SSC-42

THE DETERMINATION OF INITIAL STRESSES IN STEEL PLATES

by

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and

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SHIP STRUCTURE COMMITTEE

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ADDRESS CORRESPONDENCE TO:

SECRETARY
SHIP STRUCTURE COMMITTEE
U. S. COAST GUARD HEADQUARTERS
WASHINGTON 25, D. C.

September 15, 1958

Dear Sir:

Because it has been considered desirable to know more about the magnitude of residual stresses existing in large structures such as ships, the Ship Structure Committee has sponsored a study at Princeton University of one method for measuring these stresses. Herewith is the Final Report, SSC-42, of this project, entitled "Determination of Initial Stresses in Steel Plates," by C. Riparbelli, E. W. Suppiger, and E. R. Ward.

This project has been conducted under the advisory guidance of the Ship Structure Subcommittee.

This report is being distributed to individuals and groups associated with or interested in the work of the Ship Structure Committee. Please submit any comments that you may have to the Secretary, Ship Structure Committee.

Sincerely yours,

E. H. Thiele, Rear Admiral
U. S. Coast Guard
Chairman, Ship Structure
Committee

Serial No. SSC-42

Final Report
of
Project SR-113

to the

SHIP STRUCTURE COMMITTEE

on

THE DETERMINATION OF INITIAL STRESSES IN STEEL PLATES

by

C. Riparbelli, E. W. Suppiger and E. R. Ward

Princeton University
Princeton, New Jersey

under

Department of the Navy
Bureau of Ships Project Order NObs-47613
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Washington, D. C.
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September 15, 1958

ABSTRACT

A non-destructive method for the determination of the direction and magnitude of the principal stresses at any location in a structure, such as a ship or a bridge, has been investigated and developed as reported here, in an effort to overcome the generally destructive and costly methods presently available. This non-destructive method consists of attaching electric-wire strain gages around the point at which the stress is to be measured and then drilling a hole (1 1/8 in. to 1 in. diameter) at that point. The gage reading before and after the drilling of the hole are used to determine the stress.

Known stress values up to 15,000 psi were applied to test plates, and measurements were made by the hole relaxation method. When corrected for the stresses initially contained in the plates, this hole relaxation method checked the known applied stresses to within 1000 psi. The holes are easily and economically repaired.

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I. NOMENCLATURE

		UNITS
E	Modulus of elasticity	psi
ν	Poisson's ratio	--
a	Radius of final hole	in.
r	Distance from center of hole to center of strain gage	in.
m	Distance from edge of final hole to nearest edge of gage element oriented tangential to the hole.	in.
e	Distance from edge of final hole to nearest edge of gage element oriented radial to the hole	in.
d_1	Guide hole diameter	in.
d_2	Final hole diameter	in.
ϕ	Generalized change in strain as a result of drilling a hole	10^{-6} in. per in.
ϕ_A, ϕ_B, ϕ_C	Changes in strain indicated by strain gages A, B, and C	10^{-6} in. per in.
ϕ_1, ϕ_2	Change in strain in directions of maximum and minimum principal stresses, respectively	10^{-6} in. per in.
σ_r	Normal stress in radial direction	psi
σ_t	Normal stress in tangential direction	psi
σ_1	Maximum principal stress	psi
σ_2	Minimum principal stress	psi
σ_x	Stress in a specimen in a direction perpendicular to the axis of loading	psi

NOMENCLATURE (Continued)

Symbol	Description	UNITS
σ_y	Stress in a specimen in the direction of loading	psi
θ	Angle between x and r axes measured in counter-clockwise direction	degrees
α_A, α_B, α_C	Angle between x axis and radii to fixed strain gages A, B, and C, respectively	degrees
α_1, α_2	Angle between x axis and directions of maximum and minimum principal stresses, respectively	degrees
$\eta = \theta - \alpha_1$	Angle between r axis and direction of maximum principal stress	degrees
ϵ_r	Linear strain in the radial direction	in. per in.
ϵ_t	Linear strain in the tangential direction	in. per in.
o	Used as a subscript to denote the stress or strain in the given direction in the solid plate; e.g., ϵ_{ro}	---
λ_1	Coefficient of sensitivity for stress in the direction of the stress	10^6 psi
λ_2	Coefficient of sensitivity for stress in the direction orthogonal to the stress	10^6 psi
K_1, K_2	Constants which are functions only of the properties of the plate material and the geometry of the gage and hole arrangement	1/psi

II. INTRODUCTION

At the present time, the effects of initial stresses* upon the mechanical performance of welded structures are not entirely understood. The lack of adequate knowledge on this subject is reflected in the conflicting opinions held by recognized authorities in the field. One example is the uncertainty regarding the extent of influence of initial stresses on brittle fractures of welded steel merchant vessels. Although many persons hold the opinion that stresses locked into ship structures as the result of fabrication practices do not contribute materially to failure, contrary arguments can not be disregarded. The fundamental problem, the manner in which initial stresses influence structural performance, is one of great interest and importance, not only to shipbuilders and operators, but to all persons concerned with the use or fabrication of any welded structure where such stresses may occur.

Several methods have been used in the past to determine the magnitudes of initial stresses. Of these, the technique found most practical for use on ship structures is the trepanning method. Practical development of this method, using electric-wire resistance strain gages, was done by Meriam, DeGarmo and Jonassen.[1] In this technique, strain gages are mounted on a member at the point where the magnitudes of the initial stresses are to be measured. A section of the member containing the gages is then removed. Strain gage records taken before and after removal (trepanning) of this section indicate the relaxation of strain in the plug. The relaxation strains are then used to calculate the orientation and magnitude of the principal stresses initially present. The plug must be considerably larger than the gages mounted on it. In the determination of initial stresses in the field, removal of such a large plug is sometimes inconvenient and may also

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*For the purpose of this report, initial stresses are defined as those internally balanced stresses existing in bodies upon removal of all currently applied external loads. These stresses have been referred to in the literature as by such terms as residual stresses and locked-in stresses.

weaken the structure. Furthermore, the structure must be repaired, and the stresses obtained must be considered as average values for a relatively large area.

The present project was devoted to the development of a similar but less destructive technique for measurement of initial stresses. It utilizes the relaxation provided by the drilling of a relatively small hole in the member*. The method is considered relatively non-destructive inasmuch as the small hole required may be readily filled. The procedure employed in the field is simple, and the required computations are no more complicated than those of other similar methods.

The drilling of a hole to determine initial stresses was first proposed by J. Mathar[2, 3] in 1932. Mathar worked on rolled sections using mechanical and optical extensometers to measure the change in displacement between two points on the surface of a member when a hole is drilled between the points. Preliminary work using the drilled hole and electric wire resistance strain gages was carried out by C. Riparbelli[4] at Princeton University from 1946 to 1948 under the sponsorship of the Research Corporation. Results of this study were reported to the Society for Experimental Stress Analysis in December, 1947. Similar work was presented before the SESA in May, 1949 by W. Soete and R. Vancrombrugge[5].

The investigation reported here was sponsored by the Ship Structure Committee. The work was done between February 1, 1949 and June 30, 1950.

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*Although this method was proposed as early as 1932, it had not by 1945 reached a state of development sufficient to make it practical for extensive use. This was partly the result of the cumbersome nature of mechanical strain gages required in earlier applications.

III. THEORETICAL BASES FOR THE HOLE RELAXATION METHOD

The method reported here for determining magnitudes of initial stress depends on the measurement of strain relaxation that occurs with the drilling of a hole. The strain distribution is known for a plate with a small central hole subjected to biaxial stresses within the elastic range. Thus the relationship between strain distribution in the presence of a hole and strain distribution in a plate which is intact can be found as a function of the existing biaxial stresses. Conversely, the biaxial stresses can be calculated if the difference between strain distribution in the presence of a hole and strain distribution in an intact plate can be determined experimentally. This is the basic premise of the hole relaxation method. Only two properties of the material are involved--the modulus of elasticity and Poisson's ratio; and these do not enter into the calculation of stresses if a calibration method is used.

A complete derivation of the relationship between biaxial stresses and the strain distribution in plates with and without holes is given in Appendix A of this report. For the purpose of the following discussion of the experimental work, a brief summary of the derived relationship will be presented.

It can be shown from the theory of elasticity that, for a region of a plate subjected to a biaxial stress condition,

$$\phi = K_1(\sigma_1 + \sigma_2) + K_2(\sigma_1 - \sigma_2) \cos 2\gamma$$

where

ϕ is change in strain caused by drilling a hole in an initially intact plate as measured by a gage.

γ is the angle between the direction in which strain is measured and the direction of maximum principal stress

σ_1 and σ_2 are maximum and minimum principal stress, respectively, which existed in the plate at a point that subsequently became the center of a drilled hole.

K_1 and K_2 are constants which are a function only of the properties of the plate material and the geometry of the gage and hole arrangement.

It is apparent that for constant (but not necessarily known) values of σ_1 and σ_2, ϕ varies in a periodic manner as the angle γ is increased, the maximum and minimum values of ϕ, (ϕ_1 and ϕ_2) occurring respectively in the directions of σ_1 and σ_2.

The unknowns in the problem are three: σ_1, σ_2, and γ. Therefore, three independent measurements will be necessary for solution. These measurements are obtained by locating three strain gages at equal angles from each other and at equal (small) distances from the point where the hole is to be drilled. A zero reading is taken for each gage; the hole is drilled and final gage readings are recorded. Three strain differences, ϕ_A, ϕ_B, and ϕ_C will then have been obtained. From these strain differences, and since the geometrical considerations and principal stresses apply to all three gages equally, it is possible to obtain the values of ϕ_1 and ϕ_2 that occur in the directions of σ_1 and σ_2, respectively, and the direction of σ_1 with relation to the known direction of measurement of the strain gages. This can be done using the dyadic construction and without having to evaluate the constants K_1 and K_2.

It is now necessary to determine the values of σ_1 and σ_2. This can be done most conveniently by solving Equation (1) for $\phi_1(\gamma = 0)$ and for $\phi_2(\gamma = \frac{\pi}{2})$. The resulting expression can be rewritten in the following form:

$$\sigma_1 = \lambda_1\phi_1 - \lambda_2\phi_2$$
$$\sigma_2 = \lambda_1\phi_2 - \lambda_2\phi_1 \qquad (2)$$

For the purpose of this report, λ_1 and λ_2 are denoted as "coefficients of sensitivity for stress" and are limited in use to the restrictions implied by Equations (2).

These coefficients are functions only of the constants K_1 and K_2 and are therefore functions of the modulus of elasticity and Poisson's ratio for the plate material, the radius of the hole and the distance from the center of the hole to the strain gage center.

It should have been apparent earlier, when the values of ϕ_1 and ϕ_2 were obtained, that the principal stresses of σ_1 and σ_2 could have been computed directly on the bases of the strain differences (as shown by the three gages), material properties and the geometrical arrangement of hole and gages (evaluation of constants K_1 and K_2 and simultaneous solution of Equation 3). In the same manner, λ_1 and λ_2 in Equations (2) can be calculated and the stresses σ_1 and σ_2 obtained after ϕ_1 and ϕ_2 are derived from the gage readings.

However, in view of the finite strain gage area and the undetermined influence upon the gage reading of stresses perpendicular to the gage length, the values of λ_1 and λ_2 are obtained from calibration tests for various gage types, hole diameters, and gage configurations. This is a comparatively simple matter. If Equations (2) are rewritten as follows,

$$\lambda_1 = \frac{\sigma_1\phi_1 - \sigma_2\phi_2}{\phi_1^2 - \phi_2^2}$$

$$\lambda_2 = \frac{\sigma_1\phi_2 - \sigma_2\phi_1}{\phi_1^2 - \phi_2^2} \qquad (3)$$

then, by means of laboratory tests, values of ϕ_1 and ϕ_2 can be obtained for known values of stress, σ_1, and applied to a plate loaded in uniaxial tension ($\sigma_2 = 0$ in Equations (3)), in which a hole of given diameter is drilled at the center of a specific gage rosette.

Once the coefficients λ_1 and λ_2 are determined for a given hole-gage geometry, the stresses are obtained by using Equations (2), and inserting the values of ϕ_1 and ϕ_2 measured with a similar gage arrangement, of a similar hole drilled at the point in the structure at which the stresses are desired.

As will be seen later, practical considerations dictated the use of two holes: a "guide hole" to assist in making the necessary preparations for satisfactory determination of changes in strain; and a "final hole," which is an enlargement of the guide hole. The recorded, useful values of strain differences (ϕ_1) were those obtained in enlarging the guide hole to a final hole. The nature of the problem is such that the above discussion covering the determination of stresses is applicable to this case as long as the calibration coefficients are obtained for the equivalent case, i.e., enlarging a guide hole to final hole diameter.

IV. DEVELOPMENT OF AN EXPERIMENTAL TECHNIQUE

A. Preliminary Considerations

When an attempt is made to devise a practical method for utilizing the theoretical relations developed in Appendix A and summarized in the preceding section, several problems are immediately encountered. One assumption, upon which the hole relaxation method is based, is that the measurements of changes of strain can be made at a point. The closest approach to such measurements would be achieved through use of an X-ray strain measuring technique; this is not economical for field use. Accordingly, it was decided that electric-wire resistance strain gages should be employed. A gage element 7/64 in. wide by 1/4 in. long was selected as the best compromise between the desired minimum area and practical field application.

The biaxial stress field was assumed to be uniform in the theoretical approach. In contrast to this, the magnitudes of initial stresses may vary greatly from point to point in actual structures, e.g. in the direction perpendicular to a weld. In order to approach the assumed condition, it would therefore be desirable to make the radius of the hole and the distance between the hole and the strain gages as small as practicable. Experiments were made to determine optimum values for these distances, and it was necessary to compromise between the ideal and practicable conditions.

It is well known that the initial stresses are not uniform throughout the thickness of a plate. For the purposes of this work, it was assumed that the average of ϕ measurements on opposite sides of the plate at a given point gave a reasonable approximation to the average biaxial stress state throughout the plate thickness. The accuracy of this assumption was not tested, but indirect justification was obtained in the agreement between the applied and experimentally determined stress, as will be noted later.

B. The Method of Computation in the Laboratory and in the Field

In order that the data and discussion to be presented may be meaningful, a description will be given here of the computational procedures employed in undertaking the hole relaxation method.

Laboratory tests investigating the effects of various parameters, such as hole diameter, and tests for determination of λ values were made on stress-relieved plates loaded uniaxially. It can be said with certainty that in such tests, the principal stresses and strains are oriented along the longitudinal and transverse axes of the specimens; any deviations from this orientation are insignificant. This factor enables circumvention of the calculation of orientations and magnitudes of principal strains, if the strains are measured on the appropriate axes. The simplified computations will be discussed first; the more complex computations required for field applications, in which the direction and magnitudes of principal strains are not known, will be taken up later.

Strain gages were placed at corresponding sites on each side of the plate, and the changes in strain which resulted from drilling the hole were measured. The strain changes indicated by the gages at the same point but on opposite sides of the plate were then averaged, and the mean value was employed for all calculations. It was felt that this corrected for accidental bending in the plate.

In the calibration tests for the determination of λ_1 and λ_2 values, measuring gages were placed on the longitudinal and transverse plate axes. Strains at various applied tensile loads were then measured for the no-hole, guide hole and final hole condition. The zero applied load strains for each hole condition were then subtracted from these strains. In this manner, the influence of stresses initially present in the plate was eliminated. For any two hole condition, the difference (as measured by the two gages) in strain increment from a no-load to a given load situation was, by virtue of the gage orientations, the values of ϕ and ϕ. When equations (3) are reduced to the uniaxial stress condition, values of λ can then be calculated from the known ϕ and nominal stress values, according to the following equations:

$$\lambda_1 = \sigma_1 \frac{\phi_1}{\phi_1^2 - \phi_2^2}$$

$$\lambda_2 = \sigma_1 \frac{\phi_2}{\phi_1^2 - \phi_2^2} \qquad (4)$$

A guide hole was needed to ensure that the gages were placed at precisely corresponding locations on opposite sides of the plate. Triangulation with a far-removed hole accomplished this.

For field measurements, as indicated before, the principal values of ϕ may be found by measuring the changes of strain at three separate locations, radially equidistant from the hole and on opposite sides of the plate. Employing the three measured quantities, the principal strain differences can be determined by means of the dyadic strain-circle construction.[6] The two unknown principal stresses in the solid plate are then computed from the linear equations involving the two principal strain differences (ϕ values) and the two experimental constants (λ_1 and λ_2) found by calibration tests. Orientations of the principal stresses are assumed to be identical with the orientations of the principal strain differences, which are determined with respect to arbitrary reference axes in the dyadic strain-circle construction.

C. Materials, Instrumentation and Measuring Techniques

All tests reported here were performed on steel supplied by the Department of the Navy, conforming to grade M, Navy Department Specification 48S5. The size of plate in each test is indicated in Table I. All plates from Tests 20 through 79, with the exceptions of 69, 70, 71, 77, 78 and 79 were heat treated for the relief of initial stresses at a temperature of 1200 F for 3 hours. The temperature was then slowly lowered by putting the plates in a shut down oven until they were cool enough to be handled (140 F). The plate for Test 77 was stress relieved by heating at 1350 F for from 4 to 5 hours. It was then cooled 10° per hour until it reached a temperature of 1050 F. At this point the relay failed and the plate cooled to room temperature in about 5 hours. The plates for Tests 78 and 79 were stress relieved by heating at 1350 F for about 4 hours; they were then cooled gradually in irregular steps totaling 50° per day until they reached 700 F, and then in steps of 150°--200° per day until they reached room temperature.

Commercial electric-wire resistance strain gages (SR-4, Type A-7), having a 7/64 in. by 1/4 in. gage element, were used in all tests; it was felt that these gages represented the smallest size that could be conveniently handled in the field. The plates were carefully sanded with a power sander in the vicinity of the gage locations to remove rust, mill scale, and irregularities, after which they were smoothed with emery cloth and cleaned with acetone. The gages were then applied with commercial cement at predetermined locations on opposite sides of the plates and allowed to dry in air for 2 hours. Subsequently, the plates were heated by means of a 500-watt cone heater to a temperature of from 70--75 C

TABLE I -- INDEX OF TESTS

Legend

HRS	Commercial hot-rolled steel	T	Tension	TD	Twist drill
PS	Project steel	C	Compression	CB	Counterbore
SR	Stress relieved	B	Bending	R	Reamer
NSR	Not stress relieved	CAL	Calibration	12--1/8 in.	Twelve overlapping 1/8 in. holes
		S	Special		

Test No.	Material	Plate Size, in.	Type of Test	Type of Gage	Final Hole d_2, in.	Drilling Method	Maximum Load, lb	Edge Distance e, in.
1	Duralumin	1/8 x 24 x 48	S	AR-1	1/2, 1, 2	TD & Saw	T 15,000 B 300	--
2	HRS NSR	1/2 x 6 x 36	CAL & S T & B	A-7	1/8	TD	T 45,000 B 750	1/16
3	" "	"	CAL, T & B	"	1/4	"	"	"
4	" "	"	"	"	1/2	"	"	"
5	" "	"	"	"	1	"	"	"
6, 7	PS "	3/8 x 2 x 18	S	"	--	--	To failure	--
8	" "	1 1/4 x 6 x 12	CAL, C	"	1/2	TD	C 112,500	1/16
9	" "	"	"	"	1	CB	"	1/8
10	" "	3/8 x 6 x 12	S	"	1/8, 1/4, 1/2, 1	TD & CB	--	Various
11	HRS "	3/8 x 30 x 30	"	A-7 & AR-1	1/2 & Plugs	TD	Welded	1/8
12	PS "	3/8 x 6 x 12	"	A-7	Plugs	Saw	--	--
13	" "	1 1/4 x 6 x 12	CAL C	"	1/2	TD	C 112,500	1/8
14	" "	"	"	"	1/4	"	"	1/16
16	" "	"	CAL C	A-7	1/2	TD	C 112,500	1/8
17, 18	" "	"	"	"	1/8	"	"	1/16
19	" "	3/8 x 6 x 36	S	A-1	Plugs	Saw	--	--
20	PS SR	3/8 x 6 x 24	"	"	"	"	--	--
21, 22	" "	3/8 x 6 x 36	CAL T & B	A-7	1/8	TD	T 35,000 B 425	1/16
23	" "	1 1/4 x 6 x 12	CAL C	"	1/4	"	C 112,500	"
24	" "	3/8 x 6 x 36	CAL T & B	"	"	"	T 35,000 B 425	"
26, 27	" "	"	CAL T & B	A-7	1/2	TD	T 35,000 B 425	1/8
28	" "	"	"	"	1/4	"	T 35,000 B 425	1/16
29	" "	3/4 x 6 x 36	"	"	1/8	"	T 55,000 B 1,700	"
31, 32	" "	"	"	"	1/4	"	T 55,000 B 1,700	"
33	" "	"	CAL T	"	1/2	"	T 55,000	1/8
34	" "	"	CAL T & B	"	"	"	T 55,000 B 1,700	"
35	" "	"	"	"	1/8	"	"	1/16
36	" "	3/8 x 6 x 36	"	"	1/2	"	T 35,000 B 425	1/8
37	" "	"	S	A-7 Rosette	"	"	"	"
38, 39	" "	"	CAL T	A-7	"	"	T 35,000	"
41--44	" "	3/4 x 6 x 36	"	"	"	"	T 55,000	1/8
45	" "	3/8 x 6 x 36	"	"	"	"	T 35,000	"
46	" "	"	S	A-7 Rosette	"	"	"	"
47, 48	" "	3/8 x 12 x 72	CAL T	A-7	"	"	T 35,000	"
49	" "	3/4 x 12 x 72	"	"	"	"	55,000	
51	" "	3/8 x 12 x 72	"	"	"	12--1/8	T 55,000	"
52	" "	3/8 x 12 x 36	S	A-7 Rosette AR-1 & AR-2	" "	TD CB & R	--	"
53	" "	3/8 x 6 x 6	"	A-7	1/8--1/2	Various	--	--
54, 55	" "	3/8 x 6 x 36	CAL T	"	1/2	CB	T 35,000	3/16
56	" "	3/8 x 12 x 36	S	A-7 Rosette AR-2	" "	TD CB & R	-- --	1/8
58	" "	3/8 x 6 x 36	CAL T	A-7	"	12--1/8	T 35,000	"
59	" "	"	CAL T & S	"	"	"	"	"
60, 61	" "	"	CAL T	"	"	CB	"	1/4
62	" "	"	"	"	"	"	"	1/8
63	" "	"	"	"	"	12--1/8	"	"
64, 65	" "	"	"	"	"	"	"	3/16
66, 67	" "	"	"	"	"	"	"	1/4
68	" "	"	"	"	"	CB	"	1/8
69--71	" NSR	"	"	"	"	12--1/8	"	"
72, 73	" SR	"	"	"	"	"	"	"
74	" "	"	S	"	"	"	"	"
75	" "	"	CAL T	"	"	"	"	"
76	" "	"	"	"	"	TD	"	"
77--79	" "	"	"	"	"	12--1/8	"	"

during a minimum time limit of 3 hours; this temperature was then maintained for at least 12 hours. While the gages were still warm they were covered with a thin coat of wax to minimize effects of humidity changes on gage readings.

Lead wires from the gages were soldered to small mounting strips attached to plastic blocks cemented to the plates; this provided insulation from the plate and protected the gages from accidental damage. Leads were connected to a switchboard so that any gage could be read as either a measuring or a compensating gage. A reversing switch enabled both normal and reverse readings to be taken, thus providing checks against instrument zero shift as well as against errors in reading. Strains were read on a commercial electronic strain indicator.

Loads were applied to the tensile specimens by means of a 200,000-lb hydraulic testing machine. The loads were applied uniformly across the plates through linkages attached to the ends of the plates. To remain safely within the capacity of the linkages, the maximum load employed was 55,000 lb.

D. Investigation of Gage Arrangements

For the purposes of the laboratory calibration tests, which were made on uniaxially loaded stress-relieved plates, it was only necessary to measure ϕ values on the axes through the hole (parallel and transverse to the loading direction) in order to obtain principal ϕ values directly. In the first tests, gages were mounted as shown in Fig. 1, which enabled ϕ values to be measured by three different gage arrangements, depending on which gages were read. These possible gage arrangements are indicated in Table II.

For a given hole size, λ_1 and λ_2 values vary somewhat with different thicknesses of the plate. This variation was found to be most pronounced for Arrangement II, whereas the λ_1 and λ_2 values for Arrangement I were essentially constant for all plate thicknesses. Use of Arrangement III showed an intermediate degree of λs' dependence on thickness.

TABLE II

Gage Arrangements for Calibration Tests

(see Fig. 1)

	Arrangement I		Arrangement II		Arrangement III	
	$\lambda 1$	$\lambda 2$	$\lambda 1$	$\lambda 2$	$\lambda 1$	$\lambda 2$
Measuring Gage	1	4	2	3	1	4
Compensating Gage	Dummy*	Dummy*	Dummy*	Dummy*	2	3

*Dummy gages were mounted on unstressed plates.

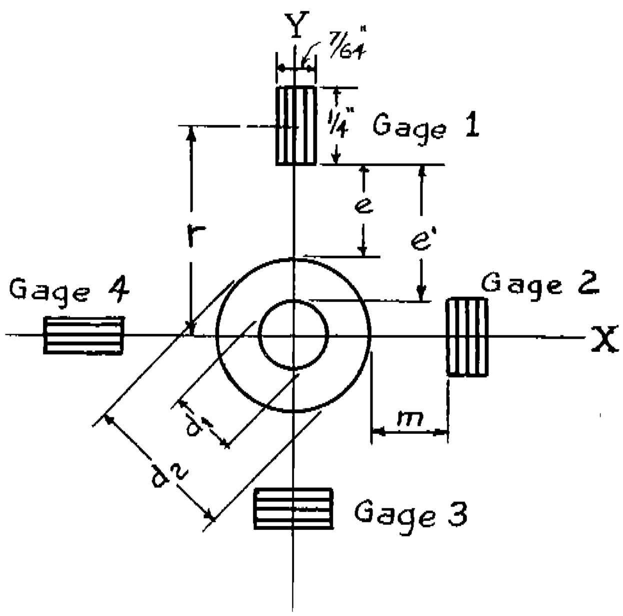

Fig. 1. Gage Location for Calibration Tests

Since it would be desirable in field applications to have one pair of λ values appropriate for all plate thicknesses of a material, it was decided to employ Arrangement I in subsequent tests. Accordingly, gages were mounted in a double Arrangement I, as depicted in Fig. 2; each set was read independently, thus making possible the use of average values, with greater confidence in the results.

In a field application, it is necessary to make three independent measurements of strain changes at radially equidistant points to enable determination of principal ϕ values. Here it is recommended that gages be placed in a manner analagous to a multiple Arrangement I, as shown in Fig. 3. The reasons for this will be discussed in the next section.

To approach the conditions assumed in the derivation of the theoretical bases of the hole relaxation method and to obtain the greatest possible sensitivity from the method, it is desirable to place the strain gages as close to the final hole as practicable. Several tests were performed in which the distance from the hole to the strain gages was varied. The results are shown in Table III for plates with final holes of 1/2 in. diameter.

Table IV compares the average λ values obtained in the sequential test procedure (no hole to final hole) for various edge distances as averaged from the test results given in Table III and calculated from Equations A-20 and A-22 given in Appendix A. Tables III and IV indicate that there is some advantage to minimizing the distance between the hole and the gages, since errors in locating the gages would then have much less effect on the values of λ.

During preparation of the test specimens, it was found difficult to prevent damage to the strain gages while drilling holes with diameters equal to or greater than 1/2 in., when the edge distance between the hole and the gages was 1/16 in. Since such damage would be even harder to avoid in field applications, it was accordingly decided to use a 1/8-in. edge distance as standard for final hole diameters of 1/2 in. or larger. An edge distance of 1/16 in. was satisfactory for final hole diameters of 1/8 in. or 1/4 in.

TABLE III

AVERAGE λ_1 AND λ_2 VALUES FOR TENSION

		No Hole to Final Hole			Guide Hole to Final Hole		
Method of Drilling	Plate Thickness in.	No. of Tests	λ_1 10^6 psi	λ_2 10^6 psi	No. of Tests	λ_1 10^6 psi	λ_2 10^6 psi
for Edge Distance e = 1/16 in.							
(a) 1/2" Twist Drill	1/2 & 1 1/4	2	-61.6	22.4	2	-61.8	19.6
for Edge Distance e = 1/8 in.							
1/2" Twist Drill	3/8	9	-62.2	24.8	9	-67.0	26.2
"	3/4	7	-61.7	23.4	7	-64.3	23.6
(b) "	1 1/4	2	-67.0	26.1	2	-70.3	26.5
"	3 Thicknesses	18	-62.5	24.4	18	-66.3	25.2
12 - 1/8" holes	3/8	14	-63.1	25.2	13	-67.5	26.3
1/2" Counterbore	"	2	-58.1	21.7	2	-60.9	21.5
All Tests		34	-62.5	24.6	33	-66.4	25.4
for Edge Distance e = 3/16 in.							
12 - 1/8" holes	3/8	2	-78.0	34.2	2	-81.1	34.0
1/2" Counterbore	"	2	-74.4	31.7	2	-78.5	32.5
All Tests		4	-76.2	33.0	4	-79.8	33.3
for Edge Distance e = 1/4 in.							
12 - 1/8" holes	3/8	2	-95.7	42.4	2	-102.0	44.0
1/2" Counterbore	"	2	-92.9	40.5	2	-96.7	40.8
All Tests		4	-94.3	41.5	4	-99.4	42.4

(a) One tension and one compression test
(b) Compression tests

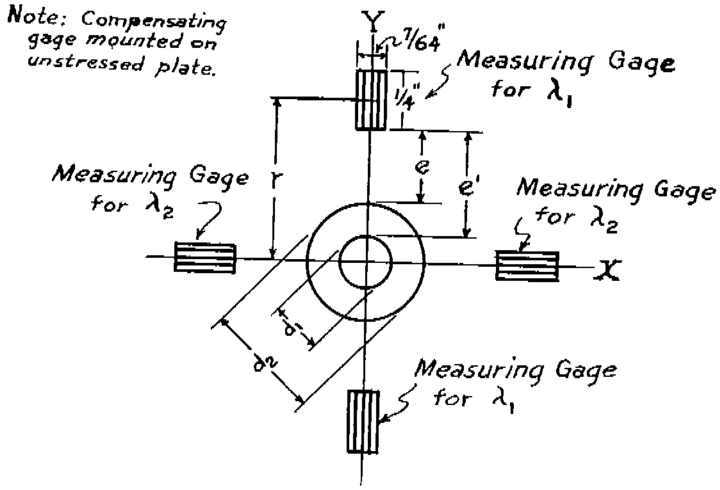

Fig. 2 . Double Arrangement I of Gages to Obtain an Average λ Value

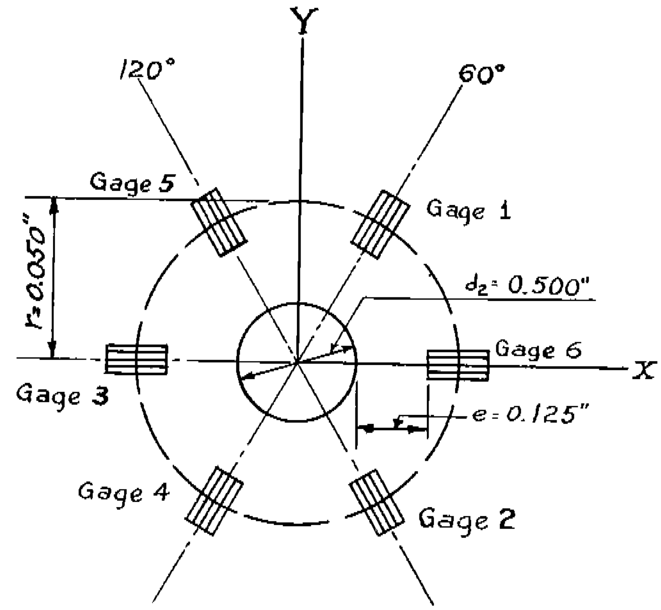

Fig. 3. Rosette for field-type test

TABLE IV

COMPARISON OF MEASURED AND CALCULATED VALUES OF λ_1 AND λ_2 FOR VARIOUS EDGE DISTANCES

Edge Distance e, in.	λ_1, psi x 10^6 Meas.*	λ_1, psi x 10^6 Calc.**	λ_2, psi x 10^6 Meas.	λ_2, psi x 10^6 Calc.
1/16	-61.6	-52.5	+22.4	+18.9
1/8	-62.5	-66.2	+24.6	+26.8
3/16	-76.2	-82.0	+33.0	+35.2
1/4	-94.3	-100.1	+41.5	+45.1

*Average values from Table III

**Calculated from Equations A-20 and A-22, Appendix A with E = 30 x 10^6 psi and ν = .290

E. Investigation of Effects of Final Hole Size and Methods of Drilling the Hole

Tests were made on a series of specimens in which the size of the final hole was varied; the diameters studied were 1/8 in., 1/4 in., 1/2 in. and 1 in. Considering the relative numbers of tests involved, the most consistent values of λ_1 and λ_2 were obtained for a diameter of 1/2 in. As may be seen from Fig. 4 insufficient data are available to assess the suitability of a 1-in. diameter final hole. As indicated previously, however, the smallest practical hole is the most desirable; for this reason, it is recommended that a 1/2-in. diameter final hole be employed in the field use of the hole relaxation method.

Since there are many ways in which the holes can be drilled, a study was made of possible effects of several different methods. For all tests reported here, both guide and final holes were drilled under a load of 35,000 lb in the plate specimen, since in field applications holes are drilled in plates under stress. Drill guides were used to ensure that the hole would be perpendicular to the plate surface. It is recommended that this precaution be observed in field use also.

To evaluate the effects of various drilling methods, 1/2-in. diameter holes were made with twist drills, with counterbores, and by overlapping twelve 1/8-in. holes to form a 1/2-in. hole with a scalloped edge. In all instances, the guide hole diameter was 1/8 in. The values of λ determined for specimens with the variously prepared holes are shown in Table III. It may be seen that for the proposed standard conditions (1/2-in. diameter final hole, 1/8-in. edge distance), there is little difference between λ values obtained by twist-drilling holes and those procured by overlapping 1/8-in. holes. The holes made with counterbores led to lower λ values for all edge distances. Additional tests were performed on stress-relieved plates in which holes were drilled at zero applied load by the methods listed below at points around which strain gages had been placed:

1. Counterbored from both sides
2. Cut with speed saw
3. Drilled with a twist drill

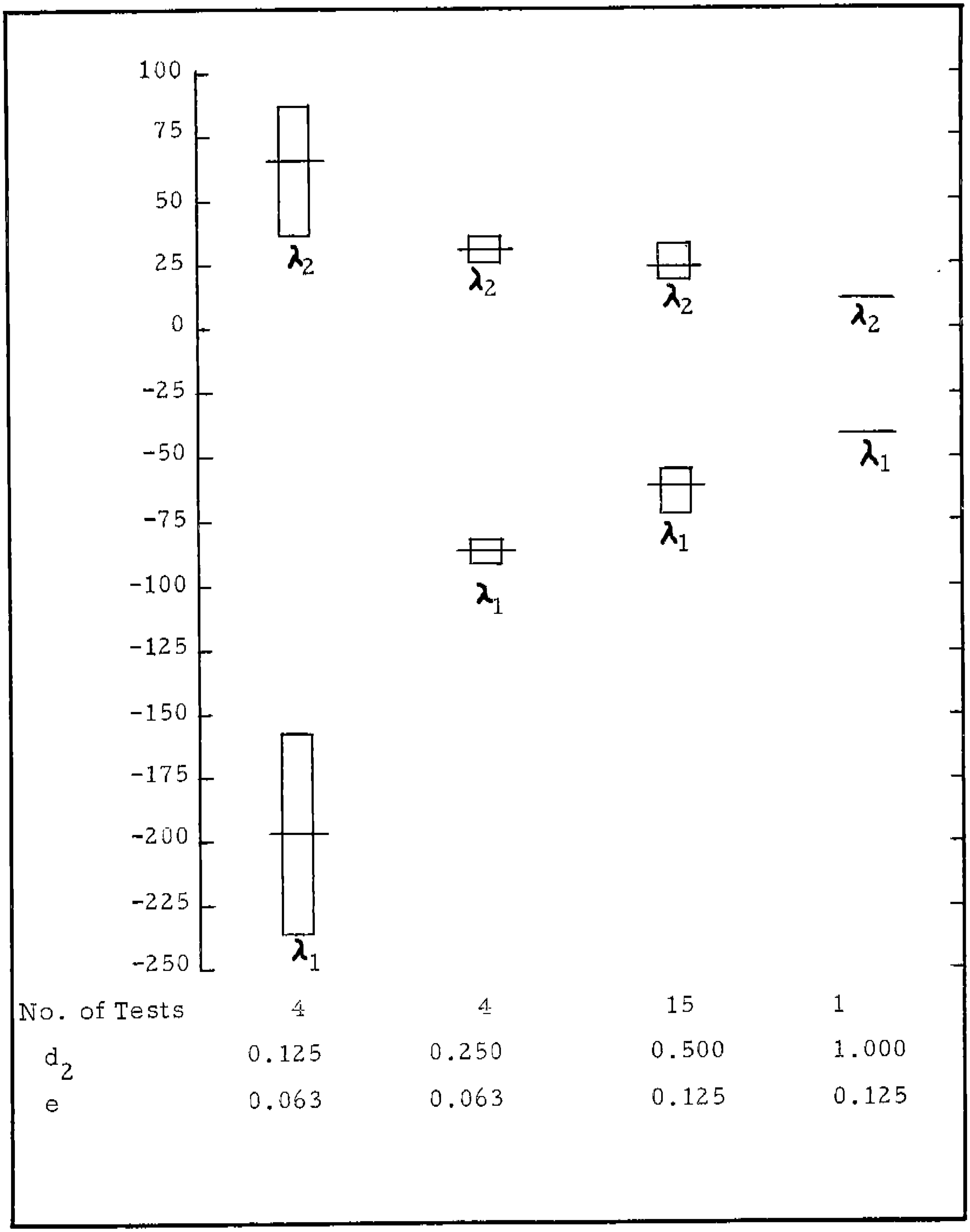

Fig. 4. λ_1 and λ_2 values for various hole diameters in Arrangement I from no hole to a final twist drilled hole. All tests were performed in tension on an approximately equal number of 3/8-in. thick and 3/4-in. thick specimens. The length of bar indicates the range of values and the horizontal line in each bar represents the average value for the indicated number of tests.

4. Drilled by overlapping twelve 1/8-in. holes to form a 1/2-in. hole.
5. Drilled and reamed
6. Attacked with acid
7. Drilled and filed

Methods 6 and 7 above, which eliminated the strain disturbances found adjacent to the hole as a result of drilling, gave negligible changes in strain readings. Method 4 also produced negligible changes. Other methods led to appreciable permanent strains.

On the basis of these results, the procedure prescribed as standard for field use is the drilling of twelve overlapping 1/8-in. holes to form a 1/2-in. hole with scalloped edges.

V. ACCURACY OF THE HOLE RELAXATION METHOD

The values of λ are functions of the modulus of elasticity, Poisson's ratio, hole diameter and the location of strain gages with respect to the final hole. Determination of the elastic modulus and Poisson's ratio for a series of plates has led to the conclusion that the λ values presented here are valid for all ship steels, so long as the gage type, location relative to the hole, and hole diameter are the same. Thus a separate determination of λ_1 and λ_2 is not required for each plate investigated.

A series of calibration tests was made under conditions similar to those that would be encountered in field use of the hole relaxation method, with the objective of evaluating the method's accuracy. Holes were made in various ways under a load of either 55,000 lb or one that would correspond to a stress of 15,000-psi in the particular size of the plate, depending on whichever was the lower. Gages were placed on the principal axes of the specimens so that principal changes in strain could be determined directly; the changes in strain for both the solid plate and the plate with a final hole were then measured using a load equal to that under which the hole was drilled and again at zero external load. From the principal ϕ values for the loaded plates and the appropriate coefficients of sensitivity for stress (λ), the principal stresses in the solid plate at particular loads were computed and compared with the stresses calculated from the external load and the cross sectional area of the specimens. The results of these tests, expressed as the differences between computed and known external stresses, are given in Table V in the columns headed "uncorrected $\Delta\sigma_1$" and "uncorrected $\Delta\sigma_2$."

It may be noted that the differences between the applied stresses and experimentally determined stresses vary from 2000 to 12,000 psi. It is believed that these are the stresses that were present in the plates as initial stresses, even though the plates were stress-relieved. The values of the initial stresses are of the same order of magnitude as those obtained for the plates that were stress-relieved at the same time and in the same manner. It has been reported by the Lukens Steel Co.[7] that after heat treatment, a body contains initial stresses which are never less than the proportional limit of the material at the stress-relieving temperature. The proportional limit of ordinary carbon steel is about 2500 psi at 1150 F. This corresponds approximately to the lower limit of the initial stresses that were actually found; stresses higher than the proportional limit probably resulted from less complete stress-relief.

The differences in strain between the solid plate and the plate with a final hole, when they are under no external load, correspond to the strain changes brought about by relief of the initial stresses at the location of the final hole. If, then, the principal strain differences measured under the applied load are corrected for the strain changes under no load, the computed principal stresses should correspond to the stresses resulting from externally applied load; by comparing the computed and the applied stresses, therefore, it is possible to assess the accuracy of the hole relaxation method. The differences between computed stresses, corrected for the initial stresses, and the actual applied loads are also given in Table V under the

TABLE V

DIFFERENCES BETWEEN COMPUTED AND APPLIED STRESSES PSI

Test No.	uncor. $\Delta\sigma_1$	uncor. $\Delta\sigma_2$	cor. $\Delta\sigma_1$	cor. $\Delta\sigma_2$	Test No.	uncor. $\Delta\sigma_1$	uncor. $\Delta\sigma_2$	cor. $\Delta\sigma_1$	cor. $\Delta\sigma_2$
for Edge Distance e = 1/16 in.									
4	7596	9073	3092	3485	8	1387	-6371	-490	-448
for Edge Distance e = 1/8 in.									
13	8731	8994	-138	-531	51	3780	1779	212	298
16	4332	4586	101	519	58	2850	1614	- 10	- 83
26	5899	3617	193	136	59	2411	1436	168	23
27	6515	3330	150	-224	62	3762	2334	452	339
33	8440	2703	-270	58	63	1929	115	-433	-614
34	6094	3950	-690	-351	68	2472	635	169	42
36	6981	3889	48	133	69	9474	-1995	182	-166
38	3378	2935	-250	- 95	70	7169	2212	147	356
39	3851	2215	83	-178	71	5857	- 498	92	-474
41	5941	4446	88	45	72	2053	1194	399	142
42	9323	7605	127	553	73	3458	1229	189	-152
43	6698	6037	- 84	-269	74	- 654	1483	820	-212
44	11531	9406	39	286	75	2554	1198	-175	-176
45	7439	4878	-398	-295	76	11988	8863	319	222
47	5823	4207	1055	697	77	2657	953	273	- 41
48	6510	5427	376	43	78	2457	0	84	-579
49	6069	4404	-394	-259	79	2730	-1526	408	- 40
for Edge Distance e = 3/16 in.									
54	4394	1970	396	249	64	2298	856	108	125
55	2108	1846	- 29	-121	65	1450	145	-127	-215
for Edge Distance e = 1/4 in.									
60	3117	708	- 67	-502	66	2300	758	582	273
61	2253	165	754	565	67	3626	1649	625	401

columns headed "corrected $\Delta\sigma_1$" and "corrected $\Delta\sigma_2$." These differences show agreement within ± 500 psi for 75% of the tests and within ± 1000 psi for all tests except one in which the edge distance was insufficient. On this basis then, it may be stated that the hole relaxation method may be used to determine stresses with a probable accuracy of 500 to 1000 psi.

Two field-type tests were made on stress-relieved plates, in which 1/2-in. diameter final holes were drilled with twist drills at a nominal uniaxial stress of 15,500 psi. In the first tests, gages were placed as indicated in Fig. 5. Using the appropriate λ values and principal ϕ values, which were determined in the dyadic strain-circle construction from the strain changes as measured on Gages 1, 2 and 3, the principal stresses were found. A comparison of the computed with the applied stresses summarized in Table VI, where θ is the angle between the transverse specimen axis and the direction of the maximum principal stress. It can be seen that when corrections were made for the difference in strain (at zero external load) between the solid plate and the plate with the final hole (i.e., the strain changes due to relief of the initial stresses), the calculated stresses were found to be in good agreement with the applied stresses.

The orthogonal Gages 4, 5 and 6 were not utilized, but could have been employed for measurements corresponding to Arrangement II.

A similar test was performed using the gage arrangement depicted in Fig. 3. This placement of gages in rosettes, corresponding to a double Arrangement I, enables the use of the average measurements of strain changes and is therefore recommended for field applications. Experimental determinations of changes in strain were corrected for relaxation of initial stresses as above, leading to the computed principal stresses given below:

	Calculated (Corrected)	Applied
σ_2	378 psi	0 psi
σ_1	15,739 psi	15,300 psi
θ	90°	90°

TABLE VI

Comparison of Computed and Applied Stress in Field Type Test

	Applied	Calculated Uncorrected	Calculated Corrected*
σ_1	15,500 psi	17,753 psi	15,842 psi
σ_2	0 psi	- 1,137 psi	502 psi
θ	90°	86° 45'	89°

*Corrected for difference in strain between solid plate and plate with final hole at zero applied load.

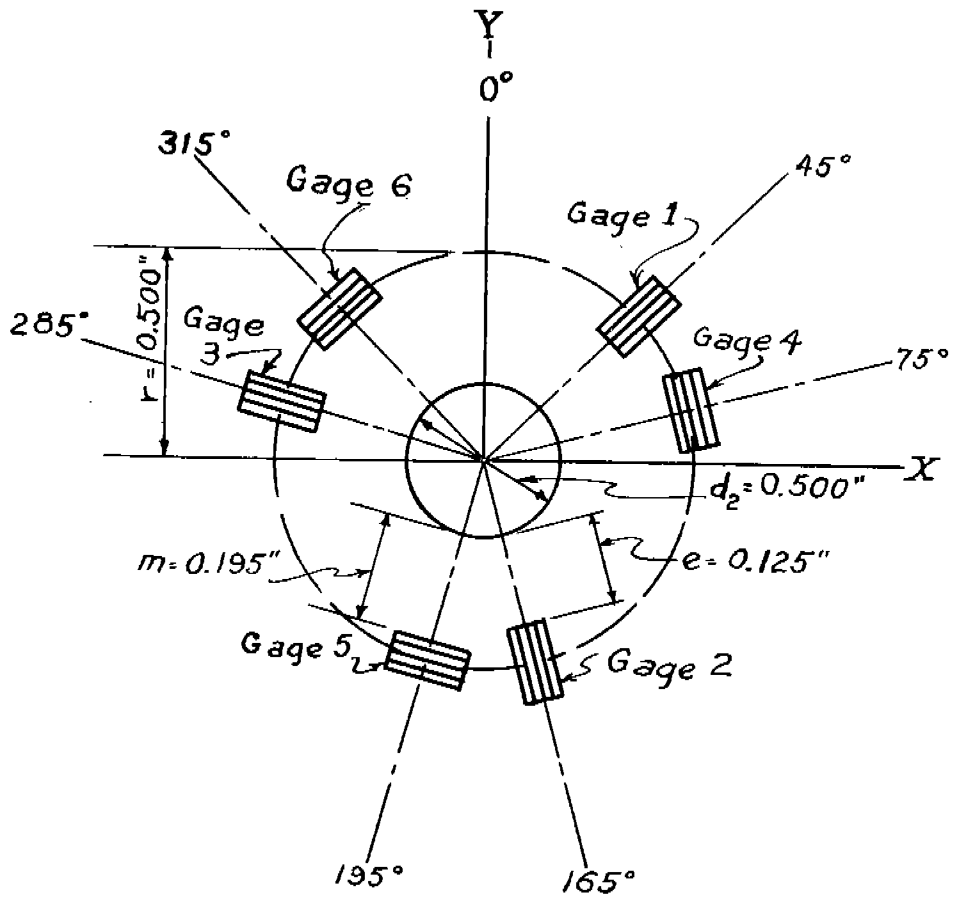

Fig. 5. Gage arrangement field type on stress-relieved plate (Test 37)

The results of the field-type tests lead to the conclusion that these tests may be carried out with the same probable accuracy (500--1000 psi) as the calibration tests.

VI. DISCUSSION OF MISCELLANEOUS TESTS

By studying several specimens, various features of the hole relaxation method were evaluated in the course of this investigation, with the following results:

Gage Performance in the Plastic Range

One specimen of reduced cross-section was loaded to a stress of 32,600 psi, with the objectives of studying the behavior of the strain gages in the plastic range and obtaining information on the technique necessary to measure plastic strains. Gages were mounted on the longitudinal and transverse axes of the specimen through a 1/8-in. diameter hole. Results show that many of the gages were rendered useless by the plastic deformation. Further studies should be made on this subject to increase the usefulness of the hole relaxation method.

Relief of Initial Stresses by Trepanning

In order to obtain accurate values of λ for a particular hole size and gage arrangement, it is necessary to know the stresses existing in the test specimen; it was therefore desirable to use plates that for the calibration tests, had been relieved of initial stresses to the maximum practical degree. Three tests were made, two on hot-rolled plates and one on a stress-relieved plate, to obtain an indication of any advantages that would accrue through use of stress-relieved plates in the calibration tests. Strain gages were mounted with parallel axes across the faces of the three plates, readings were taken, and then the plugs containing the gages were cut from the plates. The changes in strain in the three plates concomitant to relaxation of strain in the plugs are shown in Fig. 6. It may be seen that the changes in strain, and hence the magnitudes of the initial stresses in the solid plate, were significantly less for the stress-relieved material than for the hot-rolled plates. In view of this, it was decided to use stress-relieved plates for calibration tests. It is recognized that such experiments are only of qualitative value.

One additional specimen was tested so as to provide a comparison between the initial stresses found by trepanning and those found by the hole relaxation method. Rosettes of six gages each were mounted on a specimen around 2 points 6-in. apart. The gages were read with the plate under zero load, and then the plugs (3 in. x 4 in.) containing the rosettes were cut from the plate. The gages were read again, after which a 1/2-in. diameter hole was made in the center of each rosette by drilling twelve overlapping 1/8-in. diameter holes. Following this, a final set of readings was taken. By removing the plug, it was possible to calculate the following principal initial stresses in the plate:

No. 1	Plug	$\sigma_x = -405$ psi	$\sigma_y = 676$ psi
No. 2	Plug	$\sigma_x = -260$ psi	$\sigma_y = 268$ psi

Because of the relaxation provided by drilling the 1/2-in. diameter hole the following initial stresses remaining in the plug after removal from the plate were obtainable:

No. 1	Plug	$\sigma_x = 1499$ psi	$\sigma_y = 1741$ psi
No. 2	Plug	$\sigma_x = 1590$ psi	$\sigma_y = 1719$ psi

The size of the plug probably tended to exaggerate the relaxation provided by drilling the hole. (Less constraint against changes in strain is provided by such a small specimen in comparison to the test plates used in determining λ values.) Hence the plug contained apparently higher values of initial stresses than would have been found if the hole had been drilled in a solid plate. It seems reasonable to conclude that the hole relaxation method provides more complete relaxation of stress than trepanning and therefore leads to more accurate results.

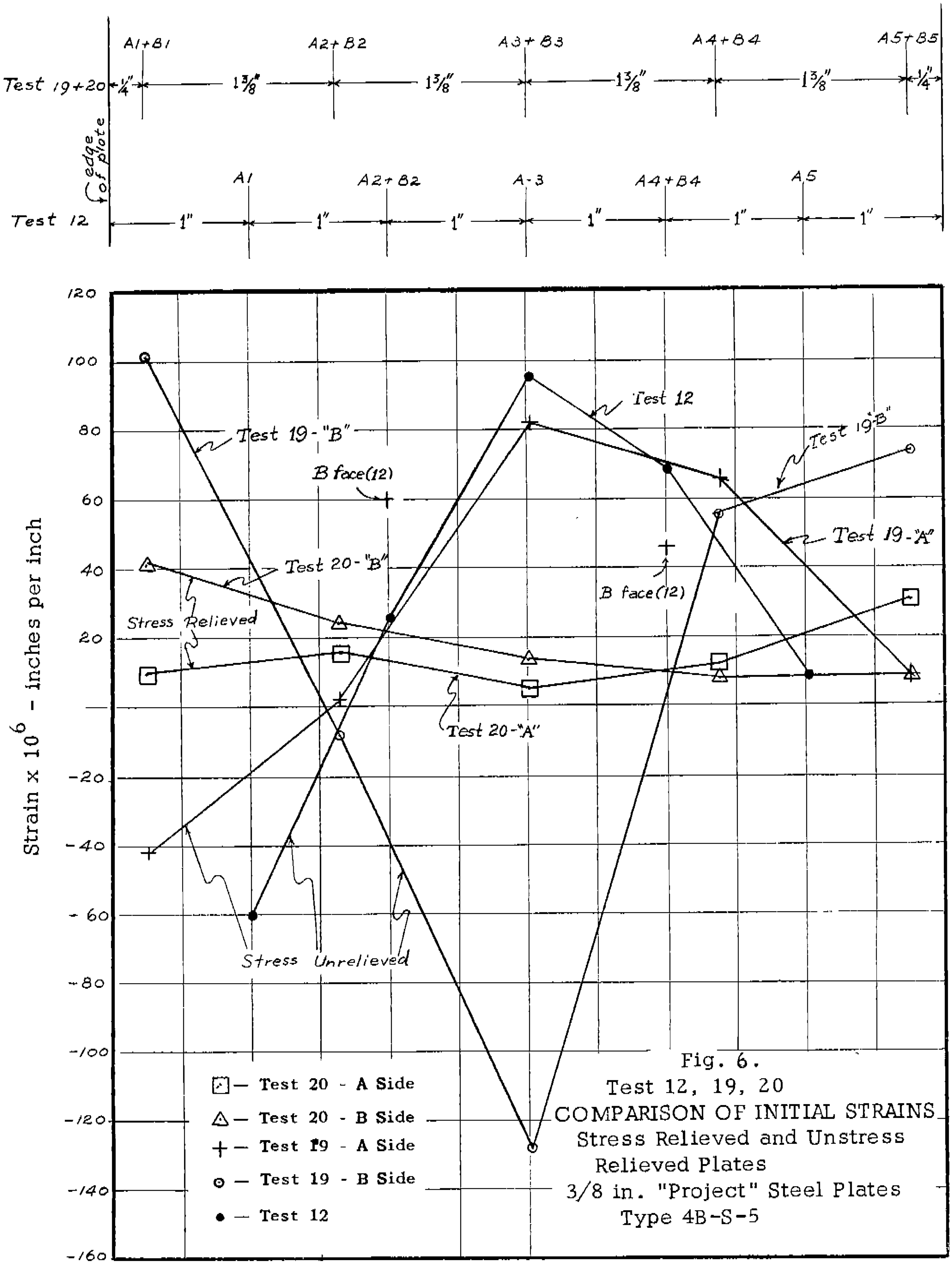

Fig. 6.
Test 12, 19, 20
COMPARISON OF INITIAL STRAINS
Stress Relieved and Unstress Relieved Plates
3/8 in. "Project" Steel Plates
Type 4B-S-5

VII. RECOMMENDED PROCEDURE FOR USE OF THE HOLE RELAXATION METHOD IN THE FIELD

It is recommended that the following techniques be followed in the field when applying the hole relaxation method to the determination of initial stresses:

1. Drill a 1/8-in. diameter guide hole at the point where the stresses are to be evaluated. This hole and all other holes (including the reference hole) should be made using a drill guide to ensure that the hole is normal to the plate surface.

2. Remove paint, rust, and mill scale on both sides of the plate, and smooth the area around the guide hole by sanding or grinding. Finish with a fine emery cloth until the surface is smooth and bright.

3. Choose a reference point on opposite sides of the plate. This can be done by drilling a hole a minimum distance of 12 in. away from the guide hole, if no other reference point is available.

4. Use the centers of the guide and reference holes to establish a reference axis on both sides of the plate; from the reference axis lay out radial lines from the center of the guide hole 60° apart. Scribe a 3/4-in. diameter circle around the center of the guide hole on each side of the plate.

5. Clean the plate with acetone. Center and cement one strain gage along each of the six radial lines on both sides of the plate, with the gage-length edge of the wire grid tangent to the 3/4-in. diameter circle. Allow the gages to air dry or bake as recommended by the manufacturer. Wax the gages lightly after moisture has been removed.

6. Solder flexible wire to gage leads, and attach free ends to the terminal barrier strips. Fasten soldered connections to small plastic blocks cemented to the plate so as to absorb any accidental pull on the gage leads. Attach wire from the gage selector switchboard to the terminal barrier strips.

7. Take a complete set of readings. Read all gages against the dummy compensating gage of the same lot number, resistance, and gage factor. Also read all of the gages on the drilling side of the plate against those on the opposite side. This second set of readings indicates a value that is double the bending strains. Read all gages in both normal and reverse positions.

8. Drill twelve overlapping 1/8-in. holes with their centers 3/16 in. from the center of the guide hole. A jig for drilling these holes is desirable.

9. Take a complete set of readings as in No. 7 above.

10. Subtract the readings for the guide-hole condition from the readings for the final-hole condition. The sign is correct for normal reading, and opposite for reverse readings.

11. Average the strain differences for normal and reverse readings with proper sign.

12. Correct these strain differences by subtracting one-half the bending strain with the proper sign from the gages on the drilling side and adding one-half the bending strain to the gages on the opposite side.

13. Compare gage circuits 180° apart and on both faces of plate. Discard any apparent erroneous strain. Average the remaining values. This gives the values ϕ_A, ϕ_B, and ϕ_C.

14. From the dyadic strain circle construction (or any other preferred procedure), find the principal strain differences ϕ_1 and ϕ_2 and their directions with respect to the reference axis. Substitute the principal strain differences and the known coefficients λ_1 and λ_2 for Arrangement I (1/8-in. edge distance, guide hole enlarged to final 1/2-in. hole) in the following equations:

$$\begin{aligned} \sigma_1 &= \lambda_1 \phi_1 - \lambda_2 \phi_2 \\ \sigma_2 &= \lambda_1 \phi_2 - \lambda_2 \phi_1 \end{aligned} \qquad (2)$$

This gives the principal stresses existing in the plate at the point being investigated.

VIII. SUMMARY

1. The hole method may be used to determine stresses with a probable accuracy of 500 to 1000 psi. In the tensile tests performed, 75% of the tests checked within 500 psi and all tests checked within 1000 psi.

2. With the exercise of proper care in the determination of the experimental coefficients of sensitivity for stress, λ_1 and λ_2, these coefficients will be independent of stresses initially present in the calibrating plate. Measurements in a field-erected structure, however, will yield the sum of the stresses resulting from fabrication and applied loads, plus the stresses initially present in the plate prior to its incorporation into the structure.

3. A preferred gage arrangement is the double Arrangement I rosette (6 gages at 60° to one another as shown in Fig. 3). Theoretical considerations indicate the desirability of using small holes and placing the gages very close to the hole. In practice, however, with the type of gage used in these studies, a 1/2-in. diameter final hole with 1/8-in. edge distance is optimum.

4. From the point of view of local stresses introduced by the drilling operation, the best method is to drill a scalloped 1/2-in. hole by overlapping twelve 1/8-in. holes. A 1/2-in. high-speed twist drill may be used, but great care should be taken in the drilling operation not to heat the gages or to damage them with chips.

5. As a result of this study, it appears that more work should be done on the determination of coefficients for smaller final hole sizes (1/4 and 1/8 in.) and on the use of the hole method for determining the values of the initial stresses in as-rolled plates of structural steel.

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APPENDIX A

DERIVATION OF THEORETICAL BASES FOR THE HOLE RELAXATION METHOD

For theoretical purposes, we consider a region of a plate in which a distribution of stresses is present. The stress distribution considered is in the plane of the plate, and in this analysis, is assumed as two-dimensional. Bending stresses and stress gradients in the direction perpendicular to the plane of the plate are deliberately neglected.

No restriction is imposed on the two-dimensional stress distribution, which is assumed unknown; the principal stresses are unknown in magnitude and direction.

The difficulty of measuring initial or residual stresses lies in the fact that they are permanent, so that it is not possible to read the strain gages before and after the application of the stresses and to work out the stress distribution in the structure from the elongation readings. The methods utilized for determining residual stresses are usually destructive. The present method, which is only partially destructive, consists of the following procedure:

1) Apply strain gages to the plate in the neighborhood of a chosen point with an opportune disposition.

2) Drill a hole at the point considered.

3) Read the gages, before and after drilling. The difference in the readings allows the determination of the direction and magnitude of the principal stresses.

The unknowns of the problem as formulated are three: the principal stresses σ_1 and σ_2, and the angle formed by the direction of one of them to a direction arbitrarily chosen for reference. Therefore, three independent measurements will be necessary.

At the point considered, assume a system of polar coordinates, with center O and angle θ measured from a pre-established direction x. (Fig. A-1).

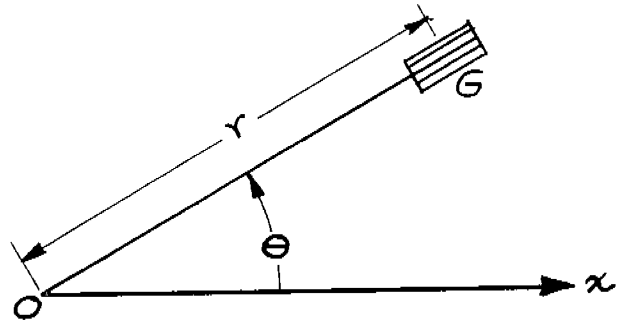

Fig. A-1. Elementary Polar Coordinate System

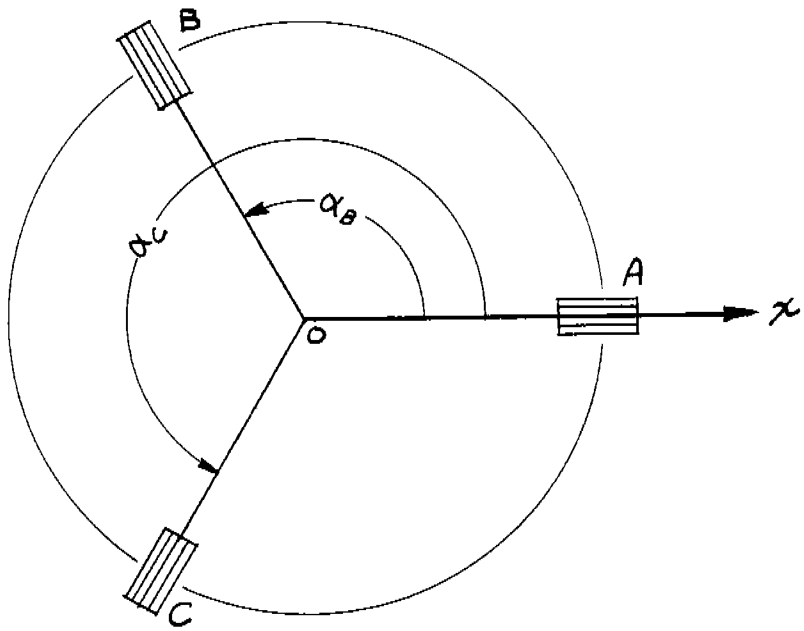

Fig. A-2. Placement of Gages for Measuring Strain in the Radial Direction for Arrangement I

At a distance r from the center O, place a strain gage G, having the wires parallel to the radial direction i.e., measuring changes in strain in the radial direction.

Now, let us imagine that we can vary θ, that is, that we are able to rotate the gage around the center O and at the same time read its indication. Furthermore, we imagine that we are able to rotate the gage about the center and simultaneously read it once before drilling the hole at O and again after drilling the hole at O.

Thus, we can establish a polar diagram of the difference between gage indications in the solid and in the perforated plate. By inspecting such a diagram, we observe the position of its maximum and minimum, obviously corresponding to the directions of the principal stresses.

The system composed of a gage measuring in the radial direction we call Arrangement I.

Obviously, in reality, we cannot rotate one gage at a given distance from the center. Therefore we choose three arbitrary directions, OA, OB and OC, forming angles $\alpha_A = 0$, α_B, and α_C, respectively, with the X- direction; we then locate radially oriented strain gages at points A, B and C. (Fig. A-2)

After reading the gages at A, B, and C, (or setting indicators at 0), we drill a hole at the center. The differences in the readings resulting from the hole we call ϕ_A, ϕ_B, ϕ_C.

From these three independent readings, knowing that the polar diagram previously described will be bisymmetric, we can construct the whole diagram. In particular, we can find the maximum and minimum values of ϕ (ϕ_1 and ϕ_2) and the angles α_1 and α_2 formed by the directions of ϕ_1 and ϕ_2 with the x-axis.

The construction using the Dyadic circle is best suited for this purpose. No analytic deductions are needed for the application of this method. Only a calibration showing a relationship between ϕ_1, ϕ_2 and the principal stresses σ_1, σ_2 is necessary. Analytic relationships can also be found for the point gage (when the strain gage is assumed to act at a point) in different arrangements as illustrated by the following method:

Arrangement I

Consider a region of a plate where the stress distribution can be taken as uniform. We assume an origin O and an x-axis from which we measure angles (Fig. A-3). θ is the variable angle, and α_1 is the angle formed by the direction of one principal stress (σ_1) with the x-axis. At radius r and angle θ (variable) with x, we consider a point gage G measuring in the radial direction (Arrangement I). The angle of the gage G with σ_1 will be $(\theta - \alpha_1)$.

α_1 is unknown in the determination. However, the stresses will have maximum and minimum values when $\theta = \alpha_1$ and when $\theta = \alpha_1 \pm \frac{\pi}{2}$. Therefore, to simplify the expressions, we put (See Fig. A-3):

$$\gamma = \theta - \alpha_1 \qquad \text{(A-1)}$$

We now compute the indication of the point gage G while γ varies in the solid plate, before the hole is drilled. We indicate with σ_{ro} the stress in the radial direction while γ varies, and with σ_{to} the stress in the direction orthogonal to the radius while γ varies (Fig. A-3).

Then*

$$\sigma_{ro} = \sigma_1 \cos^2\gamma + \sigma_2{}^2 \sin^2\gamma = \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 - \sigma_2}{2} \cos 2\gamma \qquad \text{(A-2)*}$$

and

$$\sigma_{to} = \sigma_2 \cos^2\gamma + \sigma_1 \sin^2\gamma = \frac{\sigma_1 + \sigma_2}{2} - \frac{\sigma_1 - \sigma_2}{2} \cos 2\gamma \qquad \text{(A-3)}$$

The strain in the radial direction is

$$\epsilon_{ro} = \frac{1}{E}(\sigma_{ro} - \nu\,\sigma_{to})$$

$$= \frac{1}{E}\left[\frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 - \sigma_2}{2} \cos 2\gamma - \nu \frac{\sigma_1 + \sigma_2}{2} + \nu \frac{\sigma_1 - \sigma_2}{2} \cos 2\gamma\right]$$

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*See Timoshenko, Theory of Elasticity, I, (9)

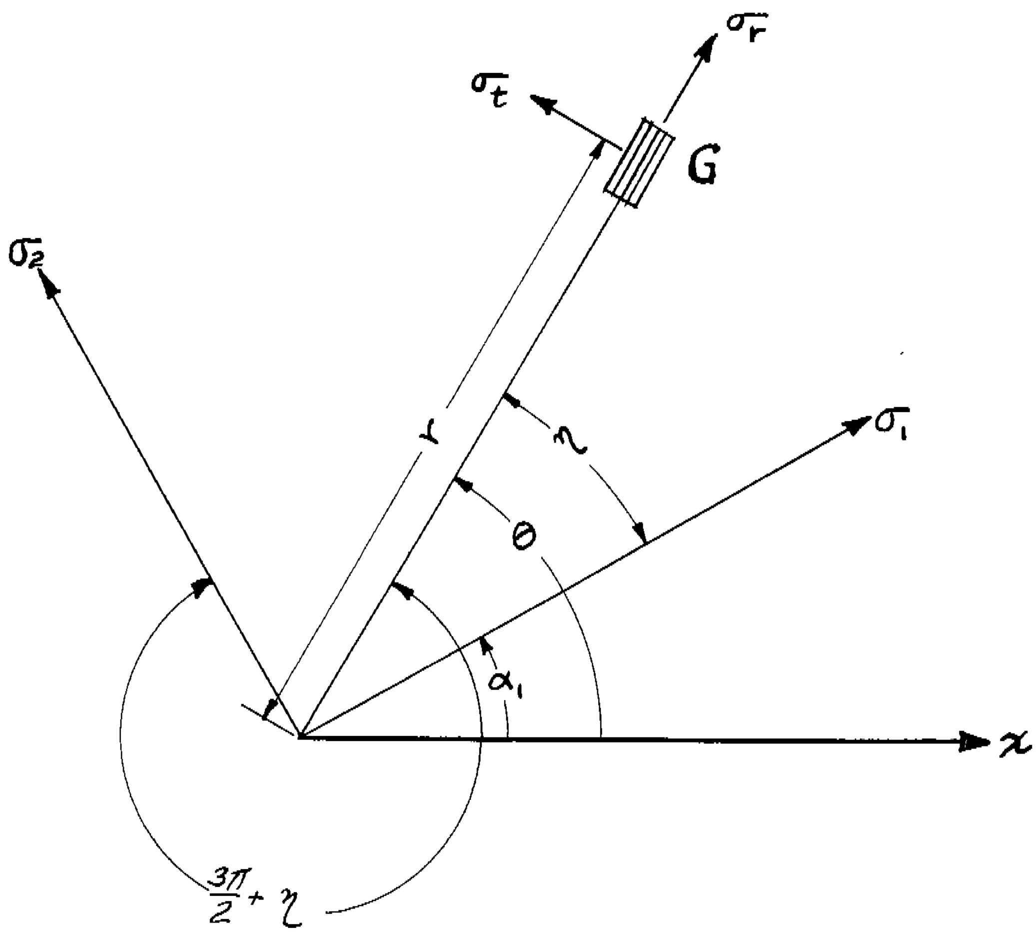

Fig. A-3. Description of Terms as Used in Arrangement I

$$\epsilon_{ro} = \frac{1}{E}\left[\frac{\sigma_1 + \sigma_2}{2}(1 - \nu) + (1 + \nu)\frac{\sigma_1 - \sigma_2}{2}\cos 2\gamma\right] \qquad \text{(A-4)*}$$

From this expression for the solid plate follows the construction to find the direction and magnitude of the principal elongations from three elongations in arbitrary directions.**

We now form the corresponding expressions for the perforated plate, with hole of radius a.

Kirsch's equations for uniaxial stress ($\sigma_1 \neq 0$, $\sigma_2 = 0$) give for stress in the radial direction:

$$\sigma_r = \frac{\sigma_1}{2}\left(1 - \frac{a^2}{r^2}\right) + \frac{\sigma_1}{2}\left(1 + 3\frac{a^4}{r^4}\right) - 4\frac{a^2}{r^2}\right) \cos 2\gamma \qquad \text{(A-5a)}$$

and for stress in the direction orthogonal to the radius:

$$\sigma_t = \frac{\sigma_1}{2}\left(1 + \frac{a^2}{r^2}\right) - \frac{\sigma_1}{2}\left(1 + 3\frac{a^4}{r^4}\right) \cos 2\gamma \qquad \text{(A-5b)}$$

For the other uniaxial stress $\sigma_1 = 0$, $\sigma_2 \neq 0$, substitute $(\frac{3\pi}{2} + \gamma)$ for γ (See Fig. A-3).
and find

$$\sigma_r = \frac{\sigma_2}{2}\left(1 - \frac{a^2}{r^2}\right) + \frac{\sigma_2}{2}\left(1 + 3\frac{a^4}{r^4} - 4\frac{a^2}{r^2}\right) \cos (3\pi + 2\gamma)$$

$$\text{(A-6)}$$

$$\sigma_t = \frac{\sigma_2}{2}\left(1 + \frac{a^2}{r^2}\right) - \frac{\sigma_2}{2}\left(1 + 3\frac{a^4}{r^4}\right) \cos (3\pi + 2\gamma)$$

---

*See also Hetenyi, Handbook of Experimental Stress Analysis, Chap. 3, Par. 5, Eq. 19

**Hetenyi, Chap. 9, Par. 34, Fig. 9--26

Substituting $\cos(3\pi + 2\eta) = -\cos 2\eta$ in Equations A-6 and adding Equations A-5a and A-5b to their respective values in Equations A-6, the expressions given below for σ_r and σ_t for both σ_1 and σ_2 different from zero are obtained.

$$\sigma_r = \frac{\sigma_1 + \sigma_2}{2}\left(1 - \frac{a^2}{r^2}\right) + \frac{\sigma_1 - \sigma_2}{2}\left(1 + 3\frac{a^4}{r^4} - 4\frac{a^2}{r^2}\right)\cos 2\eta$$

$$\sigma_t = \frac{\sigma_1 + \sigma_2}{2}\left(1 + \frac{a^2}{r^2}\right) - \frac{\sigma_1 - \sigma_2}{2}\left(1 + 3\frac{a^4}{r^4}\right)\cos 2\eta \qquad \text{(A-7)}$$

From these, the strain in the radial direction is

$$\epsilon_r = \frac{1}{E}(\sigma_r - \nu\sigma_t)$$

$$= \frac{1}{E}\left\{\left[\frac{\sigma_1 + \sigma_2}{2}\left(1 - \frac{a^2}{r^2}\right) - \nu\frac{\sigma_1 + \sigma_2}{2}\left(1 + \frac{a^2}{r^2}\right)\right] + \left[\frac{\sigma_1 - \sigma_2}{2}\left(1 + \frac{3a^4}{r^4} - 4\frac{a^2}{r^2}\right) + \nu\frac{\sigma_1 - \sigma_2}{2}\left(1 + 3\frac{a^4}{r^4}\right)\right]\cos 2\eta\right\}$$

$$= \frac{1}{E}\left\{\left[\frac{\sigma_1 + \sigma_2}{2} - \frac{a^2}{r^2}\frac{\sigma_1 + \sigma_2}{2} - \nu\frac{\sigma_1 + \sigma_2}{2} - \nu\frac{a^2}{r^2}\frac{\sigma_1 + \sigma_2}{2}\right] + \left[(1 + \nu)\frac{\sigma_1 - \sigma_2}{2} + \frac{\sigma_1 - \sigma_2}{2}\left(3\frac{a^4}{r^4} - 4\frac{a^2}{r^2}\right) + \nu\frac{\sigma_1 - \sigma_2}{2}3\frac{a^4}{r^4}\right]\cos 2\eta\right\}$$

$$\epsilon_r = \frac{1}{E}\left\{\frac{\sigma_1 + \sigma_2}{2}\left[1 - \nu - \frac{a^2}{r^2}(1 + \nu)\right] + \frac{\sigma_1 - \sigma_2}{2}\left[(1 + \nu)\left(1 + 3\frac{a^4}{r^4}\right) - 4\frac{a^2}{r^2}\right]\cos 2\eta\right\} \qquad \text{(A-8)}$$

The change in strain in the radial direction caused by drilling the hole in the stressed plate is what we have called ϕ, a function of the angle η.

$$\phi = \epsilon_r - \epsilon_{ro}$$

$$\phi = \frac{1}{E}\left\{\frac{\sigma_1 + \sigma_2}{2}\left[1 - \nu - \frac{a^2}{r^2}(1 + \nu)\right] + \frac{\sigma_1 - \sigma_2}{2}\left[(1 + \nu)(1 + \frac{3a^4}{r^4}) - 4\frac{a^2}{r^2}\right]\cos 2\gamma\right.$$

$$\left. - \left[\frac{\sigma_1 + \sigma_2}{2}(1 - \nu) + (1 + \nu)\frac{\sigma_1 - \sigma_2}{2}\cos 2\gamma\right]\right\} \qquad \text{(A-9)}$$

$$\phi = \frac{1}{E}\left\{- \frac{a^2}{r^2}(1 + \nu)\frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 - \sigma_2}{2}\left[(1 + \nu)\frac{3a^4}{r^4} - 4\frac{a^2}{r^2}\right]\cos 2\gamma\right\}$$

$$= K_1 + K_2 \cos 2\gamma \qquad \text{(A-10)}$$

where

$$K_1 = - \frac{1}{E}\frac{a^2}{r^2}(1 + \nu)\frac{\sigma_1 + \sigma_2}{2}$$

$$K_2 = \frac{1}{E}\frac{\sigma_1 - \sigma_2}{2}\left[(1 + \nu)\frac{3a^4}{r^4} - 4\frac{a^2}{r^2}\right] \qquad \text{(A-11)}$$

are constants because of our assumptions. The gage is always at the same distance r from the center while γ varies. The Dyadic circle (Fig. A-4) illustrates the relationship.

The Dyadic Construction

We start again from three abritrary directions, forming angles α_A, α_B, α_C with the x-axis. In those directions we measure ϕ_A, ϕ_B and ϕ_C, the strain changes caused by drilling the hole.

To construct the illustrative example, Fig. A-5, the strain differences are multiplied by 10^6 and the angles are:

$\phi_A = 300$ $\alpha_A = 0$

$\phi_B = 600$ $\alpha_B = 120°$

$\phi_C = 500$ $\alpha_C = 240°$

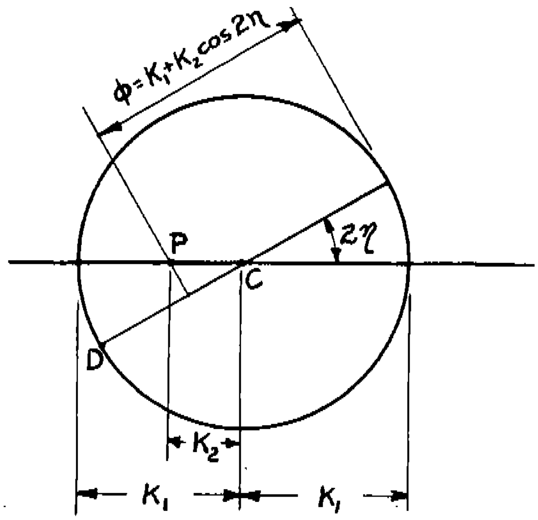

Fig. A-4. The Dyadic Circle

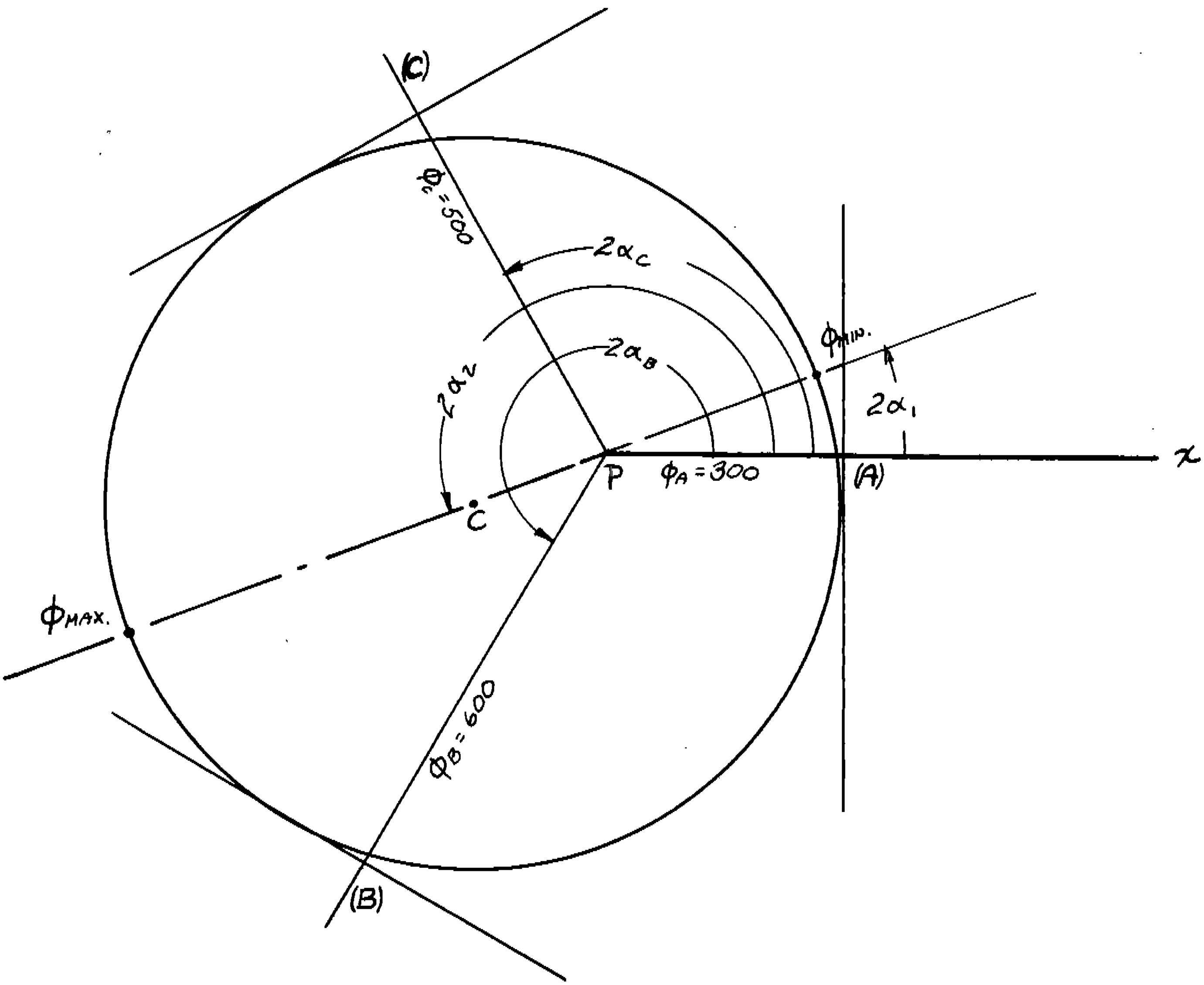

Fig. A-5. Example of the Dyadic Construction

A center, P, is assumed. With P as origin, 3 lines, A, B, and C are drawn at angles of $2\alpha_A$, $2\alpha_B$, and $2\alpha_C$ with the x-direction.

Along A, a segment is taken proportional to ϕ_A; along B, a segment is taken proportional to ϕ_B; and along C, a segment is taken proportional to ϕ_C. At the end of each segment, a normal to the line is drawn. The normal excludes the region not containing P if ϕ is positive; it excludes the region containing P if ϕ is negative. In the example, ϕ_A, ϕ_B, and ϕ_C are positive, so that the region for the subsequent construction is the triangle included within the three normals.

The Dyadic circle is drawn tangent to the three normals. A line from the center C to any point D on the circle is the radius K_1 and a perpendicular to this line from P establishes the length $K_2 \cos 2\eta$ from the center C (See Fig. A-4).

The maximum and minimum values of ϕ are determined by the diameter of which PC is a segment; the diameter also gives $2\alpha_1$ and $2\alpha_2$, or the orientation α_1 and α_2 of the principal stresses with respect to the x-direction.

In Arrangement I, the maximum indication is of opposite sign to the maximum stress measured, and it occurs for $\eta = 0$.

One may observe that the real strain gage covers a finite area, which is not negligible when compared to the hole and to the region of the plate in which we operate. It is also known that strain gages measure strains not only in the direction parallel to the grid also in the orthogonal direction. This will certainly affect the readings, to the extent that the computation of stresses in the plate from the simple indication, the assumption that the centroid of the gage is the point representing the whole gage, and the use of the deduced formulae are not valid.

These considerations, however, will not change the periodicity of the indication and the double symmetry of the polar diagram. Therefore, the construction determining the maximum and minimum values of ϕ and their orientation will still be valid. The passage from $\phi_{maximum}$ and $\phi_{minimum}$ to σ_1 and σ_2 must necessarily be done by calibration.

Arrangement II

We consider again a polar coordinate system and a radius vector rotating about the origin, carrying a point gage at constant distance from the origin and measuring strains in the direction orthogonal to the radius. The angle measured is still between an arbitrary x-axis and the measuring direction of the gage (Fig. A-6). The reason for this convention is that it permits a combination of the indications of Arrangements I and II to form Arrangement III.

With this convention, we repeat the analysis as for Arrangement I.

The strain ϵ_{to} in the direction defined by η in the solid plate is still the same as for Arrangement I (See Eq. A-4), since definition of η is the same. With relation to the hole, however, the gage measures a tangential strain:

$$\epsilon_{to} = \frac{1}{E}\left[\frac{\sigma_1 + \sigma_2}{2}(1 - \nu) + (1 + \nu)\frac{\sigma_1 - \sigma_2}{2}\cos 2\eta\right] \tag{A-12}$$

In the perforated plate, the strain measured by the gage is

$$\epsilon_t = \frac{1}{E}\left[\sigma_t(\frac{\pi}{2} - \eta) - \nu\sigma_r(\frac{\pi}{2} - \eta)\right] \tag{A-13}$$

and the stresses are

$$\sigma_t(\frac{\pi}{2} - \eta) = \frac{\sigma_1 + \sigma_2}{2}(1 + \frac{a^2}{r^2}) - \frac{\sigma_1 - \sigma_2}{2}(1 + 3\frac{a^4}{r^4})\cos 2(\frac{\pi}{2} - \eta)$$

$$= \frac{\sigma_1 + \sigma_2}{2}(1 + \frac{a^2}{r^2}) + \frac{\sigma_1 - \sigma_2}{2}(1 + 3\frac{a^4}{r^4})\cos 2\eta \tag{A-14}$$

$$\sigma_r(\frac{\pi}{2} - \eta) = \frac{\sigma_1 + \sigma_2}{2}(1 - \frac{a^2}{r^2}) + \frac{\sigma_1 - \sigma_2}{2}(1 + 3\frac{a^4}{r^4} - 4\frac{a^2}{r^2})\cos 2(\frac{\pi}{2} - \eta)$$

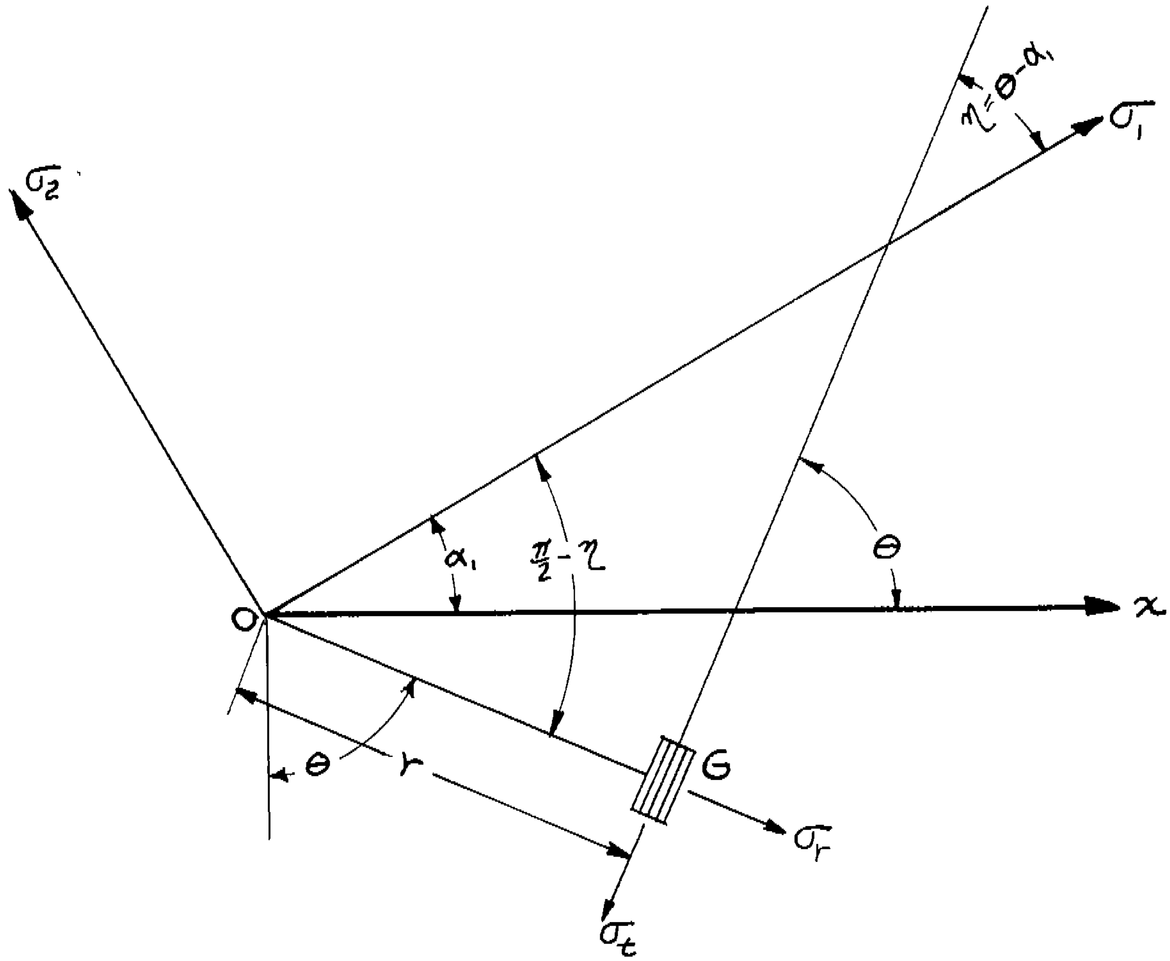

Fig. A-6. Description of Terms as Used in Arrangement II

$$= \frac{\sigma_1 + \sigma_2}{2}\left(1 - \frac{a^2}{r^2}\right) - \frac{\sigma_1 - \sigma_2}{2}\left(1 + 3\frac{a^4}{r^4} - 4\frac{a^2}{r^2}\right)\cos 2\eta$$

(A-14)
continued

(since $\cos(2\pi - \eta) = -\cos 2\eta$)

$$\epsilon_t = \frac{1}{E}\left\{\frac{\sigma_1 + \sigma_2}{2}\left(1 + \frac{a^2}{r^2}\right) + \frac{\sigma_1 - \sigma_2}{2}\left(1 + 3\frac{a^4}{r^4}\right)\cos 2\eta \right.$$

$$\left. -\nu\frac{\sigma_1 + \sigma_2}{2}\left(1 - \frac{a^2}{r^2}\right) + \nu\frac{\sigma_1 - \sigma_2}{2}\left(1 + 3\frac{a^4}{r^4} - 4\frac{a^2}{r^2}\right)\cos 2\eta\right\}$$

$$= \frac{1}{E}\left\{\frac{\sigma_1 + \sigma_2}{2}(1 - \nu) + \frac{\sigma_1 + \sigma_2}{2}\frac{a^2}{r^2}(1 + \nu)\right.$$

(A-15)

$$\left. + \frac{\sigma_1 - \sigma_2}{2}\left[\left(1 + 3\frac{a^4}{r^4}\right)(1 + \nu) - 4\nu\frac{a^2}{r^2}\right]\cos 2\eta\right\}$$

$$\epsilon_t = \frac{1}{E}\left\{\frac{\sigma_1 + \sigma_2}{2}\left[(1 - \nu) + \frac{a^2}{r^2}(1 + \nu)\right]\right.$$

$$\left. + \frac{\sigma_1 - \sigma_2}{2}\left[\left(1 + 3\frac{a^4}{r^4}\right)(1 + \nu) - 4\nu\frac{a^2}{r^2}\right]\cos 2\eta\right\}$$

the change in strain may be calculated as follows.

As before, $\phi = \epsilon_t - \epsilon_{to}$

$$\phi = \frac{1}{E}\left\{\frac{\sigma_1 + \sigma_2}{2}\left[(1 - \nu) + \frac{a^2}{r^2}(1 + \nu)\right] + \frac{\sigma_1 - \sigma_2}{2}\left[\left(1 + 3\frac{a^4}{r^4}\right)(1 + \nu) - 4\nu\frac{a^2}{r^2}\right]\right.$$

$$\left.\cos 2\eta - \frac{\sigma_1 + \sigma_2}{2}(1 - \nu) - (1 + \nu)\frac{\sigma_1 - \sigma_2}{2}\cos 2\eta\right\}$$

$$\phi = \frac{1}{E}\left\{\frac{\sigma_1 + \sigma_2}{2}\,\frac{a^2}{r^2}(1 + \nu) + \frac{\sigma_1 - \sigma_2}{2}\left[3\frac{a^4}{r^4}(1 + \nu) - 4\nu\frac{a^2}{r^2}\right]\cos 2\eta\right\} \qquad \text{(A-16)}$$

$$= C + D\cos 2\eta$$

The deductions and the construction valid for Arrangement I are also valid here. In Arrangement II, the maximum indication is of the same sign as the maximum stress measured, and it is at $\eta = 0$.

Arrangement III

In order to increase the sensitivity, Arrangements I and II can be coupled in an arrangement in which two gages are connected so that they are parallel (Fig. A-7). One of them is radial as in Arrangement I; the other is orthogonal to the radius as in Arrangement II.

The two gages are connected to opposite sides of the measuring bridge, so that the two indications can be added (Fig. A-8). Denoting by ϕ_I the indication of Arrangement I and by ϕ_{II} the indication of Arrangement II,

$$\phi_{III} = \phi_I + \phi_{II} \qquad \text{(A-17)}$$

Arrangement III is not preferred to Arrangement I: it is more expensive and lack of space may make it difficult to cement six gages around the zone of the hole.

Evaluation of the Stresses

By using the procedures described, it is possible to find the values ϕ_1 and ϕ_2 (the maximum and minimum changes in strain), as well as the orientation of their directions, which coincide respectively with the directions of the principal stresses σ_1 and σ_2. If certain considerations are ignored, such as the finite strain gage area and influence of stresses perpendicular to the gage

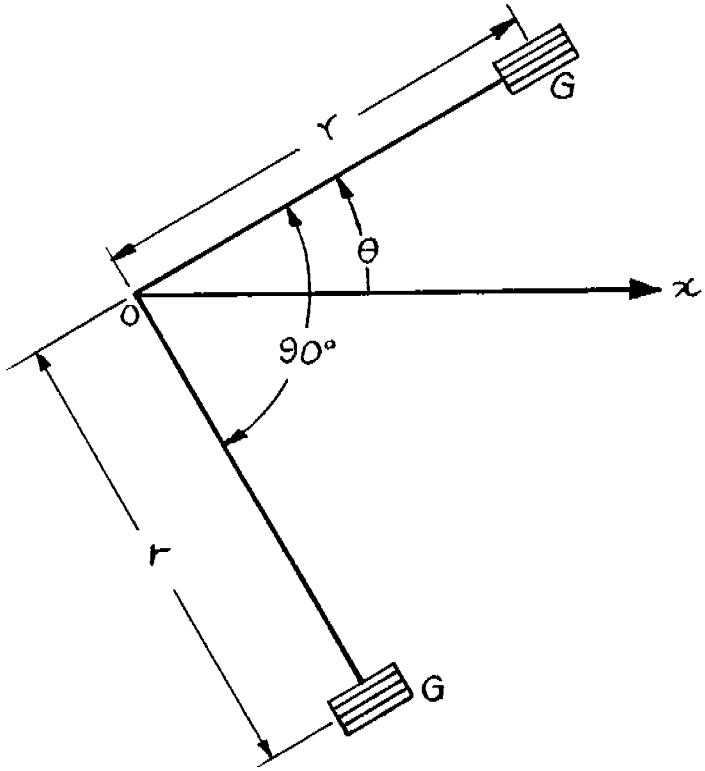

Fig. A-7. Placement of Gages--Arrangement III

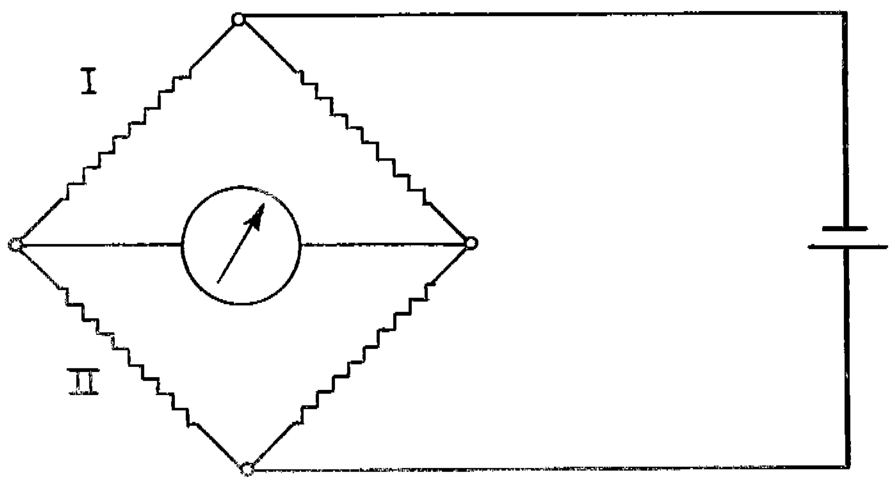

Fig. A-8. Connection of Gages in Measuring Bridge for Arrangement III length upon the gage reading, the magnitudes of the principal stresses may be calculated by solution of Eq. A-10. Using Arrangement I, for example, and rewriting Eq. A-10, first with $\phi = \phi_1, \eta = 0$, and then with $\phi = \phi_2, \eta = \frac{\pi}{2}$:

$$\phi_1 = \frac{1}{E}\left\{-\frac{a^2}{r^2}(1+\nu)\frac{\sigma_1+\sigma_2}{2} + \frac{\sigma_1-\sigma_2}{2}\left[(1+\nu)\frac{3a^4}{r^4} - \frac{4a^2}{r^2}\right]\right\}$$

$$\phi_2 = \frac{1}{E}\left\{-\frac{a^2}{r^2}(1+\nu)\frac{\sigma_1+\sigma_2}{2} - \frac{\sigma_1-\sigma_2}{2}\left[(1+\nu)\frac{3a^4}{r^4} - \frac{4a^2}{r^2}\right]\right\} \tag{A-18}$$

or:

$$\phi_1 = K_1(\sigma_1+\sigma_2) + K_2(\sigma_1-\sigma_2)$$

$$\phi_2 = K_1(\sigma_1+\sigma_2) - K_2(\sigma_1-\sigma_2) \tag{A-19}$$

where

$$K_1 = -\frac{1}{2E}\frac{a^2}{r^2}(1+\nu)$$

and

$$K_2 = \frac{1}{2E}\left[(1+\nu)\frac{3a^4}{r^4} - \frac{4a^2}{r^2}\right] \tag{A-20}$$

K_1 and K_2 are, therefore, functions <u>only</u> of the geometry of the system and the material. Solving Eq. A-19 for σ_1 and σ_2,

$$\sigma_1 = \frac{(K_1+K_2)\phi_1 - (K_1-K_2)\phi_2}{4K_1K_2}$$

$$\sigma_2 = \frac{(K_2-K_1)\phi_1 - (K_1+K_2)\phi_2}{4K_1K_2} \tag{A-21}$$

$$\lambda_1 = \frac{K_1+K_2}{4K_1K_2}$$

$$\lambda_2 = \frac{K_1-K_2}{4K_1K_2} \tag{A-22}$$

we see that for the ideal gage,

$$\sigma_1 = \lambda_1 \phi_1 - \lambda_2 \phi_2$$

$$\sigma_2 = \lambda_1 \phi_2 - \lambda_2 \phi_1 \qquad \text{(A-23)}$$

In considering the real gage, it is necessary that we obtain the constants λ_1 and λ_2 of Eq. A-23 by calibration. Referring to Eq. A-21, let λ_1 be the value of σ_1 when $\phi_1 = 1$, $\phi_2 = 0$, and $\eta = 0$ (uniaxial unit strain in direction of gage), and let λ_2 be the value of σ_2 when $\phi_2 = 1$, $\phi_1 = 0$, and $\eta = \frac{\pi}{2}$ (uniaxial unit strain orthogonal to gage). Then for $\phi_1 \neq 0$, $\phi_2 \neq 0$, by superposition,

$$\sigma_1 = \lambda_1\phi_1 - \lambda_2\phi_2$$

$$\sigma_2 = \lambda_1 \phi_2 - \lambda_2\phi_1$$

which is the same as the result obtained above.

It is thus seen that once the calibration coefficients λ_1 and λ_2 and the values of ϕ_1 and ϕ_2 are determined, the direction and magnitude of the principal stresses may be calculated.

A comparison of calculated values of λ versus experimentally determined values of λ is included in the text of the report.