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Reliability and Durability Analysis of Marine Structures

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ABSTRACT

A study of the dynamic response of offshore towers to wind-generated random ocean waves is presented. The analysis is performed in the frequency domain and the equations of motion are solved using equivalent linearization techniques. Two methods of reliability analysis are considered in this study : A statistical fatigue damage analysis using the American Welding Society model (high-cycle fatigue analysis) to demonstrate the influence of the design stress level for fatigue-critical members on the fatigue reliability and a first-passage failure probability with periodic inspections. For the latter, a crack growth analysis is performed using the fracture mechanics method to estimate the propagated crack size and corresponding residual strength under random service loading conditions. The effect of periodic inspections is very important if a fatigue crack already exists. It is also shown that there is always a limit in the number of inspections beyond which no significant improvement can be achieved.

INTRODUCTION

In order to assess the reliability and durability of an existing structure, the statistical distribution of each of the significant influencing factors, such as service loading, structural performance parameters of the material as well as of the fabricated structure, environmental conditions, inspection and repair procedures must be adequately characterized.

In this paper, a study of the dynamic response of offshore towers to wind-generated random ocean waves is presented. The analysis is performed in the frequency domain and the equations of motion are solved using equivalent linearization techniques. Then, a crack growth analysis is performed using the fracture mechanics method to estimate propagated crack size and corresponding residual strength under random service loading conditions. The residual strength is obtained as a function of crack size and other parameters of redundancy in the case of a redundant (with crack-stoppers) design. Once the residual strength is established, the failure rate is evaluated as the rate of upcrossing of the residual strength by the random stress process. The probability of structural failure is then computed on the basis of this failure rate taking the inspection procedures into consideration. At this point, the following should be acknowledged: (1) This paper deals with the fatigue-related inspection for a single location. Multi-critical location problems are an interesting subject of future research. (2) In some cases, the operational sea states may produce a significant amount of failure damage. However, the primary purpose of this study is to introduce inspection procedures into reliability analysis. Hence, the analyical procedures developed in this paper considered only the stress arising from a stormy sea state, although the stress due to an operational sea state can also be incorporated in a similar fashion.

To estimate the probabilty of fatigue failure, a second approach is also incorporated for illustrative purposes. A statistical fatigue damage analysis is presented herein using the AWS model (high-cycle fatigue analysis). Numerical examples have been worked out to demonstrate the influence of the design stress level for fatigue critical members on the fatigue reliability.

STOCHASTIC ANALYSIS OF OFFSHORE STRUCTURES

A 1075 ft (327.7 m) offshore tower has been analyzed for waves under fully- developed sea conditions for which the wave height spectrum specified by Pierson and Moskowitz is used (Pierson & Moskowitz, 1964). The analysis is performed by means of equivalent linearization techniques (Malhotra & Penzien, 1969; Okumura & Nishioka, 1974; Paliou & Shinozuka, 1986; Penzien et al., 1972; Yang & Freudenthal, 1977) so that the original nonlinear equations of motion can be solved in the frequency domain. The nonlinearities in the system are due to drag forces arising from wave-structure interaction.

The structure shown in Fig. 1 is idealized as a discrete mass system. It is assumed that the platform is above the sea surface, the tower is fixed on the ocean floor and the vertical structural displacements are negligible.

The equations of motion for a discrete mass system can be written as (Malhotra & Penzien, 1969):

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{K}\mathbf{u} = \mathbf{C}_M(\ddot{\mathbf{v}} - \ddot{\mathbf{u}}) + \mathbf{C}_D(\dot{\mathbf{v}} - \dot{\mathbf{u}})|\dot{\mathbf{v}} - \dot{\mathbf{u}}| \quad (1)$$

where $\mathbf{C}$ = structural damping matrix in air, $\mathbf{K}$ = stiffness of the structure, $\mathbf{M}$ = diagonal matrix of the lumped masses, $\mathbf{C}_M = \rho\ k_M\mathbf{V}$, $\mathbf{C}_D = \rho\ k_D\mathbf{A}$, ρ = mass density of the water, $\mathbf{V}$ = diagonal matrix indicating volume of water displaced by the structure, $\mathbf{A}$ = diagonal matix indicating area projected in direction of flow, k_M = empirical coefficient of inertia in the range 1.4 - 2.0 for linear wave theory, k_D = empirical coefficient of drag in the range 0.5 - 0.7 also for linear wave theory (Penzien et al., 1972), $\ddot{\mathbf{v}}, \dot{\mathbf{v}}$ = vectors of horizontal acceleration and velocity of wave particles, respectively and $\mathbf{u}, \dot{\mathbf{u}}, \ddot{\mathbf{u}}$ = vectors of horizontal displacement, velocity and acceleration of structure, respectively. The $|A|$ symbol refers to the absolute value of A. From Eq. 1 it is clear that the nonlinearity is caused by the velocity term. Now letting $\mathbf{r} = \mathbf{v} - \mathbf{u}$, Eq. 1 can be rewritten as

$$(\mathbf{M} + \mathbf{C}_M)\ddot{\mathbf{r}} + \mathbf{C}\dot{\mathbf{r}} + \mathbf{C}_D\dot{\mathbf{r}}|\dot{\mathbf{r}}| + \mathbf{K}\mathbf{r} = \mathbf{M}\ddot{\mathbf{v}} + \mathbf{C}\dot{\mathbf{v}} + \mathbf{K}\mathbf{v} \quad (2)$$

The wind-induced storm waves are modeled as stationary Gaussian random processes with zero mean and finite duration. This indicates that the wave height and the water particle velocity are also stationary Gaussian random processes. In order to completely define them, we use their power spectral densities (Paliou & Shinozuka, 1985). In the present study, the Pierson-Moskowitz one-sided wave height spectrum has been used.

$$S_{hh}(\omega) = \frac{\alpha_1 \cdot g^2}{\omega^5} \cdot exp\left[-\beta_1 \left(\frac{g}{\omega \cdot W}\right)^4\right] \qquad 0 < \omega < \infty \quad (3)$$

where α_1, β_1 are nondimensional constants, g is the gravity acceleration and W the average storm wind velocity at 64 ft (19.52 m) above the water surface. A plot of the spectrum for two different values of wind velocity (W = 25 ft/sec [7.6 m/sec] and 75 ft/sec [22.9 m/sec]) is given in Fig. 2. Under storm waves, the effect of the drag force increases significantly when the average wind velocity W increases, whereas it is negligible when W is below 50 ft/sec (15.25 m/sec).

The wave analysis performed in the present study uses the assumption of small amplitude (Airy) theory implying that the fluid is inviscid, incompressible and the ratio of wave amplitude to wave length is small.

To linearize Eq. 2 (Penzien et al., 1972), we rewrite it, using $\mathbf{r} = \mathbf{v} - \mathbf{u}$, in the form

$$(\mathbf{M} + \mathbf{C}_M)\ddot{\mathbf{u}} + \tilde{\mathbf{C}}\dot{\mathbf{u}} + \mathbf{K}\mathbf{u} = \mathbf{C}_M\ddot{\mathbf{v}} + \bar{\mathbf{C}}\dot{\mathbf{v}} \qquad (4)$$

where

$$\bar{C}_{jj} = C_{D_j}\sqrt{\frac{8}{\pi}}\,\sigma_{\dot{r}_j\dot{r}_j} \qquad (5)$$

$$\tilde{C}_{jj} = C_{jj} + C_{D_j}\sqrt{\frac{8}{\pi}}\,\sigma_{\dot{r}_j\dot{r}_j} \qquad (j = 1, 2, ..., 7) \qquad (6)$$

Using modal decomposition, $\mathbf{u} = \mathbf{\Phi Y}$, eq. 4 yields to the following

$$\mathbf{M}^*\ddot{\mathbf{Y}} + \mathbf{C}_0\dot{\mathbf{Y}} + \mathbf{K}^*\mathbf{Y} = \mathbf{P}^* \qquad (7)$$

where

$\mathbf{M}^* = \mathbf{\Phi}^T\mathbf{m}\mathbf{\Phi}$ = generalized mass matrix (diagonal) (8)

$\mathbf{K}^* = \mathbf{\Phi}^T\mathbf{K}\mathbf{\Phi}$ = generalized stiffness matrix (diagonal) (9)

$\mathbf{C}_0 = \mathbf{\Phi}^T\tilde{\mathbf{C}}\mathbf{\Phi}$ = generalized damping matrix (not diagonal) (10)

$\mathbf{P}^* = \mathbf{\Phi}^T(\mathbf{C_M}\ddot{\mathbf{v}} + \bar{\mathbf{C}}\dot{\mathbf{v}})$ = generalized force vector (11)

$$\mathbf{C} = \mathbf{M\Phi}\varsigma\mathbf{\Phi}^T\mathbf{M} \qquad (12)$$

$$\varsigma_n = \frac{2\xi_n\omega_n}{M_n^*} \qquad (13)$$

The essence of the approach is to alter the damping coefficients in an optimal manner by minimizing the average mean square error. The iterative solution process converges rapidly and the convergence depends upon the severity of the nonlinearities.

NUMERICAL EXAMPLE

The geometrical and material properties of the tower are given in Table 1. Furthermore, the statistical parameters α_1 and β_1 appearing in the Pierson-Moskowitz spectrum have been taken as 0.0081 and 0.74, respectively. The mass density of the water, ρ, is 2×10^{-3} kips·sec^2/ft^4 (1.03 kPa·sec^2/m^2) and the acceleration of gravity, g, is 32.2 ft/sec^2 (9.8 m/sec^2). The empirical coefficients of inertia, k_M, and of drag, k_D, were assumed to be 2.0 and 0.7, respectively. The modal damping ratio ξ_n, which is used to compute the structural damping matrix (Eq. 13) is taken to be 5%.

The standard deviations of the structural displacements and velocities are displayed in Figs. 3a and 3b, associated with the different values of the average wind velocity W. The seven natural frequencies and standard deviations of the deck displacements and deck velocities for the five examined values of windspeed W are given in Table 2. These results have been confirmed using the time domain (Monte Carlo) analysis performed in Shinozuka et al., 1977.

The iterative process involving (a) the linearized and (b) the uncoupled damping matrices, converges rapidly requiring three to four and two to three iterations, respectively.

PROBABILITY OF FATIGUE FAILURE - DAMAGE ANALYSIS

The statistical variability of most of the factors involved in offshore construction makes it necessary to use a probabilistic approach to fatigue analysis and design. The purpose of this section is to present a fatigue analysis technique to estimate the probability of failure of offshore stuctures in deep water (i.e., over 1000 ft [305 m]) under severe storm waves (Yang, 1978; Yang, 1979).

The occurrence of a storm wave is modeled as a homogeneous Poisson process with occurrence rate γ. Assume that in the time interval (0,t), there are N number of storms. Once the storm occurs, the wind-induced storm wave is modeled as a Gaussian random process with zero-mean and finite duration. The duration T of each storm is also a random variable with expected value $\bar{T}$ and coefficient of variation V_T. The Pierson-Moskowitz wave height spectrum given by Eq. 3 is used. The storm wind velocity W varies from one storm to another and is a random variable with log-normal density function

$$f_W(y) = \frac{2log\ e}{\sqrt{2\pi}\,\sigma_W y} \cdot exp\left[-0.5\left(\frac{log(C_1 y^2) - \mu_W}{\sigma_W}\right)^2\right] \tag{14}$$

The probability density function $f_W(y)$ of W has been derived from the statistical distribution of the annual expected maximum wave height Y_{1m}. The distribution function of Y_{1m} in the North Sea is characterized by a log-normal distribution, and W is related to Y_{1m} (Yang & Freudenthal, 1977a; Yang & Freudenthal, 1977b) by

$$Y_{1m} \approx C_1 W^2 \approx \frac{3.85}{2g}\sqrt{\frac{\alpha_1}{\beta_1}} \cdot W^2 \tag{15}$$

The base shear force Z(t) of the offshore structure is a Gaussian random process with zero mean and finite duration. Its standard deviation σ_1 (W) can be computed from the statistics of the structural response (Paliou & Shinozuka, 1986). The stress at the hotspot (fatigue critical point) denoted by S(t) for members below -26 ft of the water surface is produced mainly by the random vibration of the tower. The nominal stress Y(t) in the member connecting to the hotspot can be related in approximation to the base shear force Z(t) by the following relationship (Nolte & Hansford, 1976; Wirsching et al., 1977)

$$Y(t) = C \cdot Z(t) \tag{16}$$

where C is a constant depending on the particular design of the member and the tower. Assume that the design nominal stress is a% of the yield stress and the applied base shear force is b% of the total weight of the structure. Then C can be computed by

$$C = \frac{a \cdot \sigma_Y}{b \cdot G} \tag{17}$$

where σ_Y = yield stress and G = total structural weight.

Let y(j) be the absolute value of the j-th local extremum of the nominal stress Y(t) and h(j) be the absolute value of the j-th local extremum of the hotspot stress S(t). Then y(j) is related to h(j) through the relationship

$$h(j) = K_f \cdot y(j) \tag{18}$$

$$K_f = K_t \cdot K_w \tag{19}$$

where K_t = stress concentration factor in the range 2 - 2.5 and K_w = fatigue strength reduction factor due to the notch at the weld toe. K_f depends on the geometrical configuration of the joints and lies in the range 3 - 6 (Munse, 1964). As Y(t) and S(t) are random processes, so are their extreme point processes y(j) and h(j). For the estimation of the fatigue life of offshore platforms, the characteristic S-N fatigue curve is commonly used. It is expressed as

$$(\Delta S)^b\ N = K \tag{20}$$

where b = constant parameter and K = random variable to account for the statistical variation of the fatigue data (Marshall, 1976; Nolte & Hansford, 1976; Wirsching et al., 1977; Wirsching and Yao, 1976). ΔS = stress range equal to twice the stress peaks or troughs since Eq. 20 is obtained from test results under constant amplitude loading. The fatigue damage due to the i-th storm, denoted by D_i is

$$D_i = \sum_{j=1}^{NJ} \frac{1}{2N(j)} = K^{-1} \cdot \sum_{j=1}^{NJ} \frac{[2h(j)]^b}{2} = \\ = \frac{(2K_f)^b\, K^{-1}}{2} \cdot \sum_{j=1}^{NJ} [y(j)]^b \tag{21}$$

$$NJ = 2\nu T = \frac{\omega_a T}{\pi} \tag{22}$$

where T = duration of the storm, ω_a = the fundamental natural frequency and NJ = the total number of half-cycles per storm. T and therefore NJ are random variables.

The total fatigue damage in a service interval (0,t) denoted by D(t) is obtained using Eq. 21,

$$D(t) = \sum_{i=1}^{N} D_i = (2\,K_f)^b\,K^{-1}\,L \tag{23}$$

where

$$L = \sum_{i=1}^{N}\sum_{j=1}^{NJ}\frac{1}{2}[y(j)]^b = \sum_{i=1}^{N} Q_i \tag{24}$$

where N is a random variable following the Poisson distribution denoting the number of storms in (0,t).

Since K^{-1} and L are statistically independent random variables, the expected damage $\bar{D}$(t) and coefficient of variation V_D(t) are obtained from Eq. 23 as

$$\bar{D}(t) = (2\,K_f)^b \cdot \overline{K^{-1}} \cdot \bar{L} \tag{25}$$

$$V_D(t) = \sqrt{[V_{\frac{1}{K}}^2 + V_L^2 + V_{\frac{1}{K}}^2\,V_L^2]} \tag{26}$$

where

$$\bar{L} = \gamma\,t\,\bar{Q}_i \tag{27a}$$

$$V_L = \frac{V_{Q_i}}{\sqrt{\gamma t}} \tag{27b}$$

Following the analysis of Yang (1978), $\bar{D}$(t) and V_L(t) can be easily obtained as

$$\begin{aligned}\bar{D}(t) \;&=\; (2K_f)^b\,\overline{K^{-1}}\,\gamma t\,\frac{\omega_a \bar{T}}{\pi}\,2^{b/2}\,C^b\,\Gamma(1+\frac{b}{2}) * \\ &* \int_0^\infty \sigma_1^b(w) f_W(w)\,dw\end{aligned} \tag{28}$$

$$V_L^2 = \left\{ [\frac{(1+V_\sigma^2)\,2\pi\,f_1(b)}{\varsigma\omega_a \bar{T}}] + [V_T^2 + V_\sigma^2 + V_T^2 V_\sigma^2] \right\} \cdot (\frac{1}{\gamma t}) \tag{29}$$

where

$$1 + V_\sigma^2 \;=\; \frac{\int_0^\infty \sigma_1^{2b}(w) f_W(w)\,dw}{[\int_0^\infty \sigma_1^b(w) f_W(w)\,dw]^2} \tag{30}$$

where $\bar{T}$, V_T are the expected value and coefficient of variation of storm duration, ς is the damping coefficient associated with the vibration of the first mode of the structure and $f_1(b)$ is a function of b (Crandall et al., 1962).

Fatigue failure is assumed to occur when the total cumulative damage D(t) exceeds a certain value δ. δ lies generally between 0.3 and 1.6. It has been shown that δ is a random variable following a log-normal distribution with $\bar{\delta}$ and V_δ being the mean value and coefficient of variation, respectively. D(t) is also assumed to follow a log-normal distribution.

Hence, the probability of failure P_f(t) can be obtained as

$$P_f(t) \;=\; P[D(t) \;>\; \delta] \;=\; \Phi\Big(-\frac{ln\,\mu}{\sqrt{(1+V_\delta^2)\,(1+V_D^2)}}\Big) \tag{31}$$

where

$$\mu \;=\; \frac{\bar{\delta}\,\sqrt{1+V_D^2}}{\bar{D}\,\sqrt{1+V_\delta^2}} \tag{32}$$

NUMERICAL EXAMPLE

The same deep offshore tower analyzed before has been used herein in order to perform the statistical fatigue analysis. The variance matrix of the shear forces and the values of the standard deviation of the base shear force σ_1(W) for various storm wind velocities W, can be easily computed (Paliou & Shinozuka, 1986). The damping coefficient ς, associated with each vibrational mode is taken as 5%. The total weight of the structure is G = 107,967 kips (480,453 kN) and the fundamental natural frequency was found to be ω_a = 1.155 rad/sec, (0.184 Hz).

For the AWS fatigue model (Yang, 1978), the following parameter values are used: b = 4, $\overline{K^{-1}}$ = 9.302 × 10^{-12} and $V_{\frac{1}{K}}$ = 20%. The fatigue strength reduction factor K_f is considered to be 4.0, the yield stress is σ_y = 36 ksi (0.248 MPa) and an average storm duration of four hours together with the coefficient of variation 20% is used for illustrative purposes, i.e., $\bar{T} = 4$ hours and $V_T = 20\%$. It is further assumed that the values $\bar{\delta} = 1.0$ and $V_\delta = 0.3$. The occurence rate γ of stoms is equal to unity, since we used the distributions functions of the annual expected maximum wave height Y_{1m}. The statistical parameters of the log-normal distribution function of Y_{1m} for the North Sea (Yang & Freudenthal, 1977a) are equal to $\mu_W = 2.841$ and $\sigma_W = 0.1$. Furthermore, since we used the same wave height spectrum as before, the statistical parameters α_1 and β_1 remained the same. In the present analysis, the coefficient of variation V_σ of the b-th power of $\sigma_1(W)$ (Eq. 30) has been found to be surprisingly large, $V_\sigma = 1.67$. This can be explained by the fact that the dispersion of the base shear force is magnified by the power law b of the characteristic S-N curve (Yang, 1978).

Finally, the probability of fatigue failure for twenty-five years of service life associated with various design nominal stresses and various applied base shear forces, is shown in Fig. 4.

RELIABILITY ANALYSIS OF OFFSHORE STRUCTURES UNDER RANDOM LOADING AND PERIODIC INSPECTIONS

Fatigue damage is revealed in a structure by the initiation of a visible crack. It has been a practice to periodically inspect fatigue-sensitive structures in order to detect such cracks and to repair or replace cracked components. Hence, the reliability analysis of fatigue-sensitive structures under random loading and periodic inspection is of practical importance. Although the application of reliability analysis to offshore structures is emphasized, the approach discussed in the following is equally applicable to other fatigue-sensitive structures (Shinozuka, 1976; Yang & Trapp, 1974).

The expected number of upcrossings per unit time, denoted by $\nu_1^+(W)$ for the random stress S(t) of the fatigue critical point, with standard deviation $\sigma_S(W)$, over a strength level R is

$$\nu_1^+(R,W) = \frac{\omega_1}{2\pi} \cdot exp\left[-\frac{R^2}{2\sigma_S^2(W)}\right] \tag{33}$$

where

$$\sigma_S(W) = K_f C \sigma_1(W) \tag{34}$$

ω_1 is the apparent frequency of S(t), C is given in Eq. 17 and K_f in Eq. 19.

Since the response spectrum $S_{SS}(\omega, W)$ is narrow-band, the apparent frequency of the response of deep offshore towers can be approximated by the fundamental frequency ω_a of the tower. The expected number of upcrossings per storm is obtained by

$$\nu^+(R) = \int_0^\infty \left[\int_0^\infty \nu_1^+(R,w)\, f_W(w)\, dw\right] \cdot t \cdot f_T(t)\, dt \tag{35}$$

Since we are dealing with storms causing the annual expected maximum wave height, the occurrence rate of storms is equal to unity, i.e., one storm per year. Therefore, Eq. 35 gives the expected number of upcrossings per service year. This assumption has to be revised in future study since it may result in an unconservative estimate of fatigue performance.

The time t to crack initiation is assumed to be a random variable with a density function following a two-parameter Weibull distribution

$$f_C(t) = \left(\frac{\alpha}{\beta}\right)\left(\frac{t}{\beta}\right)^{\alpha-1} exp\left[-\left(\frac{t}{\beta}\right)^\alpha\right] \tag{36}$$

where α is the shape parameter and β the scale parameter. α and β may be estimated from test results (Eggwertz, 1971). If test test results are not available, β may be estimated from the S-N curve and the cumulative damage hypothesis.

Fracture mechanics theory is applied to cracks of detectable size initiated at a certain time or to preexisting cracks for the purpose of determining their propagating size under a stress history. The power law formula of crack propagation under Gaussian random loading which has been verified experimentally (Paris, 1964; Rice & Beer, 1965; Rice et al., 1965) will be used

$$\frac{da}{dn} = D\,(\Delta K)^b \tag{37}$$

where da/dn = the rate of crack propagation per cycle, a = the crack size, ΔK = the range of stress intensity factor, and D and b = material constants.

The expected value of the b-th power of the range of stress intensity factor, ΔK^b, can be related to the expected value of the b-th power of the rise and fall of the stress S(t) and crack length a as follows

$$\Delta K^b = a^2\, S^b \tag{38}$$

If the stress process S(t) is a Gaussian process with zero-mean and standard deviation σ_S, the expected value of the b-th power of the rise and fall of the random process $S^b(t)$ can be written as

$$S^b = A(2\sigma_S)^b = A(2K_f)^b\, C^b \int_0^\infty \sigma_1^b(w) f_W(w)\, dw \tag{39}$$

where A can be determined either by using the approximate analytical techniques proposed in Rice and Beer (1965) and Yang (1974) or by the method of Monte Carlo simulation (Shinozuka and Jan, 1972; Shinozuka, 1972; Shinozuka, 1974). Integrating Eq. 37 from the initial crack size a_0 to $a(t)$ after t service years and using Eqs. 38 and 39, we obtain

$$a(t) = \frac{a_0}{(1 - a_0 N_0 S^b D t)} \tag{40}$$

in which N_0 is the number of cycles per storm (or per year)

$$N_0 = \nu \bar{T} = \frac{\omega_a}{2\pi} \cdot \bar{T} \tag{41}$$

Crack-Stopper Design

In order to prevent the reduction of the critical strength from reaching an excessive level, a common practice is to introduce crack stoppers in the structure. The strength of these structures depends on the particular design and the residual strength should be determined by individual analysis and testing. The residual strength after t_n service years is assumed to be

$$R(t_n) = R_0\left[1 - (1-\xi)\left(\frac{a(t_n) - a_0}{a_S - a_0}\right)^{1/2}\right] \tag{42}$$

where a_S = maximum allowable crack size and ξ = maximum allowable strength reduction factor at crack size a_S, $0 < \xi < 1$. (The above equation was proposed in Yang & Trapp (1974) to exhibit the trend of limited data given in Hardrath & Whaley (1957) and Snider et al. (1972)).

Failure of the structure occurs when the residual strength $R(t_n)$ denoted by R_n, is exceeded by the applied random stress. Then the problem is essentially that of a first-passage probability with a variable two-sided threshold. The expected failure rate $h(t_n, R_n)$ or risk function associated with this first-passage problem can be approximated by (Lin, 1967; Shinozuka, 1976)

$$h(t_n, R_n) = \frac{\omega_a \bar{T}}{\pi} \int_0^\infty exp[-R_n^2/2\sigma_S^2(w)] f_W(w)\, dw \tag{43}$$

In this investigation, the initial ultimate strength and therefore the residual strength are considered normal random variables. The density function of the initial ultimate strength (mean R_0 and coefficient of variation V_R) is given by

$$f_{R_0}(x) = \frac{1}{\sqrt{2\pi}\, V_R R_0} exp\left[-0.5\left(\frac{x - R_0}{V_R R_0}\right)^2\right] \tag{44}$$

Considering this fact, the risk function $h(t_n)$ can be computed by

$$h(t_n) = \frac{\omega_a \bar{T}}{\sqrt{2\pi}\,\pi V_R R_0} \int_0^\infty \int_0^\infty exp\big[-0.5\frac{R_0^2\gamma_n^2}{\sigma_S^2(w)} - 0.5\big(\frac{x-R_0}{V_R R_0}\big)^2\big] f_W(w)\, dw\, dx \qquad (45)$$

where

$$\gamma_n = 1 - (1-\xi)\big(\frac{a(t_n)-a_0}{a_S-a_0}\big)^{1/2} \qquad (46)$$

The purpose of inspection is to detect the fatigue and pre-existing cracks in the structural components so that, before cracks become critical, they can be replaced by uncracked components to endure their designed initial strength at least at the time of replacement.

The probability of detecting a fatigue crack at a structural detail during a rigorous inspection depends on the probability of inspecting this cracked detail and the resolution capability of the particular inspection technique. Typical presently used NDI techniques include delta scan, shear wave ultrasonic, magnetic particle, X-ray and magnetic rubber (MRI) methods.

Define U_1 as the probability of inspecting a cracked detail and $U_2(a)$ as the probability of detecting a crack of size a. Then, the probability of detecting a crack of size a during a rigorous inspection can be obtained by

$$F(a) = U_1 U_2(a) \qquad (47)$$

where it was assumed that U_1 and $U_2(a)$ are independent.

Since, at this time, the information on the probability of U_1 is limited, it is assumed in this study that $U_1 = 1$, i.e., every critical detail will be inspected. Based on the experimental and empirical results, however, the detection probability $U_2(a)$ may be constructed in the following fashion (Yang & Freudenthal, 1977b).

$$U_2(a) = \begin{cases} 0 & a < a_1 \\ \big[(a-a_1)/(a_2-a_1)\big]^m & a_1 \le a \le a_2 \\ 1 & a_2 < a \end{cases} \qquad (48)$$

where $1/8 < m < 1/5$ for accurate NDI techniques, $m > 1/5$ for more crude NDI techniques, a_1 = minimum crack size below which a crack cannot be detected with the particular detection technique used, a_2 = maximum crack size beyond which a crack can always be detected by the same technique.

Following, Paliou & Shinozuka (1986), Shinozuka, (1976), Yang & Trapp, (1974) for the development of this section, let P_0 be the probability of failure of the tower within the intended service life T with no inspection. Then, assuming the time t_0 to crack initiation to be a random variable with a probability density function $f_C(t_0)$, the probability of failure can be expressed as

$$P_0 = 1 - exp\big[-Th_0 - (T/\beta)^\alpha\big] - \int_0^T f_C(t)\, exp\big[-th_0 - H(T-t)\big]\, dt \qquad (49)$$

where h_0 is the expected failure rate corresponding to the threshold R_0 or $h_0 = h(t_0)$ in Eq. 46, $f_C(t)$ is given in Eq. 36 and

$$H(t_n) = \int_0^{t_n} h(t)\, dt \qquad (50)$$

Assume now that the structure is subjected to a rigorous inspection at the end of T_0 service years. The probability of failure P(j) within the $[0, jT_0]$ interval (j-1 inspections) can then be obtained from

$$P(j) = P_j^* + \sum_{i=1}^{j-1} \int_0^{T_0} q_{ij}(t)\, f_C[(i-1)T_0 + t]\, dt \qquad (51)$$

where j=2,3,... and i=1,2,...,j-1 and the complicated expressions for P_j^* and $q_{ij}(t)$ can be found in Paliou & Shinozuka (1986).

NUMERICAL EXAMPLE

A numerical example is carried out using the crack initiation model under crack-stopper design. Table 3 shows the parameter values used for the numerical computation. Some of these values are assigned on the basis of experimental evidence, others are chosen to be consistent with conventional static design practice and still others are on the basis of engineering judgment.

The crack size and the residual strength as a function of time t after crack initiation are plotted in Fig. 5. The conditional failure rate h(t) as a function of time t after crack initiation and its approximation are displayed together with the cumulative failure rate H(t) in Fig. 6. P(j) is plotted vs. design service life and number of inspections in Fig. 7. Finally, the failure probability P(j) for the design service life vs. the number of inspections is given in Fig. 8.

CONCLUSIONS

Two methods of reliability analysis have been presented in this study: A statistical fatigue damage analysis and a first passage failure probability with periodic inspections. To compute the structural responses, a nonlinear dynamic analysis in the frequency domain is performed by means of equivalent linearization techniques. The nonlinear drag effect is important and if it is neglected, the design would be unconservative.

It is shown in Yang & Freudenthal (1977a) that for deep offshore structures (i.e., over 300 ft [91.5 m]), storm waves dominate the design criteria, compared to earthquake loading or the joint occurrence of both. For shorter structures, the importance of earthquake design is expected to increase, but this is a subject for further investigation.

In the development of the present reliability analyses, various assumptions have been made but it is believed that the results presented herein are representative, and it is not expected that these results would undergo qualitative changes if one or a few of these assumptions were altered or removed. For simplicity of the analysis, the soil-pile-structure interaction has been neglected, considering a firm soil foundation and no pile failure. Such an interaction can be taken into consideration, but the computation for the structural response will become much more involved, as the interaction is generally nonlinear.

The results of the fatigue damage analysis displayed in Fig. 4 clearly indicate that the expected cumulative damage and probability of fatigue failure increase as the design nominal stress, for a given applied base shear force, increases. Also, for a given design nominal stress, the average cumulative damage and the probability of failure decrease as the applied base shear force increases (Fig. 7). These results are consistent with those displayed in Yang (1978).

The results of the second reliability analysis method, i.e., first excursion probability with periodic inspections, are shown in Figs. 7 and 8. As expected, the probability of failure increases as the design service life increases. It is important to note that the curve for the failure probability under no inspections consists of two segments. In the first segment, the failure rate is essentially h_0 as it takes approximately six years for a fatigue crack to reach the maximum allowable crack size a_S (Figs. 5 and 6). The effect of periodic inspections is very little in this first segment for the purpose of improving the reliability of the structure. In the second segment, the effect of periodic inspections is very important as the fatigue crack already exists. Detection and repair of cracked details improve dramatically the reliability of the structure at a later service time. However, it is clearly shown in Figs. 7 and 8 that no significant improvement can be achieved by an excessive number of inspections thus setting a limit at twenty inspections.

Comparing the results of the two methods for a design service life of twenty-five years, the following observation is made. The first method, associated with the fatigue damage analysis, results in a probability of failure 1.2×10^{-2} for an applied base shear force equal to 0.2 G and a design nominal stress equal to 0.45 σ_y. Using the same design parameters, by the second method, associated with the probability of first failure (no inspections), the same result is obtained.

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TABLE 1. Structural Parameters

#	Depth z	Mass M	Inertia C_M	Drag C_D	Volume V	Area A
1	-75	330	0.0	0.0	-	-
2	10	101	115.0	20.5	28750	14643
3	75	146	187.5	27.2	46875	19429
4	205	383	471.0	46.9	117750	33500
5	400	537	757.0	64.8	189250	46286
6	600	665	981.0	69.8	245250	49857
7	800	1191	1795.0	88.2	448750	63071

Flexibility $K^{-1} \cdot 10^{-6}$						
756.0	622.0	531.0	374.0	210.0	98.2	30.8
622.0	568.0	491.0	357.0	209.0	102.0	33.6
531.0	491.0	464.0	344.0	207.0	105.0	35.8
374.0	357.0	344.0	321.0	205.0	110.0	40.1
210.0	209.0	207.0	205.0	203.0	118.0	46.5
98.2	102.0	105.0	110.0	118.0	126.0	53.2
30.8	33.6	35.8	40.1	46.5	53.2	58.8

Units : z = ft ; $\mathbf{M}$ = kips sec^2/ft ; $\mathbf{C}_M$ = kips sec^2/ft ; $\mathbf{C}_M$ = kips sec^2/ft^2 ; $\mathbf{V}$ = ft^3 ; $\mathbf{A}$ = ft^2 ; $\mathbf{K}^{-1}$ = ft/kips

Note : 1 ft = 0.3048 m ; 1kips sec^2/ft = 14.6 kN sec^2/m ; 1 ft^3 = 0.0248 m^3 ; 1 ft^2 = 0.093 m^2 ; 1 ft/kip = 0.685 × 10^{-4} m/N

TABLE 2. Natural Frequencies - Standard Deviation of Deck Displacement and Velocity Due to Different Average Wind Speeds.

W	25 ft/sec		50 ft/sec	
Nat. Frq. [Hz]	σ_u [ft]	$\sigma_{\dot{u}}$ [ft/sec]	σ_u [ft]	$\sigma_{\dot{u}}$ [ft/sec]
0.18	0.230	0.269	0.506	0.489
0.35	0.209	0.244	0.462	0.445
0.58	0.194	0.226	0.421	0.408
0.82	0.163	0.191	0.340	0.335
1.04	0.119	0.143	0.232	0.237
1.68	0.075	0.095	0.136	0.146
3.01	0.022	0.045	0.056	0.064

75 ft/sec		100 ft/sec		150 ft/sec	
σ_u [ft]	$\sigma_{\dot{u}}$ [ft/sec]	σ_u [ft]	$\sigma_{\dot{u}}$ [ft/sec]	σ_u [ft]	$\sigma_{\dot{u}}$ [ft/sec]
0.872	0.609	1.615	0.809	4.607	1.450
0.800	0.557	1.491	0.742	4.299	1.342
0.726	0.506	1.359	0.673	3.975	1.228
0.570	0.405	1.070	0.530	3.231	0.978
0.371	0.274	0.690	0.347	2.177	0.642
0.205	0.160	0.376	0.194	1.234	0.355
0.079	0.066	0.142	0.075	0.484	0.135

Note : 1 ft = 0.3048 m

TABLE 3. Parameter Values for Reliability Analysis Under Random Loading and Periodic Inspections

	Description of Parameters	Value
a_0	Crack size initiated at t_0	0.04 in
D	Material constant	1.9×10^{-10}
b	Power in crack prop. law	4.0
C	Parameter in Eqs. 16-17	7.5×10^{-4}/in^2
ω_a	Fundamental natural frequency	0.184 Hz
T	Average storm duration	14400.0 sec
t	Design service life	25 yrs.
R_0	Mean value of ult. strength	58 ksi
V_R	C.O.V. of ultimate strength	0.1
K_C	Crit. stress intensity factor	100 ksi/in$^{1/2}$
K_f	Strength reduction factor	4.0
α_1	Parameter of wind spectrum	0.0081
β_1	Parameter of wind spectrum	0.74
α	Shape param. of Weibull distr.	4
β	Scale param. of Weibull distr.	50.0 years
a_1	Maximum undetectable crack size	0.02 in
a_2	Minimum detectable crack size	0.3 in
m	Parameter in Eq. 48	0.125
a_S	Maximum allowable crack size	7.0 in
ξ	Residual strength ratio	0.43
g	Gravitational acceleration	386.4 in/sec^2
j	Number of inspections	25
A	Parameter appearing in Eq. 39	115

Note : 1 in = 25.4 mm ; 1 ksi = 6.89 MPa

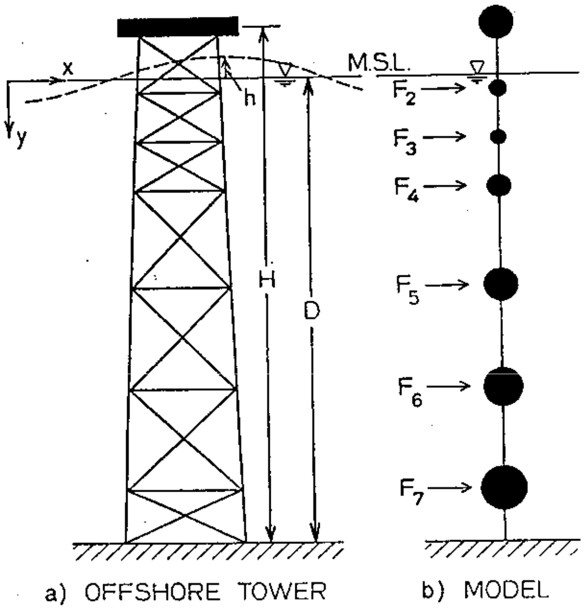

FIG. 1 A Fixed Offshore Tower and its Model

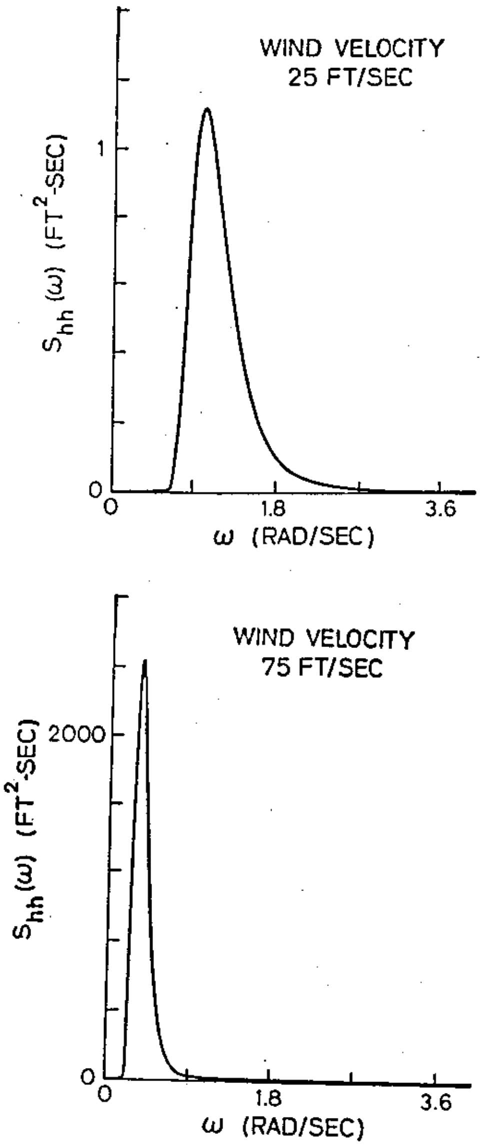

FIG. 2 One-sided Spectral Densities of Ocean Wave Elevation

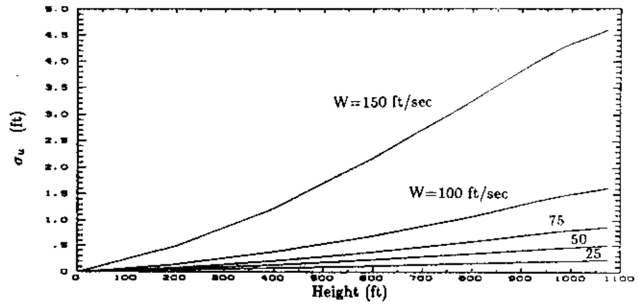

FIG. 3a Standard Deviation of Mass Velocities vs. Height

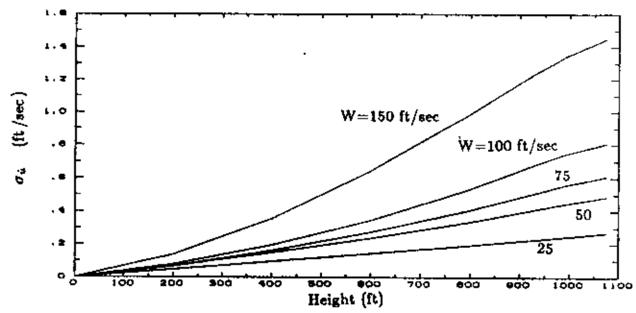

FIG. 3b Standard Deviation of Mass Displacements vs. Height

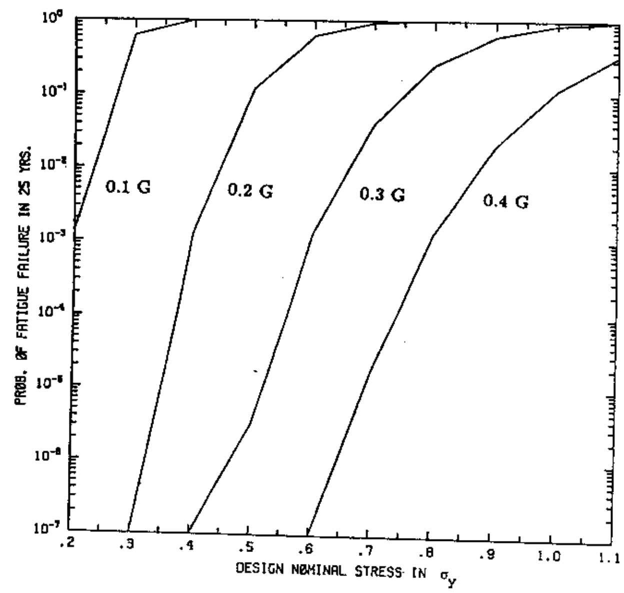

FIG. 4 Probability of Fatigue Failure P_f in 25 Years vs. Nominal Stress in σ_y

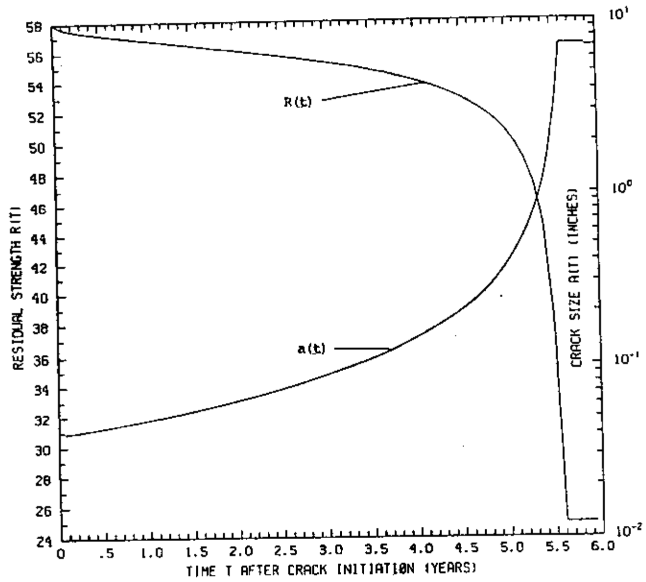

FIG. 5 Residual Strength R(t) and Crack Size a(t) vs. Time t After Crack Initiation

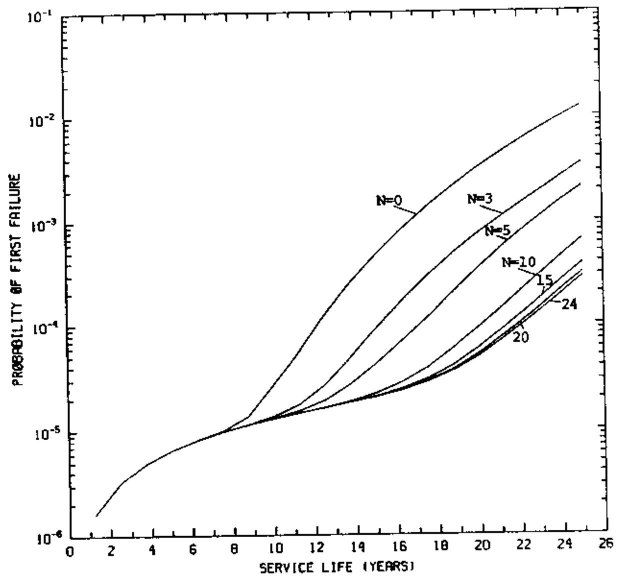

FIG. 7 Probability of First Failure as Function of Service Life and Number of Inspections

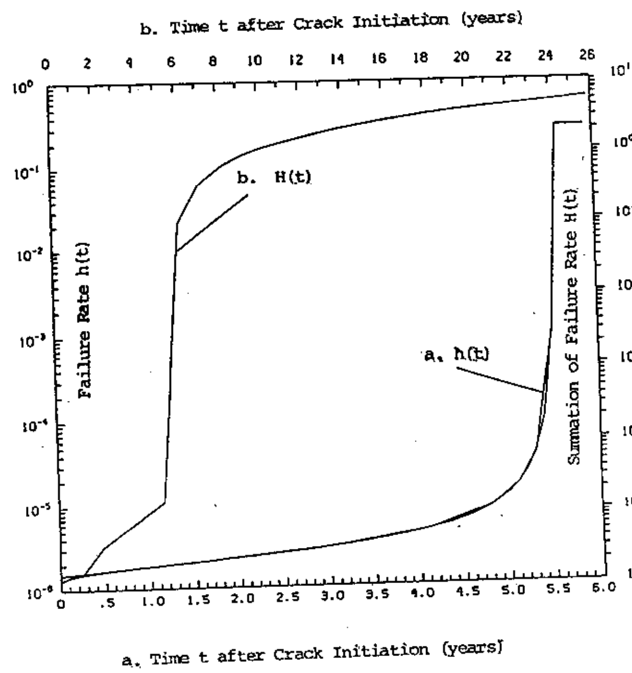

FIG. 6 Failure Rate h(t) and Summation of Failure Rate H(t) vs. Time t After Crack Initiation

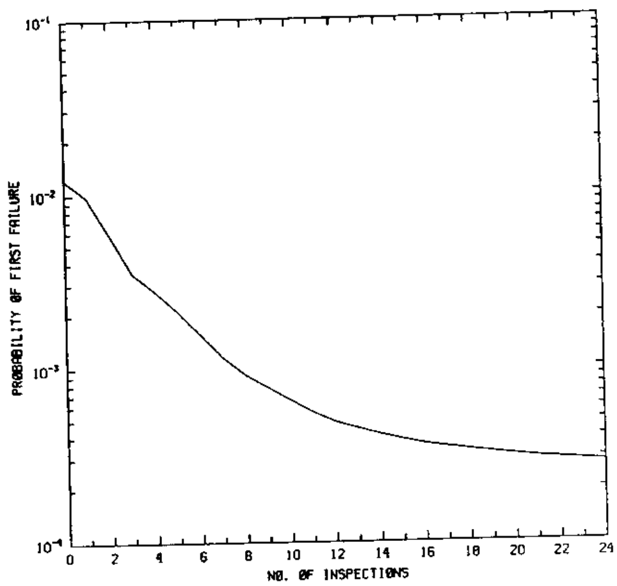

FIG. 8 Probability of First Failure at Design Service Life vs. Number of Inspections