also in view of the fact that the ships are essentially new, the effects of corrosion on the scratch data is considered minimal to nonexistent; although it is recognized that corrosion throughout the hull girder will effectively decrease the section modulus and increase strains.
Increased attention is being given to ship cyclical loading histories and fatigue life design. Nevertheless, present ship structural design and optimization methods remain primarily concerned with limit state design. Without exception, the estimate of the extreme values of ship structural response and loading expected to occur during the ship’s lifetime is a necessity in every ship design effort. Prior to the introduction of the statistical approach to seaway loads, the extreme value of midship bending moment was calculated in a deterministic fashion by mathematically poising the ship on a "design wave" of sufficiently large proportion. Ever since the 1953 Pierson-St. Denis paper (21) was presented, however, the state-of-the-art in prediction of extreme ship loading has progressed along probabilistic lines. It has generally been accepted over the last several decades, because ocean waves are irregular, that any rational estimate of loading and response must be statistically based. This concept has initiated an enormous amount of research, much of which has been performed under the auspices of the Ship Structure Committee.

The last twenty-five years of research and development has been concerned with a wide range of statistical methods as applied to ship design. There has been particular emphasis, however, on the long-term prediction of wave-induced bending loads. This is not surprising since wave-induced bending loads, for most ships, are the greatest proportion of dynamic loading. With the evidence in hand that the ocean waves could often be described as a narrow-banded Gaussian random process; that ship response was linear with respect to wave excitation; and that ship motions could be predicted in the frequency-domain using strip theory, investigators for the last fifteen years or so found that they, indeed, had the necessary tools by which to make long-term predictions of wave-induced bending loads. Other sources of loading (transient loads from slamming and green water, for example) were somewhat amenable to comparable analysis, but not nearly with the same success as found with wave-induced bending estimation.

It is within this historical context that the SL-7 Instrumentation Program, including the scratch-gauge program, was conceived and carried out. An evaluation of the scratch-gauge program, with attendant recommendations as to the disposition of the gauges, continuation of the program, etc. should be carried out within this original context, of course. It must also be evaluated, however, within the context of evolving technological capabilities. The scratch-gauge program, as an integral part of various long-term prediction methods, has value only to the extent that these particular prediction methods are relevant to state-of-the-art rational ship structural design methods. For example, the notion that the scratch-gauge program has value as an alternative data-gathering technique with respect to expensive electrical instrumentation presumes that the electrical sensor data is valuable. In effect, the value of the scratch-gauge program is conditional on the utility of the analytical methods for which it was developed to verify. Therefore, the purpose of this chapter is to assess the utility of the analytical methods used to predict ship lifetime extreme loads.
There are a variety of methods employed to predict lifetime extreme loads. These methods can be classified within two broad categories: (1) long-term prediction based on conditional probabilities of short term loads, and (2) short-term load prediction based on lifetime extreme events. The methods comprising the former category are numerous and attack the problem from widely differing approaches. The latter method is computationally simpler and has received less attention, but is potentially more powerful than the former methods. The next two sections discuss in greater detail the relevant aspects of the methods employed to predict lifetime extreme loads.

8.1 Long-Term Prediction Based on Conditional Probabilities of Short-Term Loads

Although there are markedly different techniques included in this category, from a probabilistic point of view, they are all variations of a central theme. In general, these approaches consist of the following process:

1. Predict ship response, $X$, for a number of short-term steady-state operational and environmental conditions (sea state $- w$, speed $- V$, heading $- \beta$, loading condition $- \xi$, etc.) and derive the appropriate short-term probability density function $f(x)$ characterized by $\sqrt{m_0}$, which is essentially a conditional distribution of $x$ with respect to $\sqrt{m_0}$. This prediction can be made by theoretical ship motion prediction methods or experimental data.

2. Using either empirical ocean wave data, theoretical prediction methods, estimated ship operational and weather data, or a combination of each, derive a long-term probability distribution of $\sqrt{m_0}$, which will be hereafter represented as $g(\sqrt{m_0})$.

3. The end result is a joint distribution of $x$ and $\sqrt{m_0}$:

$$f(x, \sqrt{m_0}) = f(x|\sqrt{m_0}) \cdot g(\sqrt{m_0})$$

and a marginal probability of $X$ exceeding a certain value, $X_0$:

$$P_r(X \geq X_0) = \int_0^{\infty} P_r(X \geq X_0 | \sqrt{m_0}) \cdot g(\sqrt{m_0}) \, d(\sqrt{m_0})$$

Under certain conditions, the Rayleigh probability distribution describes the peaks of the short-term random process characterizing the ship response.

† Where $m_0$ is the 0th moment or area under the response spectrum curve
Thus,
\[ P_r \{ (X \geq X_0) | \sqrt{m_0} \} = \exp \left( -\frac{X_0^2}{2 \cdot m_0} \right) \]

yields the probability of exceedance associated with the long-term extreme value, \( X_0 \):
\[ P_r (X \geq X_0) = \int_0^\infty \exp \left( -\frac{X_0^2}{2 \cdot m_0} \right) \cdot g(\sqrt{m_0}) \, d(\sqrt{m_0}) \]

The conditions under which the Rayleigh distribution is appropriate are discussed by Ochi (22). For non-narrow-banded processes, a more general distribution can be used of which the Rayleigh distribution is a limiting case. It is typically referred to as the CLH distribution after Cartwright and Longuet-Higgins (17).

Although the CLH or Rayleigh distributions are generally accepted to be adequate for the short term, it is not generally accepted that there is a preferred form for \( g(\sqrt{m_0}) \).

To summarize the investigations into methods of long-term prediction, Table 8-1 has been prepared. It shows the form of \( g(\sqrt{m_0}) \) with some brief comments on each main approach.

Jasper (4) reported in 1956 that the log-normal distribution provided a satisfactory model for the distribution of \( \sqrt{m_0} \) unconditional with respect to all operational or environmental parameters, \( \omega, v, \beta, \gamma, \) etc. This was based on data from various ship types and the U.S. Weather Bureau. Considerable discussion ensued after the presentation of Jasper's paper, some of which expressed doubt as to the "across-the-board" applicability of the log-normal distribution. The aim of several other discussions, particularly relevant to the present project, was to question the value of any distribution where the parameters could be determined only after the ship had been built and response measurements analyzed. With respect to these concerns, the end results of these discussions and author's reply were as follows:

1. The log-normal distribution was a simple distribution of practical utility that did not preclude that "mathematical distributions may be found which fit the data better. Undoubtedly, the true physical description is much more complex..." (Jasper);
2. The extrapolations of data using an appropriate statistical model cannot, obviously, be used by the designer of the original ship, but may be useful for guidance with similar type ship designs, "design of shipborne equipment, stabilization systems, fire-control equipment, and aircraft launching and landing devices." (Lewis). Further discussion of the above points as they relate to the scratch-gauge program will be presented in Chapter 9.
<table>
<thead>
<tr>
<th>CLASS</th>
<th>INVESTIGATOR</th>
<th>$g(\sqrt{\bar{\alpha}_0})$</th>
<th>REMARKS</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>Jasper (5)</td>
<td>$g(\sqrt{\bar{\alpha}_0}) = \frac{1}{\sigma \sqrt{2\pi}} \exp \left( -\frac{(\ln(\bar{\alpha}_0) - \mu)^2}{2\sigma^2} \right)$</td>
<td>log-normal</td>
</tr>
</tbody>
</table>
|       | Nordenstrom (23) | $g(\sqrt{\bar{\alpha}_0}) = \frac{1}{\sigma \sqrt{2\pi}} \exp \left( -\frac{(\ln(\bar{\alpha}_0 + C) - \mu)^2}{2\sigma^2} \right)$ | • between normal and log-normal  
• $C$ is a constant estimated for each set of data  
• for $C > 1$, little effect on probability of exceedance |
|       | Band (24)   | Same as Nordenstrom with $C = -\mu$, i.e., normal |
| II    | Bennett (25) | $g(\sqrt{\mathbf{A}}|w_I)$ | • separated $g(\sqrt{\mathbf{A}})$ into $g(\sqrt{\mathbf{A}}|w_I)$ = conditional probability density function of $\sqrt{\mathbf{A}}$ for a given weather group $w_I$  
• $Pr(w_I)$ = probability of encountering the $i^{th}$ weather group |
|     | Band        |                                                          |
|     | Nordenstrom (23) | $g(\sqrt{\mathbf{A}}|T_I)$ | • separated $g(\sqrt{\mathbf{A}})$ into $g(\sqrt{\mathbf{A}}|T_I)$ = conditional p.d.f. of $\sqrt{\mathbf{A}}$ for a given wave period group $T_I$  
• $Pr(T_I)$ = probability of encountering the $j^{th}$ wave group |
|     | Lewis (26)  |                                                          |
|     | Hoffman & Williamson (27) | $g(\sqrt{\mathbf{A}}|w_I)$ | • separated $g(\sqrt{\mathbf{A}})$ into $g(\sqrt{\mathbf{A}}|w_I)$ = conditional probability density function of $\sqrt{\mathbf{A}}$ for a given weather group $w_I$  
• $Pr(w_I)$ = probability of encountering the $i^{th}$ weather group |
|     |              |                                                          |
| III   | Ochi (20)   | $g(\sqrt{\bar{\alpha}_0}, w_I, S_j, H_k, V_j, V_m)$ | • $w_I$ = sea condition  
$S_j$ = $S(w)$: wave spectrum  
$H_k$ = heading  
$V_j$ = ship speed  
$V_m$ = loading condition  
$P_j, P_k, P_j, P_m$ for the probability of encountering each condition |

**TABLE 8-1.** Expressions for Distributions of $\sqrt{\bar{\alpha}_0}$ Used in Various Long-Term Prediction Methods
Nordenstrom (23), based on analysis of wave-induced midship bending moment data, concluded that \( g(\sqrt{m_0}) \) varied from log-normal to normal. Band (24), after analyzing wave-induced bending moments, concluded that a normal distribution of \( g(\sqrt{m_0}) \) best described his data. It became obvious that different data produced different p.d.f.'s and that there was no universally applicable distribution nor any analytical way to obtain distribution parameters. These concepts of long-term prediction were succeeded by another approach, frequently referred to as the Bennet-Band method (25). Nordenstrom (23), Lewis (26), Hoffman and Williamson (27) and others have also investigated this technique. Essentially, this approach replaces \( g(\sqrt{m_0}) \) with two probabilities--the conditional probability of \( \sqrt{m_0} \) with respect to a weather group \( w_i \)--represented as \( g(\sqrt{m_0}|w_i) \); and the probability of encountering the \( i \)-th weather group, \( Pr(w_i) \). It is also assumed that within each weather group, \( \sqrt{m_0} \) is normally distributed. The fact that the approach is not conditional on headings, speed, loading, and variations in spectral shapes within a weather group, is partially accounted for by assuming a normal distribution of \( \sqrt{m_0} \). The central limit theorems also provide a strong argument for the normal distribution. Most importantly, this technique provided a means by which a long-term distribution could be developed using theoretically predicted ship loading/response data. The approach is widely applied and is used, for example, by the American Bureau of Shipping. It is also fundamental to the analysis presented in two background Ship Structure Committee reports for this project: SSC-196 (28) and SSC-234 (6).

A third approach, presented by Ochi, (29) represents the most general approach, making no assumptions about any statistical models except for the short term. Essentially, \( g(\sqrt{m_0}) \) is separated into a number of probabilities accounting for (1) the conditional probability of \( \sqrt{m_0} \) with respect to each important operational and environmental parameter; (2) the probability of the ship encountering each particular operational and environmental state; and (3) the number of responses or cycles associated with each state. The long-term p.d.f. is normally expressed in terms of weighting factors, rather than conditional probabilities, and takes the form:

\[
f(x) = \frac{\sum\sum\sum_{ijkl} n_{ij}m_{ij}P_{ijkl}p_{ij}P_{ijkl}f_{ij}(x)}{\sum\sum_{ijkl} n_{ij}m_{ij}P_{ijkl}p_{ij}P_{ijkl}}
\]

(4-5)

where

\[
f_{ij}(x) = \text{probability density function for short-term response}
\]

\[
n_{ij} = \text{average number of responses per unit time of short-term response}
\]

\[
= \frac{1}{2\pi} \sqrt{\frac{\langle m_2 \rangle - \langle m_0 \rangle^2}{\langle m_0 \rangle^2}}
\]

\[
64
\]
The total number of responses expected in the lifetime of the ship is
\[ n = \sum_{ijk} n_i p_i p_j p_k p_l \times T \times (60)^2 \]  
(4-6)

\[ T = \text{total exposure time to sea in hours.} \]

The long-term distribution, can be calculated from \( f(x) \) as evaluated by equation (4-5).

It must be kept in mind that, although all the methods outlined above have historical significance, not all of them are appropriate for use. In particular, only those techniques which account for the number of response cycles can be considered correct.

### 8.2 Short-Term Load Prediction Based on Lifetime Extreme Events

In this technique, a short-term extreme value analysis of the response of the ship in each of a number of possible environmental conditions is carried out. The extreme response does not necessarily occur in the severest sea condition, thus necessitating evaluation of a number of sea conditions. A description of this method is given by Ochi (29) where comparison with the more widely applied long-term approach is provided.

Briefly, the method consists of the following steps:

1. For each significant wave height, determine the operation or exposure time, \( T \); the number of encounters with the specified sea in the ship's lifetime, \( k \); the ship's speed in the seaway, \( V \) and the design risk parameter \( \alpha \). (The risk parameter

\[(m_0)^* = \text{area under short-term response spectrum} \]

\[(m_2)^* = \text{second moment of short-term response spectrum} \]

\[ p_i = \text{weighting factor for sea condition} \]

\[ p_j = \text{weighting factor for wave spectrum} \]

\[ p_k = \text{weighting factor for heading to waves in a given sea} \]

\[ p_l = \text{weighting factor for speed in a given sea and heading} \]
represents the probability that the extreme response in a given sea will exceed the estimated design load.

2. For each sea condition (which may include variations in spectral shapes and parameters) and the associated operational condition, predict theoretically or experimentally determine the zeroth (m₀) and second moments (m₂) of the response spectrum.

3. Evaluate the probable extreme value and design extreme value according to the following expressions:

Probable extreme value:
\[
\hat{y}_n = \sqrt{2 \ln \left(\frac{60}{m_0} \right)} \sqrt{\frac{m_2}{m_0}}
\]  
(4-7)

Design extreme value:
\[
\hat{y}_n = \sqrt{2 \ln \left(\frac{60}{m_0} \frac{2T}{2(\sigma/k)} \right)} \sqrt{\frac{m_2}{m_0}}
\]  
(4-8)

This method is substantially simpler than the long-term prediction methods, and makes it much easier to see the effect of variations in sea spectral parameters, as well as operational parameters. Along these lines, Ochi (30) and Hoffman (31) have demonstrated significant variations in probable or design extreme values due to relatively small changes in sea spectral shape and the probability of encountering the more severe weather groups—Pr(w_i).

8.3 Philosophical Perspective

The next twenty years in ship structural response analysis, as pointed out by Caldwell (32), will likely develop in the "three areas where uncertainties currently predominate: dynamic fluid interaction, non-linear response, and capability prediction of real, imperfect structures." Caldwell further noted that it remains to be seen "whether any practical substitute for spectral analysis (with its assumption of linearity of response) can be developed which can account adequately for extreme-load conditions." Several research projects are in progress or in the planning stages which emphasize non-linear time-domain ship motion and load prediction under extreme conditions. If these techniques prove viable, then the best way to determine extreme design loads is to simulate the response of the ship in a number of severe operational and environmental conditions. The best way to
validate such techniques would be time histories of wave and full-scale ship response measurements. In lieu of this, the scratch-gauge data could provide an approximate method of validation. Since our knowledge of the exciting forces related to each measured scratch record is incomplete and very approximate, in general, some kind of assumed loading condition and sequence of exciting forces would be required as input to the theoretical simulation. Thus, from the outset, there would be difficulties in the comparison process. Also, because the minimum peak and maximum peak do not necessarily occur in the same cycle, a four-hour simulation would be needed to approximate the condition that led to the extreme sagging and an extreme hogging condition. Although the scratch-gauge data are potentially useful in this regard, the data from the electrical strain gauges are better suited for validation of time-domain simulations of extreme-load events.

To summarize, the scratch-gauge program represents over twenty ship-years of data and the data base is sufficiently large to alleviate many of the statistical problems associated with long-term extrapolations. There are other difficulties, however, as noted in this chapter. These points are further discussed in Section 9.1. From a philosophical perspective, it seems that the evolving techniques capable of realistically simulating ship behavior in a seaway, including extreme conditions, somewhat depreciate the value of the scratch data as a basis for ship lifetime extreme-load prediction.
9. UTILITY OF THE SCRATCH-GAUGE DATA

From the investigations performed, the scratch-gauge data has four potential areas of application:

- Extrapolation of the statistical model which fits the data
- Provide a data base with which to validate long-term density functions derived by theoretical means
- Provide a data base with which to validate short-term extreme value prediction methods
- Provide a long-term record of mean stresses

In each of these areas, the following questions must be answered:

1. How can the data be used for this application?
2. Is the data of sufficient quality and quantity for this application? (If not, what is required? Is re-analysis and reprocessing of the data necessary?)
3. Is the application worthwhile?

The answers to the first question have been provided in the course of the investigations reported herein. The remaining part of this chapter will answer Questions 2 and 3 as they pertain to the four potential applications. They will thus form the basis of the conclusions and recommendations offered in the last chapter.

9.1 Extrapolation of Statistical Model

Five years of scratch data provides a sufficiently large sample with which to make a strong argument for acceptance of any statistical model that fits the data. As was demonstrated, excellent fits can be obtained using the Type-I Extreme Value, Weibull and Ochi distributions.* It was also shown, however, that with each additional year of accumulated data, the parameters or long-term estimates based on the Type-I Extreme-Value Model did not demonstrate any definite trend toward some "true" value. The key question in this regard is "how many data years are enough?"

Insight into the answer to this question can be gained from attention to work that oceanographers have been carrying out for many years. In fact, the approach of fitting a statistical model to data and extrapolating to longer periods of time is largely used to predict the extreme heights of ocean waves, the "100-year" storm, the maximum floods, rainfall, temperature, gusts, etc. In the absence of an adequate physical model of the phenomenon, for which there are few with regard to natural phenomena, the recourse is data fitting and extrapolation to arrive at extreme values and events.

* (See Figures 4-9, 4-10, 5-1, and 5-4).
Concerning the appropriate length of time to gather "climatological" data, including ocean waves, various figures have been suggested in the literature. Nolte (33) indicates that an appropriate time of data collection would be 50 years. Resio (34) notes that special caution should be taken in estimating extremal distributions from 1-6 years of record and gives an example where even 20 years is not sufficient. Such large figures result from concerns about climate change; specifically, attempting to make extremal estimations under nonstationary conditions. Figure 9-1 illustrates the difficulty. The plot shows the number of storm days per year measured at Utsira, off the coast of Norway in the North Sea. It can be seen that if measurements were taken for only ten years, from 1950 to 1960, for example, a large underestimate would result with respect to the 50+ year period shown.

![Figure 9-1. Number of Storm Days at Utsira each year from 1920 to 1974 (35)](image)

This statistical fitting technique is used by designers of fixed offshore structures to estimate extreme wave crest heights in order to determine platform-to-water surface clearance. The design wave determined by this method also allows for estimation of fluid drag and inertia forces on the structural members that support the platform. Thus, this method has great use in the offshore industry. The "offshore" literature reflects the importance of this estimation procedure to platform design. References 36 to 40 are typical examples and
represent the practical applications as well as the theoretical research that is ongoing. The methods used by fixed offshore platform designers, however, do not have equal utility in the ship design process, as discussed below.

The fact that the scratch-gauge data does not represent straightforward measurements of a natural phenomenon, but include a complicated ship structural system as well as ship operator input, further complicates the situation. The master of each ship, by the decisions he makes, greatly influences the extreme stress values which the ship will experience. The decisions he makes as to the degree to which he skirts bad weather and the headings and speeds he orders when he is in severe seas, can significantly affect the parameters of the statistical model.

The utility of an extreme value derived from the extrapolation of a statistical model can also be demonstrated by the use of the following example:

A class of five instrumented commercial ships yielded a large amount of data after several years of North Atlantic and Pacific service. The ships were operated by well-qualified and experienced masters assisted by ship optimal weather routing procedures. The end result was a statistical model which describes the data very well, and could be extrapolated to longer periods of time. This information could then be used for guidance in the design and construction of sister ships, alterations to existing ships, or guidance in specifications of additional or replacement shipboard equipment or machinery.

Thereafter, the ships were required for different purposes, routes, etc., and saw the following service:

SHIP 1 was modified slightly to carry a different cargo.
SHIP 2 was placed into entirely different trade routes.
SHIP 3 carried cargo and personnel in combat areas, committed to fulfillment of military missions rather than commercial.
SHIP 4 in the course of her commercial service, was required to perform rescue operations on several occasions where lives were at stake.
SHIP 5 was sold and operated by inexperienced masters without the benefit of weather routing.

Although somewhat contrived, the example shows that the original data and statistical model cannot be relied on to predict the extreme lifetime responses of these ships if any of the mission requirements, operator priorities and capabilities, service routes, etc., are changed from those that existed when the data were acquired.
Returning to the question of whether five years of scratch data is sufficient for long-term extrapolation, the answer is that it is not. This is based on the fact that the statistical model parameters and the lifetime extreme values predicted using the statistical models do not converge. Also, five years is not long enough to account for climatic variations that may occur in a ship's lifetime.

It was noted that the technique of finding an appropriate statistical model which describes the available data, and extrapolating to longer periods of time has been successfully applied in cases where the magnitude of naturally occurring events, such as floods or hurricane generated waves, can be used as a design criteria. The method has been the object of many investigations in the ship research field over the last 25 years, some of which have been very similar to the present project. From a purely statistical point of view, the scratch-gauge data provides the foundation for a large number of interesting analyses. From a ship design perspective, a direct use may be derived in the load estimation for a ship of similar form and service. Secondary uses have been mentioned before. From a classification society perspective, lifetime extreme estimates and actual records of extreme loads experienced by a class of ships in service adds to the "experience" data base on which policies and regulations are often based.

The statistical fit/extrapolation method has minimal value, however, in the verification and support of those analytical techniques which predict ship loads and responses at the preliminary design stage. With respect to this method specifically, acquisition of more than 5 - 7 years of scratch data on the SL-7 class is not warranted.

9.2 Validation of Long-Term Prediction Methods

It was shown in Chapter 6 that the scratch-gauge data have potential use as an alternative to electrical sensor data, as suggested in SSC-234, and has limited potential as a means to validate analytical long-term prediction methods as presented in the previous chapter. Reiterating, present analytical methods produce long-term distributions of wave-induced bending strains, whereas the corresponding distributions from the scratch data represent all sources of loading. Of course, if the satisfactory means by which to probabilistically combine loads is developed, these "scratch" distributions could be used more directly. The second concern is with the future utility of the long-term predictions methods themselves. This was discussed in the previous chapter.

With respect to this particular method, acquisition of scratch-gauge data for a period greater than 5 - 7 years is not warranted.
9.3 Validation of Short-Term Prediction Methods

The scratch data, as noted in Chapter 7, can potentially be used to validate theoretically predicted extreme stresses occurring in extreme environmental conditions. The main difficulty with this application is that each scratch must be identified by date and time and then related to detailed environmental, ship loading and operational information. This requirement certainly complicates a data acquisition system whose principal advantages are large quantity, simplicity, and low cost. Additionally, the validation of short-term extreme responses is better served by accurate and detailed electrical strain records of the complex load histories which occur during extreme events. Furthermore, a tremendous quantity of data is not required and electrical strain-gauge data such as that acquired from the McLEAN may provide a superior means for validation.

Another short-term application where the scratch-gauge instruments may prove particularly helpful, however, is in the study of extreme waves. The significance of the extreme waves in ship structural design is not understood and the scratch gauge could be an inexpensive source of information. Ships transiting through the area off the Southeast coast of Africa and Cape Hatteras would be particularly good candidates for instrumentation.

9.4 Long-Term Record of Mean Stress

As was noted in Chapter 7, the scratch-gauge data do have excellent potential as a long-term record of mean stress. However, as noted, there are several difficulties associated with getting such data from the paper-tape records as they presently exist. If it is concluded from SSC Project SR-1279 (SL-7 Instrumentation Program Summary Conclusions, and Recommendations) that it would be important to track mean stress during the McLEAN's three instrumented seasons, then the scratch-gauge records could provide much of that information after re-analysis of the original data. Otherwise, the most beneficial use of the instruments would be to install the gauges on different ship types for which mean-stress data were sought.
10. CONCLUSIONS AND RECOMMENDATIONS

10.1 Summary and Conclusions

1. The measurements generated by the scratch-gauges appear valid and the data-reduction process which resulted in the histograms presented in SSC-286 introduced no significant error.

2. The principal advantage of the mechanical extreme stress recorders is large quantities of inexpensive data. However, the utility of this large data base is significantly reduced mainly as a result of the following three factors: (a) contributions from different load sources cannot be separated, (b) contributions from torsional, lateral, and vertical bending cannot be separated and, (c) there is no reasonable way to distinguish between hogging and sagging response. In precise terms, the McLEAN data are a faithful representation of the four-hour range of longitudinal strain in the 2nd longitudinal girder up from the deck in the starboard tunnel, outboard, at about frame 186. There are also similar data from the port side of the McLEAN. To transform this data into longitudinal vertical bending strains at the main deck requires some assumptions, which are summarized in the next two paragraphs.

3. To relate calculated strain response to the location of the scratch gauge to the main deck the following relationships are applied:

\[ \sigma_{\text{scratch}} = 0.77 \sigma_{\text{mn. deck}} \quad \sigma_{\text{mn. deck}} = 1.3 \sigma_{\text{scratch}} \]

These are based on the ratio of the distances of the scratch gauge and main deck from the ship's neutral axis in longitudinal vertical bending.

4. A comparison of the McLEAN's port scratch gauge to starboard gauge data samples for identical periods of time shows differences in sample statistics and long-term predictions. When the ship encounters waves from the port side, the starboard gauge response is greater than the port. The reverse is also true. Assuming a uniform distribution of ship headings over the long term, it is reasonable that the "over-responses" due to asymmetric loading will be compensated by a sufficient number of "under-responses". The five-year data base can then be considered as an approximate representation of longitudinal vertical bending.

5. Several statistical models were found to describe the data well enough to be used as a basis for statistical inference beyond the range of measured values. The Type-I Extreme-Value distribution, the Weibull distribution, and a four-parameter distribution proposed by M.K. Ochi satisfactorily represent the data in most cases. There is a particularly strong theoretical basis for the use of the Type-I Extreme-Value distribution. Consideration was given to the independent, identically distributed condition required by the Type-I Extreme-Value Model. Guided by these theoretical considerations, it is concluded that either of two approaches can be used
to infer lifetime extreme values for the SL-7 class or similar ship:

(a) Using the single maximum value from each voyage, fit the Type-I Extreme-Value Model to the data (graphically or numerically) and extrapolate to a "return period" equal to the lifetime number of voyages.

(b) Using the four-hour maxima broken down by sea severity, fit the Type-I Extreme-Value Model to the data associated with each of the more severe sea conditions. Estimate the number of four-hour watches associated with each of the sea conditions and extrapolate to a return period equal to the respective number of watches. Of these, choose the highest value. The worst sea condition may not necessarily yield the highest value due to its low number of encounters.

The scratch-gauge data easily lend themselves to these methods of statistical inference. The extreme values derived can provide guidance to design of similar ships or structural modifications to the existing SL-7 class. This is contingent upon ship service under operational and environmental conditions similar to those under which the scratch data were acquired.

6. Scratch-gauge data and electrical strain-gauge data (LVBS) from the McLEAN's second operational season were correlated. Long-term trends were developed based on this data. A comparison between the form of the curves from the scratch data, electrical strain data, and analytical calculations showed good agreement. The scratch-gauge data can be used as an alternative to electrical strain instrumentation if total combined load is sought. It should not be used to predict wave-induced loading only or to validate methods which theoretically predict only wave-induced loads.

7. The scratch-gauge data records provide a way to track mean stress over a long period of time. There are significant difficulties involved with such a procedure. To obtain mean-stress information from the present records would require substantial data remeasurement and reduction effort.

8. Another potential application of the data is to consider those particular scratches associated with the extreme situations when weather was particularly severe. As the analytical methods to realistically simulate ship response in severe conditions are developed, these "extreme-condition" scratches may be useful for validation support.

9. The SL-7 scratch-gauge program, like many experimental programs, was strongly motivated by the need for validation support for analytical methods, specifically methods to predict ship lifetime extreme loads. These methods were reviewed in order to gain a broader perspective on the possibilities for the scratch data, both present and future. The impending development of methods to
simulate non-linear ship response in extreme conditions suggests that the long-term-prediction type of validation support provided by the scratch gauge program may not be a high priority. These methods would be best validated by time history measurements, e.g. electrical strain-gauge data.

10.2 Recommendations

1. The SL-7 Scratch-Gauge Program should be discontinued at the convenience of the Ship Structure Committee.

2. Use of the gauges for studies of long-term mean stress variations is feasible. Specific recommendations concerning the disposition of the instruments for this purpose should result from SSC Project SR-1282 "In-Service Still-Water Bending Moment Determination." This still-water bending moment project and other studies of this type still must be concerned with the amount of data or the number of years of data taking required. Should the data be considered valuable, acquisition should continue until the addition of each year's data to the aggregate sample causes an insignificant change in the relevant statistics, e.g. mean, median, etc. Obviously, the key word is insignificant and this must be evaluated within the context of a particular project.

3. Use of the gauges for studies relating to ship response in extreme environmental conditions, episodic waves, "freak-waves", etc. is recommended. In order to assess the severity of the loading on the main hull girder and local structural components in extreme waves, it is recommended that instruments be mounted in ships whose regular service routes take them through those areas with the most reported incidents of encounters with abnormal waves, e.g. south-east African coast and Cape Hatteras. Specific recommendations concerning the disposition of the gauges for this purpose should result from SSC Project SR-1281 "Ship Structures Loading in Extreme Waves."

4. Further use of the gauges for studies relating to long-term distribution of ship response and loads is not recommended. The data base as it presently exists, although limited, provides sufficient data for validating analytical methods aimed at long-term distribution of loads or response.

5. Remeasurement or reprocessing of the data in order to associate scratch marks with various parameters of interest, such as Beaufort Number, is not recommended. Although this type of information has value for some of the potential applications discussed herein, the major effort required to derive this kind of information would not be warranted.
6. Future research may find that the scratch-gauge data as they presently exist may be valuable for some specific application which may not have been identified in the present investigation. Although it is not possible to foresee the particular requirements of such use, it is reasonable to believe that the ship's weather log or any descriptive information that would identify the environmental and operational conditions experienced would be valuable. It is recommended that an effort be made to obtain this type of information, if possible, from the eight instrumented ships. Many of the McLEAN voyages are already documented.

ACKNOWLEDGEMENTS

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The cooperation of Mr. Ed Booth of Teledyne Engineering Services, is very much appreciated. The assistance of Ms. T. Sholtes in the preparation of plots, artwork and computer data files is gratefully acknowledged.
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APPENDIX A

DATA MEASUREMENT AND REDUCTION PROCEDURES

PROCEDURE FOR USING DIGITIZING TABLET TO REMEASURE SCRATCH RECORDS

Step 1. Place folder on tablet and tape four corners. For measuring marks on each length of paper in the folder, two "fixes" are required. On each paper length, there is usually a "centerline" on the tape. This is used as a reference from tape to tape and is also used to define the apparent 'x' axis for measuring, even though the folder may not be perfectly aligned with the grid structure of the digitizing tablet. The two "fixes" are obtained by placing the crosshairs over the centerline on the left end of the paper and pressing the (8) button on the cursor. Then place the crosshairs over the centerline on the right end, press (8). The two sets of coordinates thus defined are later used to define the equation of the reference line for each paper type.

Step 2. Begin measuring marks. Place crosshair over the top of the mark, press (1). Place crosshairs over the bottom of the scratch, press (2). Continue this sequence.

Step 3. When there is no scratch because of no dynamic strains, place the cursor over the trace at the location indicated by the template guide, and press (4).

Note: If an error is made, note it on the printout, reinput the correct data, and remove the incorrect entry from the file-by-file editing procedures. Always compare the printout of coordinates to the record. Ensure proper number of entries, 2 "fixes" for each tape, proper sequence of flag numbers e.g. 1, 2, 1, 2, 1, 2, etc.
Title: SGAGE 1

Purpose: To calculate the p-to-t stress, maximum, and minimum stress associated with each scratch in sequence.

Description: The program reads coordinates in a data file created by the digitizing tablet. Using the "fix" points for each paper tape, a reference line is used from which to measure "absolute" locations of each scratch. Using a conversion factor of 0.005 in to each digital unit, the coordinates of scratches are converted to lengths. Using the scale factor provided by SSC-286 for each instrument, these scratch lengths are converted to stresses.

Title: SGAGE 2

Purpose: To calculate frequency distributions (histograms) of stress using voyage data files created by the digitizing tablet.

Description: The same algorithms and procedure contained in SGAGE 1 is used to get the sequential list of stresses. These stresses are then used to form histograms.

Title: SGAGE 3

Purpose: To acquire Beaufort Number, or other parameter of interest interactively from the user for the stresses associated with each scratch mark in the sequential listing for each voyage.

Description: Algorithms and procedure contained in SGAGE 1 is used to get the sequential list of stresses. These stresses are then, one-by-one, shown to the user, at which point he will enter the parameter value, e.g. Beaufort Number "6". The program then organizes the stresses by parameter value and creates a file to store this information. The output file is combined with others using SGAGE 4.

Title: SGAGE 4

Purpose: To combine output files generated by SGAGE 3 and generate histograms for each parameter value, e.g. by Beaufort Number.

Description: The program requests the file names to be combined; reads the files, accumulates the values, and generates the frequency distribution for each parameter value.
Title: SGAGE 5

Purpose: To select the maxima of a selected number of adjacent stresses from the voyage sequential list. This is carried out for the complete list, and a new list is created and stored on file.

Description: The program requests the name of a voyage data file created by the digitizing tablet. It also requests the group size from which the maxima will be selected. The program reads through the sequence of stresses, placing the selected list of maxima in a file, to be combined with other files in SGAGE 6.

Title: SGAGE 6

Purpose: To combine files of maxima created by SGAGE 5 and generated histograms.

Description: The program requests the names of the files storing the maxima lists created by SGAGE 5. These files are read, values accumulated, and a histogram is printed out.
NUMERICAL METHODS TO ESTIMATE PARAMETERS OF TYPE-I AND TYPE-III EXTREME VALUE DISTRIBUTION

The first asymptotic distribution to be considered is to be designated Type-I and appears as:

\[ G(y_n) = e^{-\alpha (y_n - u)} \quad -\infty < y_n < \infty \]  

(1)

The second type to be considered is known as a Type-III:

\[ G(y_n) = e^{b - y_n k} \quad -\infty < y_n < b \]  

(2)

The third type to be considered will be called Type III - Modified. The difference is that the modified Type III describes a line which will fit to only three points, as chosen by the evaluator. The modification is necessary because, in many cases, the parameters for the unmodified Type III do not exist because of the nature of the data. The modified Type-III is evaluated graphically.

For Type I, the parameters \( \alpha \) and \( u \) are found from:

\[ \alpha = \frac{\pi/\sqrt{6}}{\sqrt{\text{Var}(y_n)}} \]  

(3)

\[ u = E(y_n) - \frac{0.577}{\alpha} \]  

(4)

Where \( E(y_n) \) is the mean of the data

\( \text{Var}(y_n) \) is the variance of the data

For Type III, \( v \) is equal to the \( y_n \)-value for which \( G(y_n) = 0.368 \).
The upper bound value of \( y_n \) is \( b \), and \( k \) is a positive constant. They can be calculated by solving the following two equations:

\[
E(y_n) = b - (b-v) \Gamma \left(1 + \frac{1}{k}\right)
\]

\[
E(y_n^2) = b^2 - 2b(b-v) \Gamma \left(1 + \frac{1}{k}\right) - (b-v)^2 \Gamma \left(1 + \frac{2}{k}\right)
\]

The computer programs perform these calculations for the Type-I and Type-III models. The gamma function values are calculated using a polynomial approximation and recurrence formulas.
The following procedure is extracted from "Statistics for Prediction of Ship Performance in a Seaway," by M.K. Ochi and W.E. Bolton:

The value of the parameter 'v' is read from the plot by locating the value of $H(y_n) = 0.368$ and finding the corresponding value of $y_n$. The parameter 'v' is equivalent to that value of $y_n$. Then, read $H(y_n)$ from the probability distribution curve for two arbitrarily chosen value of $y_n$. Designate these two values as $y_{n_1}$ and $y_{n_2}$. Then choose two arbitrary $b$-values, and enter the following equations:

$$k = \left\{ \frac{\ln\left(\frac{-\ln H(y_{n_1})}{b - y_{n_1}}\right)}{\ln \left(\frac{b - y_{n_1}}{b - v}\right)} \right\}$$

$$k = \left\{ \frac{\ln\left(\frac{-\ln H(y_{n_2})}{b - y_{n_2}}\right)}{\ln \left(\frac{b - y_{n_2}}{b - v}\right)} \right\}$$

The result will be four coordinate pairs which can be plotted as shown in the Figure above. Connecting these coordinate pairs with straight lines, an intersection may be found which will give the desired $b$ and $k$ values.

*This method was originally presented in "On statistical estimation of maximum bending stress of ship's hull", Publications No. 411 and No. 429, Soc. Nav. Arch. of Japan, 1963 and 1965, by Y. WATANABE.

† International Shipbuilding Progress, Vol. 20, No. 229, September 1973
INTRODUCTION

The extreme value is defined as the largest value expected to occur in a certain number of observations or in a certain period of time. Let us consider the structural response of a marine system in a seaway. Here, the marine system includes ships, marine vehicles, and offshore structures, etc., operating in random seas. The response of a marine system, designated by $X$, is a random variable and has the probability density function, $f(x)$, as well as the cumulative distribution function, $F(x)$. Let us denote the extreme value (the largest value) in $N$ responses in a seaway as $Y_n$. The extreme value is also a random variable and it follows its own probability law (see Figure 1).

The probability density function and the cumulative distribution function of the extreme value, $Y_n$, can be derived by applying order statistics to the probability function of the random variable $X$. These are,

Probability density function of the extreme value:

$$g(y_n) = N \left[ f(x) \{ F(x) \}^{N-1} \right]_{x = y_n} \tag{1}$$

Cumulative distribution function of the extreme value:

$$G(y_n) = \left[ \{ F(x) \}^N \right]_{x = y_n} \tag{2}$$

The underlying probability functions, $f(x)$ and $F(x)$, are often called the initial probability density function and the initial cumulative distribution function, respectively.
Various statistical properties of the extreme value can be evaluated from Equations (1) and (2). For instance, the extreme value which is most likely to occur in $N$-observations, denoted by $\bar{Y}_n$, can be obtained as the modal value of the probability density function $g(\gamma_n)$. That is, the probable extreme value in $N$-observations is given as a solution of the following equation:

$$\frac{d}{d\gamma_n} g(\gamma_n) = 0$$

(3)

Thus, if the initial probability density function, $f(x)$, is precisely known, the probable extreme value can be evaluated from Equations (1) and (2) without any assumption. This will be discussed in more detail in Section 1.

In many practical problems, however, it is rather difficult to determine the initial probability distribution precisely. This is particularly true when the probability function is determined from results of observed data. In this case, the extreme value may be predicted by applying the approximate method which is applicable for any initial probability function. This subject will be discussed in Section 2.

It is often necessary to estimate the extreme value of the response during the lifetime of a marine system through accumulation of data of the largest value observed in a specified period of time; for example, the largest value in a day. In this case, the estimation can be achieved by applying the concept of asymptotic distributions of the extreme values. Application of this concept will be presented in Section 3.

SECTION 1 ESTIMATION OF EXTREME VALUE BASED ON THE EXACT INITIAL DISTRIBUTION

As is stated in the Introduction, when the initial probability distribution is precisely known, the probable extreme value in $N$-observations can be evaluated from Equations (1) and (3). In this case, the formula for evaluating the probable extreme value, $\bar{Y}_n$, may not necessarily be expressed in a simple closed form depending on the initial distribution. For example, if the initial distribution is the Rayleigh distribution, which is applicable for evaluating responses of a conventional-type ship in a seaway, the probable extreme value in $N$-observations can be evaluated by:

$$\bar{Y}_n = \sqrt{2} \ln N \sqrt{\mu_0} \quad \text{for amplitude}$$

(4)

where, $\mu_0$ = area under the response spectrum.
On the other hand, the probable extreme value expected to occur in T-hour observation is given by,

$$\bar{Y}_n = \sqrt{\frac{2\ln (60)^2 T}{2\pi}} \sqrt{\frac{m_2}{m_0}} \sqrt{m_0}$$

for amplitude

where, $m_2 = 2$nd moment of the response spectrum.

However, if the initial distribution is more complicated such as in Figure 2, where the generalized gamma distribution representing responses of a surface effect ship (SES) in a seaway is shown, the probable extreme value cannot be given in a simple closed form. Instead, it is evaluated only through numerical computations (Reference 1). In any case, in evaluating the extreme value through Equations (1) and (2), the initial probability distribution has to be precisely known.

To amplify the above statement, Figure 2 shows an example of the peak-to-trough excursion of the longitudinal bending strain measured in SES trials. The figure shows the comparison between (a) an observed histogram, (b) Rayleigh probability distribution, (c) Weibull probability distribution, and (d) generalized gamma probability distribution. As can be seen in the figure, the Rayleigh does not show substantial agreement with the observed histogram. On the other hand, the Weibull and the generalized gamma probability distributions represent reasonably well the observed data. It should be noted, however, that the largest value of 84.8 micro-inch/inch measured during the 224-second observation is not sufficiently covered by the tail portion of the Weibull distribution, as can be seen in Figure 2. This results in a significant discrepancy between the predicted most probable extreme value and the measured maximum value as is demonstrated in Figure 3. It can be seen in the figure that neither the Weibull nor the Rayleigh distribution is satisfactory, and that only the generalized gamma distribution reasonably predicts the extreme values as far as this example is concerned. It is obvious, judging from Figure 3, that these three distributions will result in a considerable difference in the extreme values for design consideration.

SECTION 2 APPROXIMATE METHOD TO ESTIMATE EXTREME VALUE FROM INITIAL DISTRIBUTION

The initial distribution is usually determined from results of analysis made on either observed data or computations. Although it is highly desirable to represent the initial distribution precisely by some known probability function, a satisfactory representation over the entire range of the cumulative distribution is rather difficult in practice.
As an example, Figure 4 shows the cumulative distribution function of significant wave height observed at Station I in the North Atlantic plotted on log-normal probability paper, while Figure 5 shows it plotted on Weibull probability paper (2). As can be seen in these figures, the data are satisfactorily represented by the log-normal probability distribution over the range for the cumulative distribution up to 0.99, but the representation is poor for higher cumulative distributions. On the other hand, the data for higher cumulative distributions are well represented by the Weibull probability distribution, but the representation of the data by the Weibull is unsatisfactory over the entire range of significant wave height. This is evidenced in Figure 6 in which the comparison between the histogram and the two probability density functions is shown. As this example indicates, it is often difficult to represent the initial probability distribution adequately by some known probability function.

One way to overcome this difficulty is to evaluate the extreme value by an approximate method that is applicable for any probability distribution if certain conditions are met. The principle of this approximate method is as follows:

The probable extreme value in N-observations can be evaluated from Equation (3) which can be expressed in terms of the initial distribution as,

\[
\frac{d}{dx} f(x) P(x) + (N - 1)\{f(x)\}^2 = 0 \quad x = y_n
\]  

(6)

If the initial cumulative distribution function is assumed to be in the form of,

\[ F(x) = 1 - e^{-q(x)} \]  

(7)

where, \(q(x)\) is a positive real function. Then, from Equations (6) and (7),

\[
\frac{q''(y_n)}{[q'(y_n)]^2} \{1 - e^{-q(y_n)}\} + Ne^{-q(y_n)} - 1 = 0
\]  

(8)

Since the first term is small in comparison with other terms for large \(N\), Equation 8 yields,

\[ e^{-q(y_n)} = \frac{1}{N} \]  

(9)
Thus, the probable extreme value, $y_n$, for large $N$ is approximately given by,

$$y_n = q^{-1}(\ln N)$$  \hspace{1cm} (10)

where, $q^{-1}(x)$ is the inverse function of $q(x)$. However, for evaluating the extreme value in practice, it is not necessary to know the function $q(x)$ in Equation (7). As can be seen from Equations (7) and (9), the extreme value for large $N$ is given as the $x$-value for which the probability of exceeding $x$ is equal to $1/N$. That is,

$$\frac{1}{1 - F(y_n)} = \frac{1}{N}$$  \hspace{1cm} (11)

As an example of application of Equation (11), the data shown in Figures 4 and 5 are plotted as open circles in Figure 7. The vertical scale of the figure is the left-hand side of Equation (11), which is often called the return period, in logarithmic form. The solid circles in Figure 7 are for the data observed at Station J in the North Atlantic. Since there is no appreciable difference between the cumulative distribution function for high significant wave heights obtained at Stations I and J, the results are expressed by a single line as shown in the figure. However, it is noted, in drawing the line, that the open circles for high significant wave heights are discarded. This is because the frequency of occurrence of significant wave heights higher than approximately 11 m (36 ft.) is extremely unreliable as is evidenced in Table 1 taken from the original data. The data shown in Figure 7 are based on random samples from 10 years of observation and the total number $N = 2,400$ for Station I and $N = 1,440$ for Station J. Using these numbers, it can be estimated from Figure 7 that the probable extreme significant wave heights in 10 years are 15.4 m (50.0 ft.) for Station I and 14.6 m (47.9 ft.) for Station J.

It should be noted that the magnitude of probable extreme significant wave heights thus estimated from Figure 7 are much smaller than the values known to date. This is because Draper's data shown in Figure 7 are for the number $N = 2,400$ and 1,440 in 10 years. This implies that the sampling rates for the data presented in Figure 7 are one observation in every 37- and 61-hour interval, respectively. Hence, it is obvious that very severe seas which do not persist for a long period are missed in the data.

In order to estimate the probable extreme significant wave heights at Station I and J more precisely, the analysis carried out using Walden's data is available (2). The probable extreme heights expected in 10 years estimated using the Walden's data are tabulated as follows:
<table>
<thead>
<tr>
<th>Station</th>
<th>Sample Size in 10 Years</th>
<th>Probable Extreme Significant Height</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>23,620</td>
<td>18.7 m (61.5 ft)</td>
</tr>
<tr>
<td>I (Winter)</td>
<td>5,510</td>
<td>18.9 m (62.0 ft)</td>
</tr>
<tr>
<td>J</td>
<td>24,947</td>
<td>19.2 m (62.9 ft)</td>
</tr>
</tbody>
</table>

The number of observations in these data is so large to provide information of the sea condition approximately every 3.5-hour intervals, sufficient to cover all severe sea conditions. The probable extreme significant height estimated using the data obtained during the winter season at Station I is also included in the table. The extreme value from the winter season data agrees very well with that estimated from the all-year-round data. This indicates the significance of the sample size in estimating the extreme values.

SECTION 3 ESTIMATION OF EXTREME VALUE FROM OBSERVED MAXIMA

In the foregoing two sections, it was assumed that information of the initial distribution is known. That is, the initial distribution is either expressed by a known probability function or it is given numerically. However, even if the initial distribution is unknown, the extreme value can still be predicted by applying the concept of asymptotic distribution of the extreme values for a large number of observations.

Since the knowledge of the initial distribution is not required in applying the asymptotic distribution of the extreme values, one of the most useful applications of this method is to estimate the extreme value of the response expected to occur in the lifetime of a ship through observations of the largest value in a specified period of time. For example, estimate the extreme structural response in waves expected in the lifetime of a ship (30 years service) from the accumulation of daily observed maximum responses.

There are three different types (Type I through III) of asymptotic extreme value distributions. However, for most problems in naval and ocean engineering, the following two types (Type I and Type III) are sufficient to be considered.

\[
G(y_n) = e^{-\alpha(y_n - u)} \quad -\infty < y_n < \infty
\]  

(12)
where,
\[
\alpha = \frac{\pi}{\sqrt{6}} \frac{\text{Var}[y_n]}{\sqrt{\text{Var}[y_n]}}
\]
\[u = E[y_n] - 0.577 \frac{\alpha}{\alpha}
\]

Type III:
\[G(y_n) = e^{-\left(\frac{(\omega - y_n)k}{\omega - v}\right)} \quad \omega < y_n \leq \omega
\]  \hspace{1cm} (13)

where, \(v\) is equal to the \(y_n\) value for which \(G(y_n) = e^{-1} = 0.368\). \(\omega\) is the upper bound value of \(y_n\), while \(k\) is a positive constant. They can be determined from the following two moments of \(y_n\):

\[E[y_n] = \omega - (\omega - v) \Gamma(1 + \frac{1}{k})
\]
\[E[y_n^2] = \omega^2 - 2\omega(\omega - v) \Gamma(1 + \frac{1}{k}) + (\omega - v)^2 \Gamma(1 + \frac{2}{k})
\]

It is noted that \(\omega\) and \(k\) can also be determined graphically from the information given in the cumulative distribution function of the observed maxima.

As an example of application, Figure 8 taken from Reference (5) shows the cumulative probability distribution of the maximum stress-range observed each day. The Line A was obtained by Yuille (5) following the Type I distribution, while the Line B was obtained by Watanabe (6) following the Type III distribution. Unfortunately, it is not possible to carry out further analysis to estimate the design value from Figure 8, since no information is given in Reference (5) necessary to perform the estimation.

One way to estimate the extreme value for design consideration from data similar to that shown in Figure 8 is presented below.

Prior to carrying out the estimation, it is definitely necessary to prepare various information required for analysis. This information includes,

- Confirmation that the data are representing fairly well the average all-year-round operation condition of a ship (or a marine structure). If the data are taken during the winter season only, the appropriate modification should be made in the analysis.
- Obtaining the time interval during which the maximum values are recorded.
- Obtaining the total number of observations or the total time period of observations.

For convenience, let us assume that the data consist of the daily maximum value (24-hour time interval) observed for 200 days representing the average all-year-round operation condition of a ship (or a marine structure). Then, the probable extreme value in the lifetime (assuming S-years) is given by,

\[
1 - G(Z_n) = \frac{1}{200S}
\]

(14)

Note that the lifetime extreme value is denoted by \(Z_n\), while the daily extreme value is denoted by \(Y_n\). From the asymptotic extreme value distributions, Equation (14) yields,

\[
\hat{Z}_n = \begin{cases} 
  u + \frac{1}{\alpha} \ln(200S) & \text{for Type I} \\
  \omega - (\omega - v) \left( \frac{1}{200S} \right) \frac{1}{k} & \text{for Type III}
\end{cases}
\]

(15)

Next, let \(Z_n\) be the extreme value for the design consideration for which the following relationship holds,

\[
\hat{Z}_n = \begin{cases} 
  u + \frac{1}{\alpha} \ln T & \text{for Type I} \\
  \omega - (\omega - v) \left( \frac{1}{T} \right) \frac{1}{k} & \text{for Type III}
\end{cases}
\]

(16)

where, \(T > 200S\), but is unknown at this stage. Since the probability that the daily maximum value will exceed \(Z_n\) is \(1/T\) as an average, we have,

\[
\Pr\{Y_n < \hat{Z}_n\} = 1 - \frac{1}{T}
\]

Hence,

\[
\Pr\{Z_n < \hat{Z}_n\} = \Pr\{Y_n < \hat{Z}_n \text{ in S-years}\} = (1 - \frac{1}{T})^{200S} \approx 1 - \frac{200S}{T}
\]

(17)
By letting this probability be $\gamma$ (0.99 for example), we have,

$$T = \frac{200S}{1-\gamma} \quad (18)$$

The extreme value for design consideration can then be estimated by substituting Equation (18) into Equation (16).

REFERENCES


Fig. 1 Explanatory sketch of initial probability density function $f(x)$ and extreme value probability density function $g(y_n)$.

Figure 2 Comparison between observed histogram and various probability density functions: SES 100B longitudinal bending strain [1].

Figure 3 Comparison of probable extreme values predicted from three different probability functions: SES 100B longitudinal bending strain. (Solid circle is the observed value) [1]
Figure 4 Cumulative distribution of significant wave height plotted on log-normal probability paper [2] (Data from Ref. 3)

Figure 5 Cumulative distribution of significant wave height plotted on Weibull probability paper [2] (Data from Ref. 3)

Figure 6 Comparison between histogram of significant wave height and probability density functions [2]

Figure 7 Probable extreme significant wave heights in 10 years at Stations I and J [2] (Data from Refs. 3 & 4)
Figure 8: Cumulative distribution function of the measured reduced variate.

Table 1: Statistical data on wave height and period.

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
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Note: The table data are not fully visible in the image.
SL-7-1, (SSC-238) - Design and Installation of a Ship Response Instrumentation System Aboard the SL-7 Class Containership S.S. SEA-LAND McLEAN by R. A. Fain. 1974. AD 780090


SL-7-14, (SSC-278) - Wavegauge Data Reduction Method and Initial Data for the SL-7 Containership by J. F. Dalzell. 1978. AD-A062391.


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