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Probabilistic Fatigue Crack Growth Analysis of Offshore Structures, with Reliability Updating Through Inspection

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ABSTRACT

A stochastic model for fatigue crack growth is applied, which accounts for uncertainties in loading, initial and critical defect sizes, material parameters including spatial variation, and in the uncertainty related to computation of the stress intensity factor. Failure probabilities are computed by first- and second-order reliability methods and sensitivity factors are determined. Model updating based on in-service inspection results is formulated within the first-order reliability method. Updated failure probabilities are computed and the distributions of the basic variables are updated. Two types of in-service inspection results are used to update the computed failure probabilities. Inspections which do not detect a crack are used and the inspection uncertainty is included in terms of the distribution of nondetected crack sizes by the specific inspection method. Inspections which detect a crack are also included and the inspection uncertainty is included through the uncertainty in the measured crack size. The formulations are presented for updating based on one or more inspections. A similar formulation for reliability updating after repair is provided within the same framework.

INTRODUCTION

In offshore steel structures flaws are inherent due to, e.g., notches, welding defects and voids. Macro cracks can originate from these flaws and under time varying loading grow to a critical size causing catastrophic failure. The conditions governing the fatigue crack growth are the geometry of the structure and crack initiation site, the material characteristics, the environmental conditions and the loading. In general, these conditions are of random nature. The appropriate analysis and design methodologies should therefore be based on probabilistic methods.

In recent years considerable research efforts have been reported on probabilistic modeling of fatigue crack growth based on a fracture mechanics approach, see, e.g., [1-8]. In particular, stable crack growth due to cyclic loading has been studied. This paper presents a stochastic model for this crack growth phase for which linear elastic fracture mechanics is applicable. A common model is formulated for constant and variable amplitude loading. The model is developed for a cracked panel and has been shown to be in good agreement with experimental test results. A generalization to a semi-elliptical surface crack is straightforward and has been successfully implemented. Uncertainties in the loading conditions, in the computation of the stress intensity factor, in the initial crack geometry, and in the material properties are included. In particular the material resistance against crack growth is modeled as a spatial random process thus accounting for material variations within each specimen.

The probability that the crack size exceeds a critical size during some time period is of interest. It is demonstrated how this event is formulated in terms of a limit state function with a corresponding safety margin and how the probability of failure can be calculated by a first- or second-order reliability method. The critical crack size may refer to growth through the thickness or to a size where a brittle fracture or plastic collapse occur. The critical crack size can be modeled as a deterministic or as a random quantity.

Inspections are frequently made for structures in service. Some inspections result in the detection of a crack while others give no detection. The size of a detected crack is measured either directly or indirectly through a non destructive inspection method, where the measured signal is interpreted as a crack size. Neither the measurement nor the interpretation are possible in an exact way and the resulting inspection result is consequently of random nature. When the inspection does not reveal a crack this does not necessarily mean that no crack is present. A detectable crack is only detected by a certain probability depending on the size of the crack and on the inspection method. Whether or not a crack is detected, the inspection provides additional information which can be used to update the reliability and/or the distribution of the basic variables. This can lead to, e.g., modifications of inspection plans, change in inspection method, or a decision on repair or replacement. The paper describes inspection results in terms of event margins and formulates the updating in terms of these event margins and the safety margin. The use of first-order reliability methods to perform the calculations is demonstrated.

When a repair of a detected crack is made and a new reliability analysis is performed, it is important that the new analysis accounts for the information that a repair was necessary. Often it is not possible to determine if the unexpected large crack size has been caused by a large initial size, by material properties poorer than anticipated, or by a loading of the crack tip area larger than anticipated. The paper demonstrates how information obtained in connection with a repair is introduced.

For welded structures a crack is generally assumed to be present after fabrication. The analysis method can, however, in a simple manner include a random crack initiation period for which a separate model can be formulated.

FATIGUE CRACK GROWTH MODEL

In a linear elastic fracture mechanics approach the increment in crack size, Δa, during a load cycle is related to the range of the stress intensity factor, ΔK, for the load cycle. A simple relation which is sufficient for most purposes was proposed by Paris and Erdogan, [9]

$$\Delta a = C\,(\Delta K)^m, \quad \Delta K > 0 \tag{1}$$

The crack growth equation is used without a positive lower threshold on ΔK below which no crack growth occurs. The equation was proposed based on experimental results, but is also the result of various mechanical and energy based models, see, e.g., [9,10]. C and m are material constants. The crack increment in one cycle is generally very small compared to the crack size and (1) is consequently written in a 'kinetic' form as

$$\frac{da}{dN} = C\,(\Delta K)^m, \quad \Delta K > 0 \tag{2}$$

where N is the number of stress cycles. The stress intensity factor K is computed by linear elastic fracture mechanics and is expressed as

$$K = \sigma\, Y(a)\sqrt{\pi a} \tag{3}$$

where σ is the far-field stress and $Y(a)$ is the geometry function. To explicitly account for uncertainties in the calculation of K, the geometry function is written as $Y(a) = Y(a, \mathrm{Y})$, where Y is a vector of random parameters. Inserting (3) in (2) and separating the variables leads to the differential equation

$$\frac{da}{Y(a,\mathrm{Y})^m (\sqrt{\pi a}\,)^m} = C\,(\Delta\sigma)^m dN, \; a(0)=a_0 \tag{4}$$

where a_0 is the initial crack size. The equation is applied both for constant and for variable amplitude loading, thus ignoring possible sequence effects. Also a possible effect of the mean stress or R-ratio is ignored.

Eqs.(1-4) describe the crack size as a scalar a, which for a cracked panel is the crack length. For a surface or embedded crack a description of the crack depth, crack length and crack shape is necessary. It is common practice to assume a semi-elliptical initial shape for a surface crack and to assume that the shape remains semi-elliptical during the crack growth. In that case the crack depth a and the length $2c$ describe the crack. The differential equation (2) is replaced by a pair of coupled equations, see e.g. [11].

Solutions to (4) are smooth curves which do not intermingle. This is in contrast to experimental results as reported in, e.g., [12]. As a consequence the crack growth model is randomized as, [7]

$$\frac{da}{dN} = \frac{C_1}{C_2(a)}(\Delta K)^m \tag{5}$$

where C_1 is a random variable modeling variations in C from specimen to specimen, while $C_2(a)$ is a stationary log-normal process modeling variations in C within each specimen. The expected value of $C_2(a)$ is taken as one. The random model in (5) has three properties, which are experimentally observed in the test results reported in [12]:

- sample curves of a versus N are irregular and not very smooth,
- sample curves of a versus N become more smooth for larger values of a,
- sample curves of a versus N intermingle, in particular for smaller values of a.

To estimate the correlation properties of the random process $C_2(a)$ a statistical analysis of the test data from [12] has been carried out, [7]. The correlation function $\rho_2(\Delta a)$ for $C_2(a)$ is shown to decrease to zero very rapidly with Δa. The correlation radius r_C is defined as

$$r_C = \int_{-\infty}^{\infty} \rho_2(x)\,dx \tag{6}$$

and has been estimated as 0.12 mm for the aluminum alloy in the experiments of [12]. The variance of C_2 has been estimated as 0.062 for the same data. The variance is expected to be significantly larger for crack growth in material in the heat affected zone or in the weld material. Non-proprietary data are, however, not available for estimation of the variance in these circumstances.

A damage function $\Psi(a)$ is introduced from (4) as

$$\Psi(a) = \int_{a_0}^{a} \frac{C_2(x)}{Y(x,\mathrm{Y})^m (\sqrt{\pi x}\,)^m}\,dx \tag{7}$$

The stress ranges are denoted $S_i = \Delta\sigma_i$ and solution of (4) gives

$$\Psi(a) = C_1 \int_0^N S^m\, dN = \begin{cases} C_1 S^m N & \textit{constant amplitude} \\ C_1 \sum_{r=1}^{N} S_r^m & \textit{variable amplitude} \end{cases} \tag{8}$$

The difference between the two cases of constant and variable amplitude loading therefore only concerns the loading statistics. In what follows, constant amplitude loading is considered, while variable amplitude loading is considered again at the end of the paper.

In the presentation it has so far been assumed that a crack is present at the time the loading is applied. With an initial crack initiation period before the crack reaches a size a_0, for which fracture mechanics can be applied with some confidence to describe the fatigue crack growth, the solution to (4) is

$$\int_{a_0}^{a} \frac{C_2(x)}{Y(x,\mathrm{Y})^m (\sqrt{\pi x})^m} dx = C_1 S^m (N - N_0) \quad (9)$$

where N_0 is the (random) crack initiation period for which a separate model can be formulated.

The second moment statistics for the damage function conditioned upon (a_0, Y, m) are

$$E[\Psi(a) | a_0, \mathrm{Y}, m] = \int_{a_0}^{a} \frac{1}{Y(x,\mathrm{Y})^m (\sqrt{\pi x})^m} dx \quad (10)$$

$$Var[\Psi(a) | a_0, \mathrm{Y}, m] \approx \int_{a_0}^{a} \frac{r_{C_2} Var[C_2] dx}{Y(x,\mathrm{Y})^{2m} (\pi x)^m} \quad (11)$$

$$\rho[\Psi(a_1), \Psi(a_2) | a_0, \mathrm{Y}, m] \approx \quad (12)$$

$$\frac{\int_{a_0}^{\min\{a_1, a_2\}} \frac{1}{Y(x,\mathrm{Y})^{2m} (\pi x)^m} dx}{\left(\int_{a_0}^{a_1} \frac{dx_1}{Y(x_1,\mathrm{Y})^{2m} (\pi x_1)^m}\right)^{1/2} \left(\int_{a_0}^{a_2} \frac{dx_2}{Y(x_2,\mathrm{Y})^{2m} (\pi x_2)^m}\right)^{1/2}}$$

The approximations for the variance and the correlation function are justified by the short correlation length of $C_2(a)$ compared to crack size increments of interest. The random variable $\Psi(a) | a_0, \mathrm{Y}, m$ is essentially the sum of many independent random variables of approximately the same variance. The distribution is therefore well approximated by a normal distribution.

The failure criterion is taken as exceedence of a critical crack size a_C in a time period with N stress cycles,

$$a_C - a_N \leqslant 0 \quad (13)$$

where a_N is the crack size after the N stress cycles. $\Psi(a)$ is monotonically increasing and the failure criterion (13) can be written as

$$\Psi(a_C) - \Psi(a_N) = \quad (14)$$

$$\int_{a_0}^{a_C} \frac{C_2(x)}{Y(x,\mathrm{Y})^m (\sqrt{\pi x})^m} dx - C_1 S^m N \leqslant 0$$

The safety margin M is therefore defined as

$$M = \int_{a_0}^{a_C} \frac{C_2(x)}{Y(x,\mathrm{Y})^m (\sqrt{\pi x})^m} dx - C_1 S^m N \quad (15)$$

and the failure probability P_F is

$$P_F = P(M \leqslant 0) \quad (16)$$

EVENT MARGINS FOR INSPECTION RESULTS AND REPAIR

Two types of inspection results are considered

$$a(N_i) \leqslant A_{di}, \quad i = 1,2,\cdots,r \quad (17)$$

$$a(N_j) = A_j, \quad j = 1,2,\cdots,s \quad (18)$$

In the first case, (17), no crack was found in the inspection after N_i stress cycles, implying that the crack size was smaller than the smallest detectable crack size A_{di}. A_{di} is generally random since a detectable crack is only detected with a certain probability depending on the crack size and on the inspection method. The distribution of A_{di} is the distribution of non-detected cracks and the distribution function is identical to the pod (probability of detection) function for the inspection method. Information of the type (17) can be envisaged for several times. If A_{di} is deterministic, however, and the same for all inspections, the information in the latest observation contains all the information of the previous ones. In the second case, (18), a crack size A_j is observed after N_j stress cycles. A_j is generally random due to measurement error and/or due to uncertainties in the interpretation of a measured signal as a crack size. Measurements of the type (18) can also be envisaged for several times corresponding to several values of N_j.

For each measurement (17) an event margin M_i can be defined similar to the safety margin (15) as

$$M_i = C_1 S^m N_i - \int_{a_0}^{A_{di}} \frac{C_2(x)}{Y(x,\mathrm{Y})^m (\sqrt{\pi x})^m} dx \leqslant 0 \quad (19)$$

These event margins are negative due to (17). For each measurement (18) an event margin can similarly be defined as

$$M_j = \int_{a_0}^{A_j} \frac{C_2(x)}{Y(x,\mathrm{Y})^m (\sqrt{\pi x})^m} dx - C_1 S^m N_j = 0 \quad (20)$$

These safety margins are zero due to (18).

The situation is envisaged where no crack is detected in the first r inspections at a location, while a crack is detected by the $r+1$'th inspection and its size is measured at this and the following $s-1$ inspections. The updated failure probability is in this case

$$P_F = P(M \leqslant 0 | M_1 \leqslant 0 \cap . \cap M_r \leqslant 0 \cap M_{r+1} = . = M_{r+s} = 0) \quad (21)$$

A more general situation involves simultaneous consideration of several locations with potentially dangerous cracks for which inspections are carried out. The updating procedure still applies when due consideration is taken to the dependence between basic variables referring to different locations.

Assume now that a repair takes place after N_{rep} stress cycles and a crack size a_{rep} is observed. The event margin M_{rep} is defined as

$$M_{rep} = \int_{a_0}^{a_{rep}} \frac{C_2(x)}{Y(x,\mathrm{Y})^m (\sqrt{\pi x})^m} dx - C_1 S^m N_{rep} = 0 \quad (22)$$

The crack size present after repair and a possible inspection is a random variable a_{new} and the material properties after repair are m_{new} and $C_{1,new}$. The safety margin after repair is M_{new}

$$M_{new} = \int_{a_{new}}^{a_C} \frac{C_2(x)}{Y(x,\mathrm{Y})^{m_{new}} (\sqrt{\pi x})^{m_{new}}} dx - C_{1,new} S^{m_{new}} (N - N_{rep}) \quad (23)$$

and the failure probability after repair is

$$P_F = P(M_{new} \leqslant 0 \mid M_{rep} = 0) \quad (24)$$

This updated failure probability is then of the same form as (21).

The crack size at repair is not necessarily measured, but the decision of repair is based on an observed size larger than a limiting value a_{rep}. The event margin M_{rep} in (22) is then negative. In (24) for the updated failure probability, the condition $M_{rep}=0$ is then replaced by $M_{rep} \leqslant 0$ and the expression is still of the form covered by (21).

RELIABILITY METHOD

The reliability method used in this paper is the first-order reliability method which is here briefly reviewed for parallel systems. For a more thorough description see [13]. Each element in the parallel system is described by a safety margin $M_i = g_i(\mathrm{Z})$ in terms of the vector of basic variables Z. The safety margins are defined with $M_i \leqslant 0$ corresponding to failure in the i th element, and $g_i(\mathrm{z})=0$ defining the limit state surface for the i th element. The failure probability of a parallel system with k elements is

$$P_F = P(M_1 \leqslant 0 \cap M_2 \leqslant 0 \cap \cdots \cap M_k \leqslant 0) \quad (25)$$

The failure probability is computed efficiently and to a good accuracy by a first-order reliability method. The first step in the computation is a transformation of the vector of basic variables into a vector of standardized and independent normal variables U. The transformation is denoted T and the transformed space is called the normal space.

$$\mathrm{U} = \mathrm{T}(\mathrm{Z}) \quad (26)$$

A good choice for T is a transformation, which uses the conditional distribution functions $F_i(z_i \mid z_1,..,z_{i-1}) = P(Z_i \leqslant z_i \mid Z_1 = z_1,..,Z_{i-1} = z_{i-1})$ of the basic variables, [14]

$$\begin{aligned} U_1 &= \Phi^{-1}(F_1(Z_1)) \\ U_2 &= \Phi^{-1}(F_2(Z_2 \mid Z_1)) \\ &\cdot \\ U_i &= \Phi^{-1}(F_i(Z_i \mid Z_1, Z_2, \cdots, Z_{i-1})) \\ &\cdot \\ U_n &= \Phi^{-1}(F_n(Z_n \mid Z_1, Z_2, \cdots, Z_{n-1})) \end{aligned} \quad (27)$$

Here $\Phi(\,)$ denotes the standardized normal distribution function. The limit state surfaces for the individual elements are expressed in terms of **u** as

$$g_i(\mathrm{z}) = g_i(\mathrm{T}^{-1}(\mathbf{u})) = g_{u,i}(\mathbf{u}) = 0, \quad i=1,2,..,k \quad (28)$$

The second step in a first-order reliability analysis consists in determining the joint design point $\mathbf{u}^*$, which is the point on the limit state surface closest to the origin. $\mathbf{u}^*$ is thus found as the solution of a constrained minimization

$$\begin{aligned} &\min |\mathbf{u}| \\ &g_{u,i}(\mathbf{u}) \leqslant 0, \quad i=1,2,\cdots,k \end{aligned} \quad (29)$$

provided that $g_{u,i}(0)>0$ for at least one $i \in \{1,...,k\}$. Standard optimization techniques can be applied to solve this problem. All constraints are not necessarily active at the joint design point, i.e., $g_{u,i}(\mathbf{u}^*)=0$ is not necessarily valid for all i. Let $l \leqslant k$ denote the number of active constraints.

The third step in a first-order reliability method consists in a linearization of the safety margins at the joint design point $\mathbf{u}^*$ formulated in the normal space. In normalized form the linearized safety margins are

$$M_i = \beta_i - \alpha_i^T \mathrm{U} \quad (30)$$

where α_i is a unit vector and $\beta_i = \alpha_i^T \mathbf{u}^*$ is the first-order reliability index for element i of the parallel system linearized at the joint design point. The correlation coefficient ρ_{ij} between the safety margins M_i and M_j is

$$\rho_{ij} = \rho[M_i, M_j] = \alpha_i^T \alpha_j \quad (31)$$

The failure probability of the parallel system is now estimated as

$$P_F \approx \Phi_l(-\beta;\rho) \quad (32)$$

where $\beta = \{\beta_i\}$, $\rho = \{\rho_{ij}\}$ and only the l active elements are included. The asymptotic result as $|\mathbf{u}^*| \rightarrow \infty$ is, [15]

$$P_F \sim \Phi_l(-\beta;\rho)\,[\det(\mathrm{I}-\mathrm{D})]^{-1/2}, \quad |\mathbf{u}^*| \rightarrow \infty \quad (33)$$

where **I** denotes the unit matrix and **D** is a matrix determined by the coordinates of the design point and the gradients and second order derivatives of the limit state functions at the design point.

The reliability index β_R for the system is defined as

$$\beta_R = -\Phi^{-1}(P_F) \quad (34)$$

For a single element the asymptotic result for β_R is derived in [16]:

$$\beta_R \sim \beta, \quad \beta = |\mathbf{u}^*| \rightarrow \infty \quad (35)$$

A generalization of this result to a parallel system yields

$$\beta_R \sim -\Phi^{-1}(\Phi_l(-\beta;\rho)), \quad |\mathbf{u}^*| \rightarrow \infty \quad (36)$$

The failure probability in (16) is calculated directly by (32) or (33) with $k=l=1$. The updated failure probability in (21) is rewritten as

$$P(M \leq 0 \mid M_1 \leq 0 \cap . \cap M_r \leq 0 \cap M_{r+1} = . = M_{r+s} = 0) \quad (37)$$

$$= \frac{\dfrac{\partial^s P(M \leq 0, M_1 \leq 0 .. M_r \leq 0, M_{r+1} \leq x_{r+1} .. M_{r+s} \leq x_{r+s})}{\partial x_{r+1} \cdots \partial x_{r+s}}}{\dfrac{\partial^s P(M_1 \leq 0 .. M_r \leq 0, M_{r+1} \leq x_{r+1} .. M_{r+s} \leq x_{r+s})}{\partial x_{r+1} \cdots \partial x_{r+s}}}$$

where the partial derivatives are evaluated at x=0. Two parallel systems must thus be analyzed, but the optimization problem is cast in a slightly different form than (29) since the constraints corresponding to the detected crack sizes are changed to equality constraints. In addition, linearized safety margins for inactive constraints are included as described in [17]. The vector of reliability indices and the correlation matrix for the normalized safety and event margins in the numerator are

$$\begin{bmatrix} \beta \\ \beta_1 \\ \beta_2 \end{bmatrix}, \quad \begin{bmatrix} 1 & \rho_1^T & \rho_2^T \\ \rho_1 & \rho_{11} & \rho_{21}^T \\ \rho_2 & \rho_{21} & \rho_{22} \end{bmatrix} \quad (38)$$

where β refers to the safety margin, an index 1 to the normalized event margins for no detection and an index 2 to the normalized event margins for a detected crack and measured crack size. The dimension of β_1 is r (since inactive constraints have been included) and the dimension of β_2 is s. The vector of reliability indices and the correlation matrix for the denominator are similarly

$$\begin{bmatrix} \beta_1' \\ \beta_2' \end{bmatrix}, \quad \begin{bmatrix} \rho_{11}' & \rho_{21}'^T \\ \rho_{21}' & \rho_{22}' \end{bmatrix} \quad (39)$$

The joint design point for the parallel system in the denominator is generally different from the design point for the parallel system in the numerator. This is emphasized by the use of a prime in the denominator. The dimension of β_1' is r and the dimension of β_2' is s.

In [18] the asymptotic result for the partial derivative of β_R for an element has been derived with respect to a distribution or limit state function parameter p:

$$\frac{\partial \beta_R}{\partial p} \sim \frac{\partial \beta}{\partial p}, \quad |\mathbf{u}^*| \to \infty \quad (40)$$

For the failure probability then follows

$$\frac{\partial P_F}{\partial p} = \frac{\partial \Phi(-\beta_R)}{\partial p} = -\phi(\beta_R)\frac{\partial \beta_R}{\partial p} \quad (41)$$

$$\sim -\phi(\beta)\frac{\partial \beta}{\partial p}, \quad |\mathbf{u}^*| \to \infty$$

Generalizing this result to the parallel system in the numerator of (37) yields

$$\left.\frac{\partial^s P(M \leq 0, M_1 \leq 0 .. M_r \leq 0, M_{r+1} \leq x_{r+1} .. M_{r+s} \leq x_{r+s})}{\partial x_{r+1} \cdots \partial x_{r+s}}\right|_{x=0}$$

$$\sim \frac{\partial^s \Phi_{r+s+1}\left(-\begin{bmatrix} \beta \\ \beta_1 \\ \beta_2 \end{bmatrix}; \begin{bmatrix} 1 & \rho_1^T & \rho_2^T \\ \rho_1 & \rho_{11} & \rho_{21}^T \\ \rho_2 & \rho_{21} & \rho_{22} \end{bmatrix} \right)}{\partial \beta_{r+1} \cdots \partial \beta_{r+s}} \quad (42)$$

$$= \phi_s(-\beta_2; \rho_{22})\, \Phi_{r+1}\left(-\left\{ \begin{bmatrix} \beta \\ \beta_1 \end{bmatrix} - \begin{bmatrix} \rho_2^T \\ \rho_{21}^T \end{bmatrix} \rho_{22}^{-1} \beta_2 \right\}; \left\{ \begin{bmatrix} 1 & \rho_1^T \\ \rho_1 & \rho_{11} \end{bmatrix} - \begin{bmatrix} \rho_2^T \\ \rho_{21}^T \end{bmatrix} \rho_{22}^{-1} [\rho_2 \;\; \rho_{21}] \right\} \right)$$

where standard results for the conditional multivariate normal distribution have been applied since the vectors of linearized safety margins are joint normally distributed. Furthermore $\partial \beta_i / \partial x_i = -1$ has been used, which is valid since $Var[M_i] = 1$. For the conditional probability in (37) one obtains:

$$P(M \leq 0 \mid M_1 \leq 0 \cap . \cap M_r \leq 0 \cap M_{r+1} = . = M_{r+s} = 0) \quad (43)$$

$$\sim \frac{\phi_s(-\beta_2; \rho_{22})}{\phi_s(-\beta_2'; \rho_{22}')} \times$$

$$\frac{\Phi_{r+1}\left(-\left\{ \begin{bmatrix} \beta \\ \beta_1 \end{bmatrix} - \begin{bmatrix} \rho_2^T \\ \rho_{21}^T \end{bmatrix} \rho_{22}^{-1} \beta_2 \right\}; \left\{ \begin{bmatrix} 1 & \rho_1^T \\ \rho_1 & \rho_{11} \end{bmatrix} - \begin{bmatrix} \rho_2^T \\ \rho_{21}^T \end{bmatrix} \rho_{22}^{-1} [\rho_2 \;\; \rho_{21}] \right\} \right)}{\Phi_r\left(-\beta_1' + \rho_{21}'^T (\rho_{22}')^{-1} \beta_2';\ \rho_{11}' - \rho_{21}'^T (\rho_{22}')^{-1} \rho_{21}'\right)}$$

The updating of the reliability has been demonstrated. If the interest is on updating the distribution of the basic variables the same procedure is followed. Instead of the safety margin (15) an event margin M for basic variable Z_i is defined as

$$M = Z_i - z_i \quad (44)$$

With the safety margin replaced by this event margin the value of the cumulative distribution function for Z_i at the argument z_i is updated. The procedure can be repeated for different arguments z_i and the complete distribution function thereby be updated. Even when the basic variables are initially independent the updating procedure generally introduces dependence. It may thus be more relevant to update the joint distribution function. The safety margin M is then replaced by a vector of event margins $\{Z_i - z_i\}$, $i = 1, \cdots, n$ and the updating of the vector is performed as described above.

EXAMPLE.

Consider a panel with a center crack as in the experiments of [12]. The loading is a constant amplitude loading leading to a far-field stress range S. The geometry function is modeled as

$$Y(a,\mathrm{Y}) = \exp(Y_1(\frac{a}{50})^{Y_2}) \qquad (45)$$

The geometry function takes the value one for $a=0$. Lengths are measured in mm and stresses in N / mm ^{2}. The distribution of the basic variables is taken as

$$\begin{cases} S \in N(60, 10^2) \\ Y_1 \in LN(1, 0.2^2) \\ Y_2 \in LN(2, 0.1^2) \\ a_0 \in EX(1) \\ a_C \in N(50, 10^2) \\ (\ln C_1, m) \in N_2(-33.00, 0.47^2, 3.5, 0.3^2; -0.9) \end{cases} \qquad (46)$$

$N(\mu,\sigma^2)$ denotes a normal distribution with mean value μ and variance σ^2. Similarly $LN(\mu,\sigma^2)$ denotes a log-normal distribution with mean value μ and variance σ^2. $N_2(\mu_1,\sigma_1^2,\mu_2,\sigma_2^2;\rho)$ denotes a bivariate normal distribution with mean values μ_1 and μ_2, variances σ_1^2 and σ_2^2 and correlation coefficient ρ. $EX(\mu)$ denotes an exponential distribution with mean value μ. The negative correlation between $\ln C_1$ and m is not reflecting a physical dependence, but is introduced by the form of the crack growth equation (2). Statistics for $C_2(a)$ are taken as those reported in [7], see section 2 of this paper. The example has eight basic variables and the transformation into standardized and independent normal variables has been described in [13,19,20].

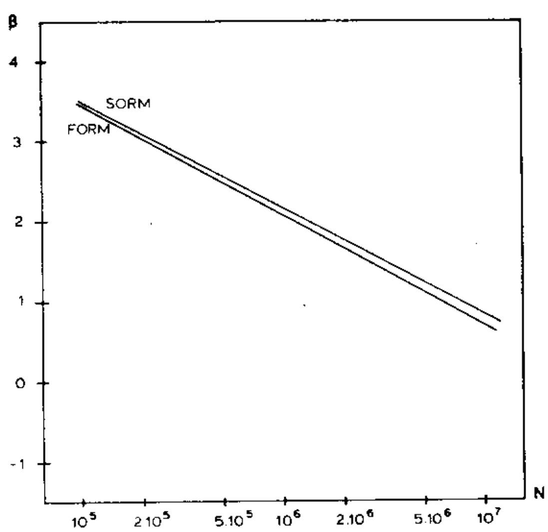

Figure 1. First- and second-order reliability index from design calculation.

The first-order and improved second-order approximations to the reliability index are shown in Fig.1 for various life times expressed in terms of the number of stress cycles N. The two approximations are close implying that the curvatures of the limit state surface are moderate at the design point. Statistics for the distribution of life time T can be directly approximated from the results of Fig.1. For the mean life time the approximation is

$$E[T] = \int_0^\infty (1-P(T \leq t))\,dt \approx \int_0^\infty \Phi(\beta(t))\,dt \qquad (47)$$

For $N=1.5\cdot10^6$ cycles the reliability index and the sensitivity factors are shown in Table 1. α_i^2 can be interpreted as the fraction of the total uncertainty due to uncertainty arising from basic variable U_i. The major contribution to the overall uncertainty is thus in this case from the uncertainty in the material parameters. The critical crack size uncertainty is of little relative importance in this case, and the same is concluded in almost all cases where the critical crack size is significantly larger than the initial crack size. The uncertainty in the geometry function contributes very little to the total uncertainty in this case. This is because the value for $a=0$ is completely known. When this initial value is not known the uncertainty is comparable to the uncertainty in the loading. The uncertainty contribution from the uncertainty in the change in the geometry function from the initial value is generally found to be low. For tubular joints, where the geometry function is approximately proportional to $a^{-1/2}$ for large values of a, this statement may not be true in all cases.

TABLE 1 Reliability index and sensitivity factors

$N=1.5\cdot10^6$	$\beta=1.816$	
Variable	α_i	α_i^2
a_0	0.5513	30%
a_C	-0.0001	0%
S	0.3577	13%
m	-0.6141	38%
$C \mid m$	0.4362	19%
Y_1	-0.0248	0%
Y_2	0.0085	0%
$\Psi(a_C) \mid a_0, a_C, \mathrm{Y}, m$	-0.0060	0%

Based on the results in Table 1 and results for the parametric sensitivity factor (40), [13,18], the sensitivity of the reliability index to a change in a distribution parameter can be determined. For the mean value μ_S of the normally distributed loading variable S, the sensitivity factor is

$$\frac{\partial\beta}{\partial\mu_S} = -\frac{\alpha_S}{\sigma_S} = -\frac{0.358}{10} = -0.0358 \qquad (48)$$

An increase in μ_S by 10 MPa thus leads to an change in β of approximately $(-0.0358)10=-0.358$.

Next, the situation where a crack is found in the first inspection is considered. It is envisaged that the inspection is carried out after $N_1=10^5$ stress cycles and a crack length of 3.9 mm is measured. The measurement error is assumed to be normally distributed with standard deviation σ_A. Figure 2 shows the updated reliability index as a function of σ_A, when (43) has been applied with $(r,s)=(0,1)$. The result is almost independent of σ_A in this example as the uncertainty in the initial crack size is dominating the uncertainty in A_1. When the crack is detected, a decision has to be made and two options are present. It may be decided to repair the crack now or to leave the crack as it is and base a decision on repair on more inspection results. With just one inspection it is not possible to determine if the crack was initially large but grows slowly enough that repair is not needed, or the crack was initially fairly small but is growing fast and must be repaired. If a requirement on the reliability index in a period without inspections is formulated, e.g., $\beta_R \geqslant 2$, the latest time of the next inspection is determined from Fig.2.

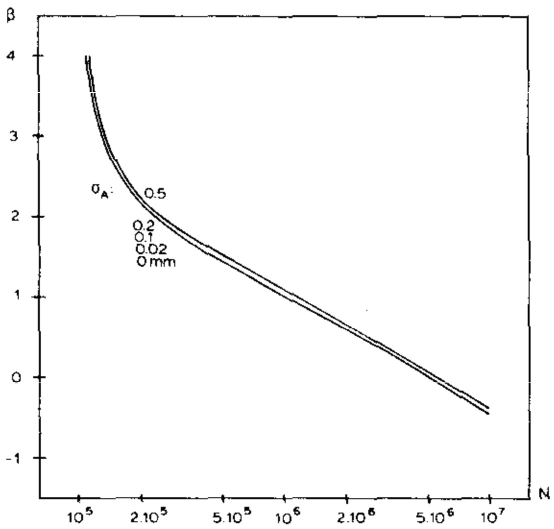

Figure 2. Updated first-order reliability index after first inspection with crack measurement 3.9 mm.

Assume that the crack is not repaired but a second inspection at $N=2\cdot10^5$ stress cycles is required. Let the inspection method be the same as in the first inspection and let the measured crack size be 4.0 mm. The measurement error is again assumed to be normally distributed with standard deviation σ_A and the two measurement errors are assumed to be statistically independent. Figure 3 shows the updated reliability index after this second inspection. Different inspection qualities now lead to very different results. With $\sigma_A=0$ the negative slope of the reliability index curve becomes very large demonstrating that the crack growth behavior is basically determined by two

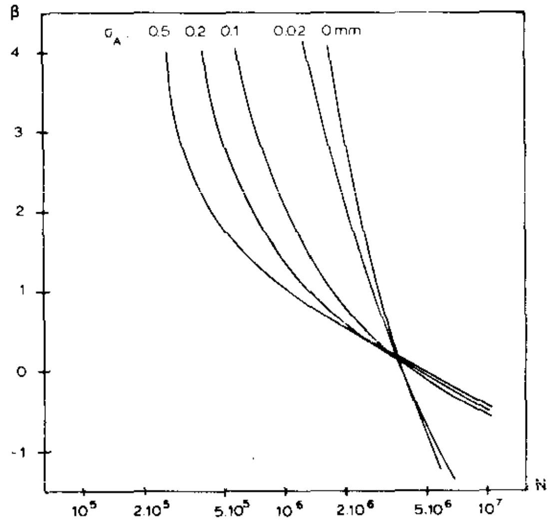

Figure 3. Updated first-order reliability index after second inspection with crack measurements 3.9 mm and 4.0 mm.

combinations of the basic variables. With a large measurement uncertainty there is an immediate and large increase in reliability, but after some time the curve becomes almost identical to the curve resulting after the first inspection. Due to large uncertainty in both inspections only little information is gained on the crack growth rate. If the inspection quality is very high it may be possible to state that the crack does not grow to a critical size within the design life time. Repair and further inspections are then unnecessary. For a poorer inspection quality a time period until the next inspection can be determined and the decision on repair be further delayed.

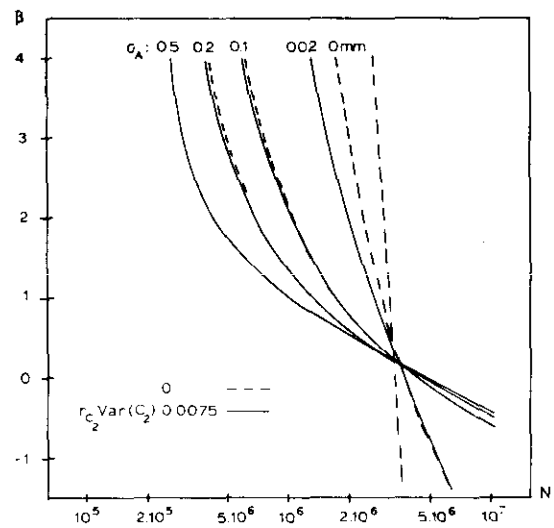

Figure 4. Updated first-order reliability index after second inspection with crack measurements 3.9 mm and 4.0 mm, importance of inhomogeneity.

Figure 4 shows the results of Fig.3 together with similar results for a homogeneous material. It is observed that only for very small inspection uncertainty does the material inhomogeneity significantly affect results. The estimates for material inhomogeneity used in this example are for base material and the conclusion may be somewhat different for crack growth in weld material or in base material in a heat affected zone.

Figure 5 presents results similar to those in Fig.3, but for the case where a crack size of 5 mm is reported in the second inspection. Together, the two inspection results now indicate that a large and fast growing crack is present. Repair is therefore necessary within a short period.

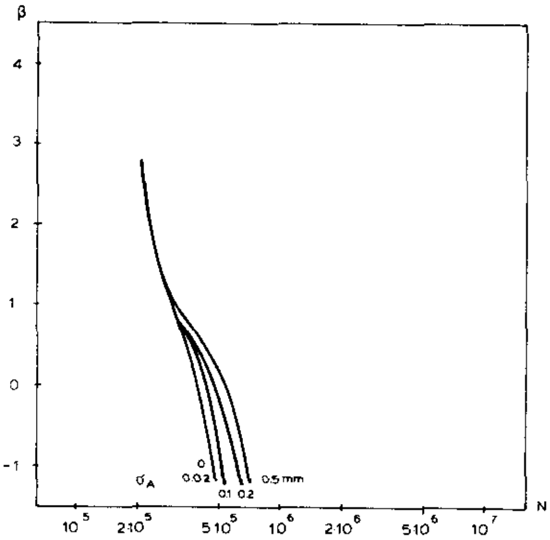

Figure 5. Updated first-order reliability index after second inspection with crack measurements 3.9 mm and 5.0 mm.

Consider now different situations where the inspections do not result in crack detection. An attempt is made to illustrate possible means to achieve a required reliability. Let the reliability requirement be $\beta_R \geq 3.0$ and let the design life time correspond to $1.5 \cdot 10^6$ stress cycles. Figure 6 shows the reliability index as a function of number of stress cycles for two plate thicknesses. With a plate thickness t the reliability requirement is fulfilled for the design life time and no inspections are needed. With a plate thickness of only 60% of t the reliability requirement is fulfilled for the period until $N = 2 \cdot 10^5$ stress cycles, where an inspection is needed. The quality of the inspection is reflected in the distribution of non-detected cracks. An exponential distribution is assumed with a mean value λ. Cracks initially present are cracks which have passed the inspection at the production site either because they were not detected or because they were below the acceptance level. If no cracks were accepted in fabrication, the fabrication inspection therefore corresponds to $\lambda = 1$.

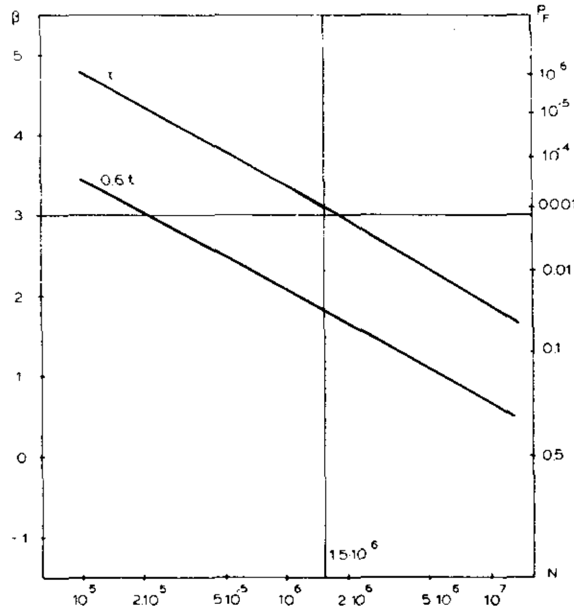

Figure 6. First-order reliability index for two plate thicknesses.

Figure 7 shows the initial reliability index and updated reliability indices for three inspection qualities. The best inspection quality $\lambda=0.3$ is better than the fabrication inspection quality and if no crack is found with this method the increase in reliability is sufficient to make further inspections unnecessary. For the two other inspection qualities, periods are determined until the next inspection.

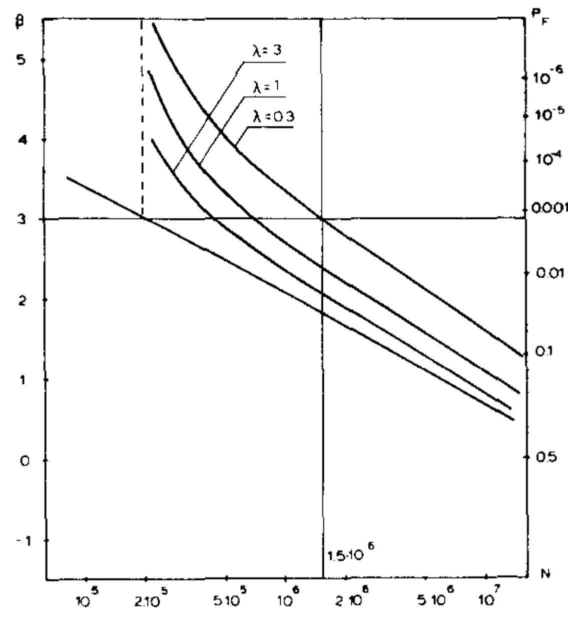

Figure 7. Updated first-order reliability index after first inspection with no crack detection.

Figure 8 shows the total inspection requirement for λ=1 when no crack is detected in any inspection. For this case two inspections are needed. Finally, Fig.9 shows the total inspection requirement for λ=3 when no crack is detected in any inspection, and for this case five inspections are needed. It is thus demonstrated that different strategies on design and inspection planning can be used to achieve a required reliability.

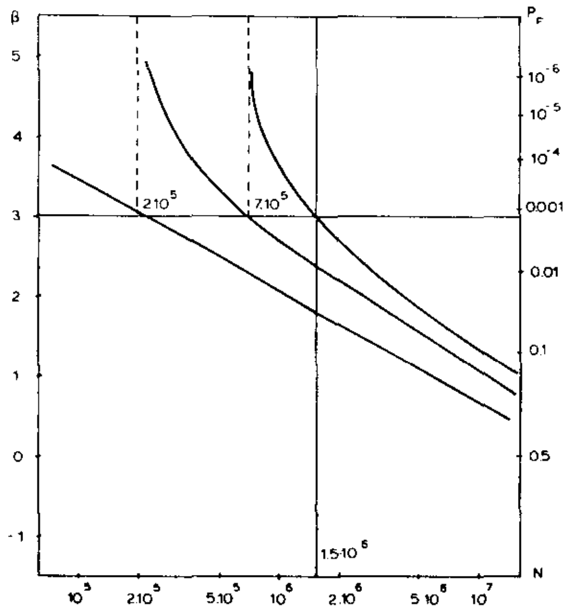

Figure 8. Updated first-order reliability index after inspections with no crack detection, mean size of non-detected cracks 1 mm.

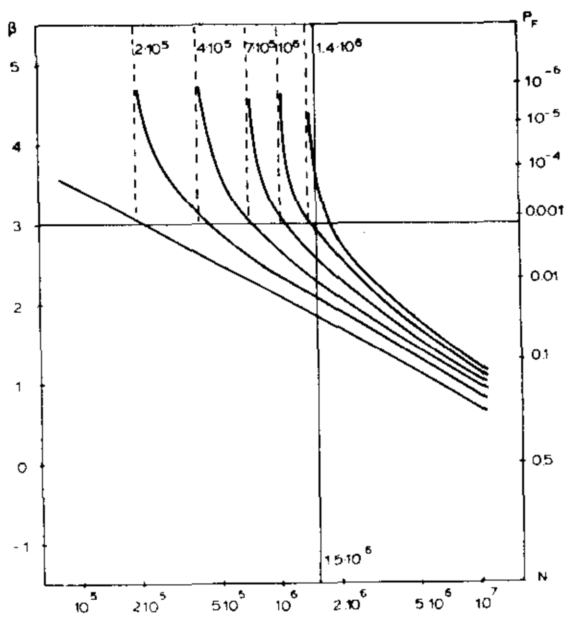

Figure 9. Updated first-order reliability index after inspections with no crack detection, mean size of non-detected cracks 3 mm.

The results of a reliability analysis following a repair of a detected crack is illustrated in Fig.10. It is assumed that a crack size of a_{rep}=8 mm is repaired after $N_{rep}=2 \cdot 10^5$ stress cycles. The distribution of the initial crack size after repair a_{new} is taken as an exponential distribution with a mean value of 1 mm, i.e., as the same initial distribution as after fabrication. Two situations are considered with either identical or independent material properties before and after repair. When independent properties are assumed the same distribution is used for the properties before and after repair. If follows from the results that there is an immediate increase in reliability after repair, but the reliability quickly drops to a level below the level obtained for the calculations before repair. This reflects the possibility that the cause for the large repaired crack size is a larger than anticipated loading of the crack tip, which is also acting after the repair.

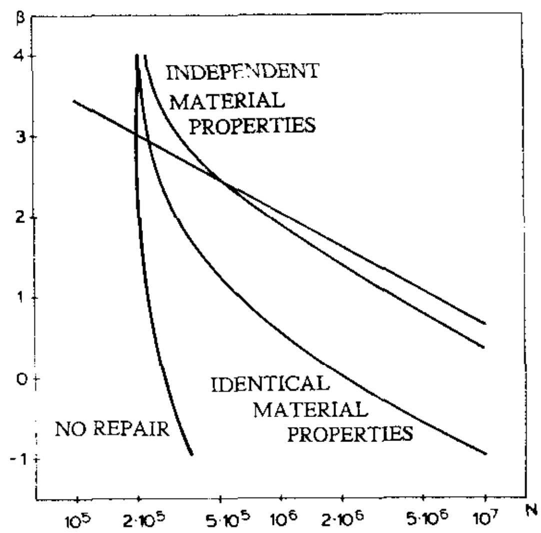

Figure 10. Updated first-order reliability index after repair of an 8 mm crack at $N=2 \cdot 10^5$ stress cycles.

The results presented in this example have been for a constant amplitude loading. For offshore structures a long term stress range distribution is generally applied in fatigue analyses. Due to uncertainty in the environmental statistics, load models, global structural analysis and local stress analysis, the parameters of the long term distribution should be modeled as random variables. A Weibull distribution is often used

$$F_S(s) = 1-\exp(-(s/A)^B), \quad s>0 \qquad (49)$$

where A and B are random variables. A calibration of the statistics for A and B, based on an uncertainty modeling for the above mentioned sources, can be performed by a modification of the probabilistic fatigue analysis presented in [21]. The factor $\sum_{r=1}^{N} S_r^m$ in (8) is replaced by the expected value, which for Weibull distributed stress ranges becomes

$$E[\sum_{r=1}^{N} S_r^m] = E[N]E[S^m] = E[N]A^m\,\Gamma\left(1+\frac{m}{B}\right) \quad (50)$$

The expected value is random due to the random distribution parameters, but the uncertainty in the sum for fixed distribution parameters is neglected. This is reasonable due to the large number of random variables with little correlation in the summation.

For variable amplitude loading it is also of interest to study the effect of a non-zero threshold value for ΔK in (2), which becomes

$$\frac{da}{dN} = C(\Delta K)^m, \quad \Delta K > \Delta K_{thr} \quad (51)$$

Eq.(4) is then replaced by

$$\frac{da}{Y(a,\mathrm{Y})^m(\sqrt{\pi a})^m} = C\,(\Delta\sigma)^m\,1_{\Delta\sigma > \frac{\Delta K_{thr}}{Y(a,\mathrm{Y})\sqrt{\pi a}}}\,dN \quad (52)$$

where 1 denotes the indicator function. Replacing as an approximation

$$(\Delta\sigma)^m\,1_{\Delta\sigma > \frac{\Delta K_{thr}}{Y(a,\mathrm{Y})\sqrt{\pi a}}}$$

by its expected value, [22], yields

$$\frac{da}{Y(a,\mathrm{Y})^m(\sqrt{\pi a})^m G(a)} = C\,E[(\Delta\sigma)^m]\,dN \quad (53)$$

The reduction factor $G(a)$, $0 \leqslant G(a) \leqslant 1$ depends on the long term stress range distribution as $(S = \Delta\sigma)$

$$G(a) = \frac{\int_{\frac{\Delta K_{thr}}{Y(a,\mathrm{Y})\sqrt{\pi a}}}^{\infty} S^m f_S(S)dS}{\int_0^{\infty} S^m f_S(S)dS} \quad (54)$$

$$= \frac{\Gamma\left[1+\frac{m}{B};\left(\frac{\Delta K_{thr}}{AY(a,\mathrm{Y})\sqrt{\pi a}}\right)^B\right]}{\Gamma\left[1+\frac{m}{B}\right]}$$

where the last expression is valid for the Weibull distribution in (49).

CONCLUSIONS

The following conclusions can be stated:

1) A stochastic model for fatigue crack growth has been applied which accounts for uncertainties in loading, initial defects, critical crack size, material parameters including spatial variation, and in the computation of the stress intensity factor. Based on the crack growth model and a load model a safety margin has been defined.

2) Two types of inspection results have been considered and the inspection uncertainty has been modeled. Event margins have been defined for both types of inspection results. Updated reliabilities have been expressed in terms of the safety margin and the inspection event margins. A similar analysis has been performed for a structure after repair.

3) A brief discussion of first-order reliability theory applied to parallel systems has been presented. It has been demonstrated that the updating after inspection and repair can be carried out in a simple way by use of first-order reliability methods. Updating of the reliability and/or of the distribution of the basic variables have been considered.

4) The analysis has been presented for an example panel with a center crack. The reliability index has been computed based on information at the design stage and has been updated based on inspection results both resulting in crack detection and in no detection. The effect of material inhomogeneity for the selected base material has been demonstrated to be insignificant. Different inspection qualities have been considered resulting in different effects on the updated reliability index.

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