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A Reliability-Conditioned Approach for the Fatigue Design of Marine Structures

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ABSTRACT

One of the areas of marine structural design which could benefit greatly from introducing reliability-based design methods is the design against fatigue failure. Recently, the authors have introduced a new reliability-based design method for fatigue. That method is based on the recently developed Reliability-Conditioned (RC) method and the Load and Resistance Factor Design (LRFD) code format. The approach utilizes a probabilistic treatment of available S-N fatigue data to generate partial safety factors for use in a simple design equation.

In this paper, the Reliability-Conditioned fatigue design approach will be further discussed and demonstrated with practical examples. In particular, the means for choosing the "most likely failure point", and thus the partial safety factors for the LRFD format, is further detailed. The development of a probability density function for the equivalent constant amplitude stress ranges from the existing stress records of full scale trials will also be shown. This development is similar to that currently being investigated under the auspices of the American Association of State Highway and Transportation Officials (AASHTO) for estimating fatigue design loads of steel highway bridges. And finally, the means by which the Reliability-Conditioned approach could be implemented in a design code and calibrated to that code is illustrated.

INTRODUCTION

There are a number of reasons for fatigue cracking of structural details. These include; poor workmanship in fabrication, poor welding practices, and poor design. In very many cases it has been found that poor design represented the root cause of cracking and failure. It would seem then, that what is needed is a better means to evaluate the design of structural details in fatigue. However, fatigue is a result of cyclic loading. For a ship that loading is the sea, a completely random system. To be truly useful, any proposed design method should be able to take into account the random cyclic loading as well as the uncertainty in fabrication, stress analysis methods, material properties, etc. The best manner in which to attempt this is to utilize the concepts of reliability-based design. Recently, many engineering code development organizations faced with similar types of random loading have begun to investigate and implement fatigue design requirements which are based on the concepts of structural reliability [1,2,3].

In this paper, the authors' recently introduced Reliability-Conditioned (RC) method for fatigue [4,5] is further discussed. That concept is combined with an approach for utilizing available ship stress history data to demonstrate practical applications for marine designers. In particular, the ability to rate proposed fatigue details on their effectiveness is demonstrated. The RC method of fatigue design shows considerable promise as an easy to use (and understand) approach to allow designers to quickly and accurately design structures to resist fatigue damage.

THE RELIABILITY-CONDITIONED FATIGUE DESIGN MODEL

Background

The major goal of engineering design is to produce a system which meets or exceeds both the performance and safety requirements for a given period of time and/or under a specified loading condition. However, the absolute safety of the system cannot be guaranteed due to the number of uncertainties involved. In structural design these uncertainties can be due to randomness of loadings, simplifying assumptions in the strength analysis, variability in material properties, etc. It is possible though, through a probabilistic analysis, to limit the risk of unacceptable consequences. The major benefit of a reliability-based design approach which utilizes probabilistic analysis is that a designer will be able to generate an engineering system which is both efficient and reliable to the level specified.

While the Reliability-Conditioned fatigue design model is explained in some detail in reference [4], it is felt that some of that material should also be presented here. This will allow the reader of this work to better understand the discussion of the applications of the Reliability-Conditioned fatigue design method in the following sections. To begin the discussion of the RC method a few concepts will be clarified and some terminology will be introduced.

Load and Resistance Factor Design (LRFD). The implementation of reliability-based design methods does not mean that all engineers and designers need to be deeply versed in probability theory. Rather, the design criteria they use should be developed in a format which is both familiar to the users and which should produce desired levels of uniformity in safety among groups of structures. This should be accomplished without departing drastically from existing general practice. One of the more popular formats for including probabilistic information in structural design is the Load and Resistance Factor Design (LRFD) format as recommended by the National Bureau of Standards [6]. This approach uses load amplification factors and resistance reduction factors (partial safety factors) and can be expressed as:

$$\phi R \geq \sum_{i=1}^{n} \gamma_i L_i \qquad (1)$$

In the above equation R is the resistance of the structure as expressed in a limit-state equation. That might be the resistance in, for example, ductile yielding, buckling, or the case we are interested in, fatigue. The L_i term in equation (1) represents the i^{th} load effect, e.g., due to stillwater loads, wave loads, slamming loads etc.. The coefficients ϕ and γ_i are the resistance reduction factor and the i^{th} partial load effect amplification factor, respectively. The total number of load effects considered in the linear limit state design equation is given by the value of n.

The implementation of an LRFD format for reliability-based fatigue design has been investigated by Albrecht [7,8] for the case of highway bridges. Much of the following discussion is based on his work.

Resistance Curves. For the case of the fatigue of structural details, the resistance is usually represented in terms of the mean and standard deviation of the number of cycles to failure at a given stress range. This information typically comes from constant amplitude fatigue test data for the type of detail being investigated. A number of these tests are conducted and the results are provided in the form of stress range vs. life (S-N) curves. Figure 1 gives an example of an S-N curve. Wirsching [9] and Albrecht [7] have found that a Log-Normal distribution about the mean value of number of cycles to failure can adequately represent the data points at each stress range. This distribution can be shown as a probability density function (PDF) as seen in Figure 1. The line labeled resistance in Figure 1 represents a least-squares fit of the mean values of life at each stress range. This best fit line has the log-log linear form

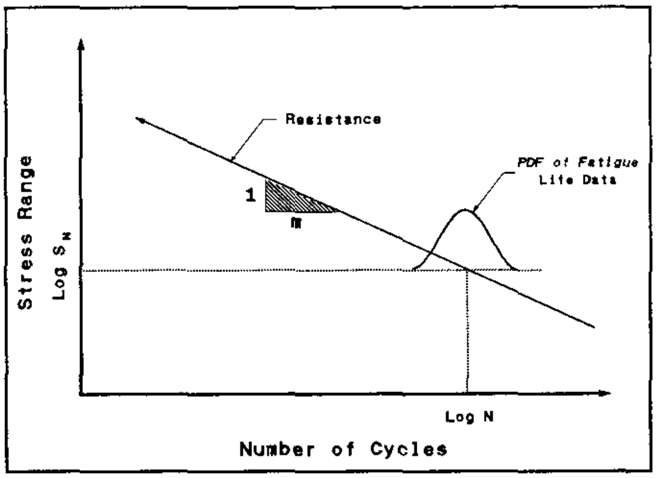

Figure 1. Example S-N Curve

$$\log N = \log C - m \log S_R \qquad (2)$$

where S_R is the constant amplitude stress range at N cycles to failure; the regression coefficients are the slope, m, and the intercept, log C. This equation is also commonly expressed in terms of stress range as

$$S_R = (C/N)^{1/m} \qquad (3)$$

The standard deviation of the fatigue life data can easily be found, however, the scatter of the data about the mean fatigue line is not the only uncertainty involved in the S-N analysis. A measure of the total uncertainty (coefficient of variation) in fatigue life, v_R, is usually developed to include the uncertainty in fatigue data, errors in the fatigue model, and any uncertainty in the individual stresses and stress effects. Ang and Munse [10] suggested that the total COV in terms of fatigue life could be given by:

$$v_R^2 = v_N^2 + v_F^2 + v_C^2 + (mv_S)^2 \qquad (4)$$

where

- v_R = total COV of resistance in terms of cycles to failure
- v_N = variation in fatigue test data about mean S-N line
- v_F = variation due to errors in fatigue model and use of Miner's rule
- v_C = variation due to uncertainty in mean intercept of the regression line; includes effects of fabrication, workmanship, and uncertainty in slope
- v_S = variation due to uncertainty in equivalent stress range; includes effects of error in stress analysis
- m = slope of mean S-N regression line

Values of m and v_N can be obtained from sets of S-N curves for the type of detail being investigated; the values of which are tabulated by Munse in reference [11]. Reasonable values for the remaining uncertainties are available in the literature [3,10,11]. Typically, v_S is taken to be 0.1, v_C is assumed to be 0.4, and v_F is taken as 0.15.

The tools of probability as used in reliability-based design apply only if the load and resistance of equation (1) are expressed in terms of the same basic quantities, i.e. either stress or cycles to failure. Typically, the resistance is provided in the form of a Probability Density Function (PDF) representing the the results of a series of constant amplitude fatigue tests on the specimen in question. As can be seen in Figure 2 this resistance is in terms of cycles to failure. The load data however, usually comes from load or stress data and generally is presented as a PDF of stress range, plotted at a specified design life, N_d. Obviously, one of the two curves must be transformed. In other words, the two curves in Figure 2 need to be plotted along the same axis.

For the reliability-based design of structures we typically express all of the variables in terms of stress. This facilitates the design process by giving a target value of stress to which the structure can be designed. Therefore one would more likely transform the resistance from cycles to failure into

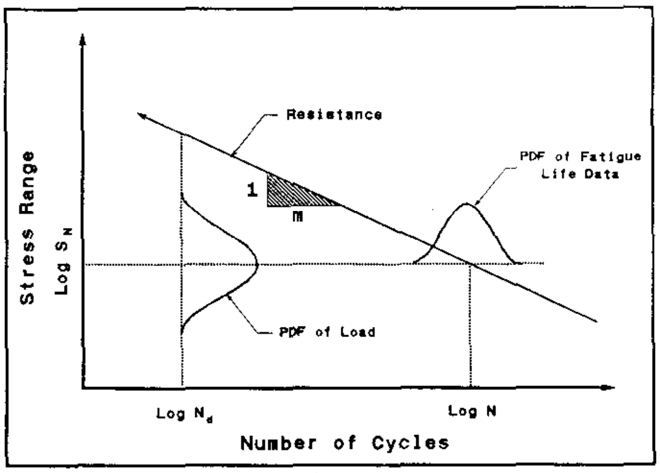

Figure 2. Load and Resistance Distributions

stress. It can be shown [12] that by using the S-N curve relationship and knowing the distribution and statistics of the resistance in terms of fatigue life, a distribution and statistics in terms of stress may be found. For this case, where there is a functional dependence between stress and fatigue life, the stress distribution has the same form as the life distribution. In other words, they have the same PDF. There is, however, a difference in the standard deviations. The relationship between standard deviations is based on the slope of the S-N curve, and is given by

$$\sigma_R' = \sigma_R / m \qquad (5)$$

where the prime indicates values in terms of stress. This relationship is shown graphically in Figure 3.

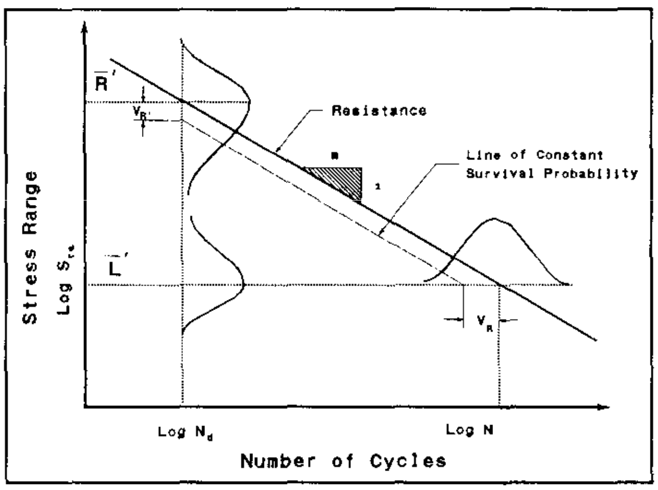

Figure 3. Transformation of Resistance [8]

The COV of the total resistance in terms of stress range is found using equation (4). These values are expressed in terms of cycles to failure and need to be converted to total COV of resistance in terms of stress range to be useful in the proposed design equations. Using the properties of log-normal distributions, equation (5), and the resistance transformation concepts of reference [9], the coefficient of variation in terms of stress range is given by

$$v_R' = \left[(1 + v_R^2)^{1/m^2} - 1 \right]^{1/2} \qquad (6)$$

<u>Equivalent Stress Range Concept</u>. For most real marine structures the loading does not take the form of a cyclic constant amplitude applied stress. Rather the loading is a random sequence of variable amplitudes and frequencies which do not repeat themselves. This type of loading can best be expressed as a continuously distributed random variable, S_L. The statistics of the variable S_L are derived from recorded stress histories or estimated from wave records. The results are usually expressed as a probability density function (PDF) of stress range for each stress or wave height record (see Figure 4). However, in order to use the S-N fatigue data, a relationship between a characteristic value of the wave induced random stress and the constant amplitude stress of the S-N curve is needed. This is accomplished by using the Palmgren-Miner hypothesis to find an equivalent constant amplitude stress for the random load distribution.

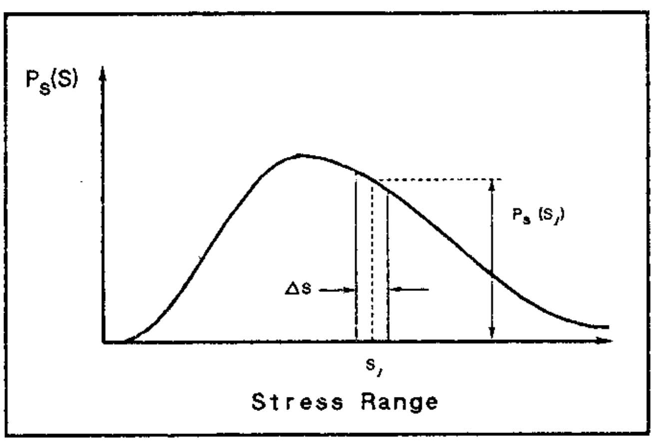

Figure 4. Example Probability Density Function for Wave Induced Stress

The Palmgren-Miner's (P-M) cumulative damage hypothesis is based on the concept of strain energy. It states that failure occurs when the total strain energy due to n cycles of variable amplitude loading is equal to the total strain energy from N cycles of constant amplitude loading. This can be written in the following form:

$$D = \sum_{i=1}^{B} \frac{n_i}{N_i} \qquad (7)$$

where
- B = the number of stress range blocks
- n_i = the number of stress cycles in stress block i
- N_i = the number of cycles to failure at constant stress range
- D = the damage ratio, which equals 1 at failure.

It can be shown [4] that the expression for the total damage due to the random load can then be written as

$$D = \frac{N}{C} E[S^m] \qquad (8)$$

where
- N = the total number of cycles in the life of a structure
- C = the y intercept log-log linear regression coefficient from the S-N curve analysis
- $E[S^m]$ = the mean or "expected value" of the stress range raisedto the m^{th} power

From the S-N analysis as given in equation (3), we can replace part of the right side of equation (8) with the following:

$$\frac{N}{C} = \frac{1}{S_R^m}$$

If D is assumed to be equal to one, as is often done, then equation (8) states that the expected value of the random stress range raised to the m^{th} power is equal to the constant amplitude stress range at N cycles raised to the m^{th} power. Thus an equivalent constant amplitude stress range can be found for a random variable amplitude stress range by using the following:

$$S_{re} = E[S^m]^{1/m} \qquad (9)$$

For $m = 2$ the above equation would represent the root-mean-square (RMS) value of the random load. In the more typical case for steels, where $m \approx 3$, the equivalent constant amplitude stress range would be the root-mean-cubed (RMC) value of the random load.

<u>The Reliability-Conditioned Approach</u>

The Reliability-Conditioned approach consists of two parts. The first part is the determination of an acceptable level of loading for a structural detail and a desired level of safety. The second part consists of determining the Most Likely Failure Point and then the partial safety factors for the "design" limit state.

When attempting to determine the unknown level of the load, several pieces of information must be known. These include the statistics and distribution of the known variable, the level of safety desired, and the design or limit-state equation. Then, using Monte Carlo Simulation with Variance Reduction Techniques (VRT), the unknown value is found. The Simulation with VRT has been fully described and analyzed by the authors in reference [13].

<u>The Most Likely Failure Point</u>. In the implementation of the LRFD format for reliability-based design for fatigue the resistance will be represented by a PDF based on the S-N fatigue data. This distribution is transformed from one in terms of cycles to failure to one in terms of stress range as described earlier. The loading will also be represented by a PDF in terms of stress range. The development of the load curve will be discussed in detail later. The limit-state equation can then be expressed as

$$\phi \bar{R}' \geq \gamma \bar{L}' \tag{10}$$

where

$\bar{R}'$ = mean of the resistance distribution
$\bar{L}'$ = mean of the load distribution
γ = load amplification factor
ϕ = resistance reduction factor

Variables expressed in terms of stress ranges have a prime in their superscript while those without primes are expressed in terms of number of cycles.

The partial safety factors are a measure of the safety of the design because they represent the separation between the mean strength and some characteristic minimum strength, or the mean load and some characteristic maximum load. The sum of the separations is the distance between the means of the load and resistance distributions and that separation indicates the level of safety. This is illustrated in Figure 5.

In order to determine the partial safety factors, a point $\underline{X}^* = (R^*, L^*)$ of special characteristics needs to be defined in the space of the basic random variables. Then the partial safety factors can be determined using the following:

$$\phi = \frac{R^*}{\bar{R}} \quad ; \quad \gamma = \frac{L^*}{\bar{L}} \tag{11}$$

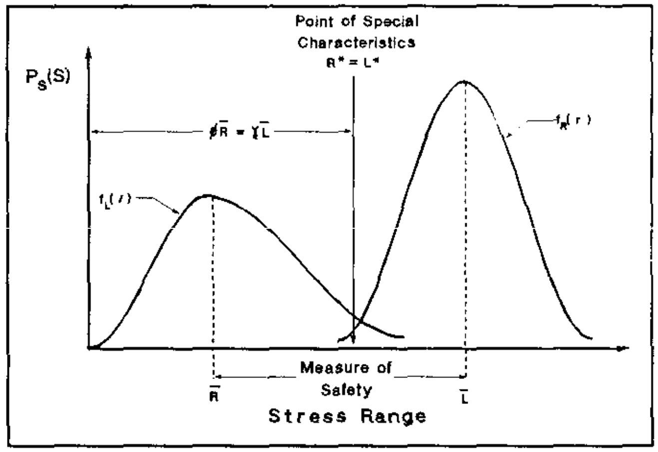

Figure 5. Separation of Means as a Measure of Safety

The point $\underline{X}^*$ can realistically be chosen as any point between the mean values of the load and resistance. It is merely being used as a reference point from which to measure the total separation. However, the most logical way to consistently chose an appropriate point would be to select that point on the failure surface where failure is most likely to occur. That is the "Most Likely Failure Point".

Using the concept of "conditional probabilities", the most likely failure point can be defined. In that approach the random variables R and L are assumed statistically independent with cumulative distribution functions and probability density functions of $F_R(r)$, $F_L(\ell)$, $f_R(r)$ and $f_L(\ell)$, respectively. The probability density function of the resistance R" of the structures that <u>fail</u> is given by

$$f_{R''}(r'') = \frac{f_R(r'')\ (1 - F_L(r''))}{P_f} \tag{12}$$

The probability density function of the load effect L" which <u>causes failure</u> is given by

$$f_{L''}(\ell'') = \frac{f_L(\ell'')\ F_R(\ell'')}{P_f} \tag{13}$$

These density functions are illustrated in Figure 6. For the case of only two random variables, the most likely failure point can be defined as the intersection of the conditional probability density functions given by equations (12) and (13). That point represents the point <u>on the failure surface</u> ($R^* = L^*$) which has the combination of the most likely resistance given failure and the most likely load to cause failure. This point can be easily evaluated by solving the following equations:

$$f_{R''}(R^*) = f_{L''}(L^*) \tag{14}$$

$$R^* = L^* \tag{15}$$

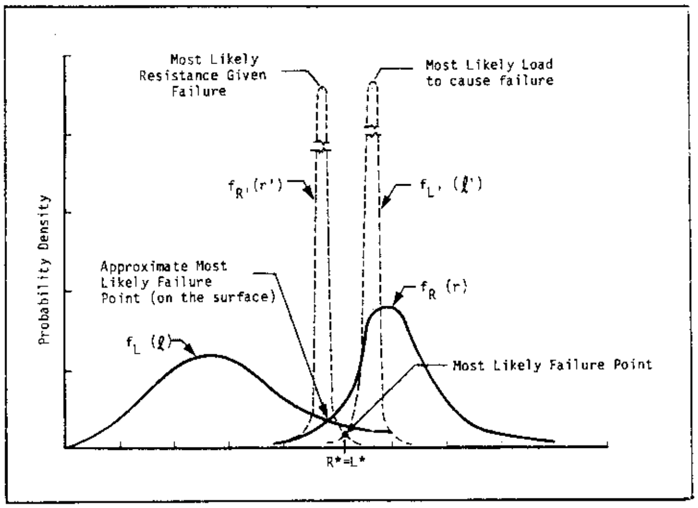

Figure 6. Most Likely Failure Point Definitions

When both the load and resistance are normally distributed and have the same standard deviation, the most likely failure point as defined by equations (14) and (15) is also the intersection point of the load and resistance PDF's. Mansour [15] and Ayyub and White [4,5,14] have used this intersection of the load and resistance PDF's to approximate the most likely failure point. While it does provide a consistent location from which to evaluate the partial safety factors, it is not actually the most likely failure point. It should be noted however, that in the case of multiple random variables, equations (12) and (13) become considerably more complex. At this writing a simple and effective means for identifying the most likely failure point for multiple random variables is still being developed. In the interim, the approximate method given in references [5] and [14] has been shown to generate partial safety factors which provide engineering systems with the desired level of safety. For the case of only two random variables it is not any significant increase in difficulty to find the point identified by equations (14) and (15), consequently for this analysis the more rigorous approach will be used.

<u>The RC Fatigue Design Approach</u>. The first step in the approach is to determine the mean value of the load which provides the desired level of reliability based on the resistance information from the S-N data for the detail of interest.. For fatigue design the mean value of the load would be an equivalent constant amplitude stress range. This requires a number of pieces of information:

1. The mean value of the constant amplitude stress range at N_d cycles from the S-N curve for the detail of interest. This can be found by specifying the m and C values for the detail, the number of cycles in the design life N_d, and then solving equation (3).
2. The distribution type to be used for the fatigue life data and the coefficient of variation of the that data. These will be used to construct the resistance PDF in terms of stress range and the total Coefficient of Variation of stress range. The COV of the total stress range in terms of fatigue life is found using equation (4) and transformed to stress range using equation (6).
3. The Coefficient of Variation and distribution type for the load data. The loading will be represented by a distribution of equivalent constant amplitude stress ranges derived from ship loading histories. The next section of this paper discusses the generation of the load curve.
4. The level of reliability sought in the design process. This can come from the level implied by existing design methods or from comparison to what levels are being used in other fields for the same type of problem. Usually it will be expressed as the probability of failure in the design lifetime.

The information provided above is used in a computer program which solves for the mean value of the load distribution necessary to provide the level of reliability desired. The program uses a Monte Carlo simulation with Variance Reduction Techniques as described in reference [13]. The newly found mean value of the load distribution represents an equivalent constant amplitude stress range, S_{re}, which the detail may experience and still maintain the desired level of reliability.

If every designer had the computer program described above and enough computer time or money to run the program for each design case, there would be no need to continue to the second part of the RC method. However, it is the intention of the authors to use the information found from the program to develop a set of partial safety factors for an LRFD format design equation. These factors will allow a designer to correctly find the design stress range without the time or expense of the Monte Carlo Simulation.

The second step of the RC method uses the distribution types and the values of the first two moments (mean and standard deviation) of the load and resistance. These values are used to iteratively solve equations (14) and (15) to find the "most likely failure point" on the failure surface. The values found for R^* and L^* are then used, with the mean values of R and L, in equation (10) to find the partial safety factors ϕ and γ. In order to be truly useful for design, the partial safety factors must be developed to cover a wide range of detail types and design lives. A designer would only have to "lookup" the detail type of interest, then knowing the design life find the appropriate partial safety factors. The result of applying the partial safety factors to the design equation and solving for the mean value of the load is the equivalent constant amplitude stress range, S_{re}.

One of the advantages of the RC method is that it will always provide a level of safety equal to that specified when developing the partial safety factors. That is because the first step finds the mean values of the load based on the desired level of safety. The method also makes use of the LRFD format and is capable of handling any distribution type for the load or the resistance. The variability of the load is accounted for through the use of the distribution of equivalent stress ranges and its moments. The resulting equivalent stress range is related to the design stress range using Munse's random load factor.

<u>Random Load Factor</u>. The stress range developed so far by the design procedure is an equivalent constant amplitude stress range which is the mean value of the load curve. It is based solely on the characteristics of the S-N curve and an estimated coefficient of variation and distribution type for the load curve. Equation (9) provides a way to relate this equivalent stress, S_{re}, to the loading expected. From that equation the equivalent stress is equal to m^{th} root of the m^{th} moment of the random load distribution. Since structural elements are designed to extreme loadings, it would be convenient if a design relationship could be introduced to relate the constant amplitude equivalent stress range for the loading to the once in a lifetime extreme stress. Munse [11] proposed a means of doing this by introducing the following:

$$S_{re} = E[S^m]^{1/m} = \frac{S_{rd}}{\xi} \qquad (16)$$

where

S_{rd} = the maximum stress range in a random loading expected only once in the vessel's lifetime

ξ = Random Load Factor

$$= \frac{S_{rd}}{E[S^m]^{1/m}}$$

The random load factor represents the distance, along the vertical axis, between the equivalent stress range for the loading, S_{re}, and the "once in a lifetime" stress range S_{rd}. The key to finding the distance is to find the equivalent stress range in terms of the once in a lifetime stress. The definition of $E[S^m]^{1/m}$ for the load distribution type, in this case a Weibull distribution, is [11]:

$$E[S^m]^{1/m} = S_{rd}\{-\ln[P_f(S_{rd})]\}^{-1/k}\,\Gamma[(m/k)+1]^{1/m} \qquad (17)$$

The $P_f(S_{rd})$ term in equation (17) is the probability that the once in a lifetime stress range will be exceeded; k is the Weibull shape parameter for the load distribution; and $\Gamma[.]$ is the gamma function. All of the other terms in the equation are as defined before. In the design of ship structures the number of load cycles in the life of a ship is generally considered to be 10^8. Then the once in a lifetime stress range is that stress range which appears once in 10^8 cycles. The probability of exceeding that stress range is thus $1/10^8$, or 10^{-8}. Since the definition of the random load factor has an S_{rd} term in the numerator and it has the right hand side of equation (17) in the denominator, the S_{rd} terms cancel. The random load factor is simply a function of the number of load cycles expected in the lifetime, the Weibull shape parameter for the load, and the slope of the S-N line.

For ship structures, Mansour [15] has shown that the long term distribution of the stress ranges are better approximated by an exponential distribution vice the Weibull distribution. For exponential distributions the equation for the random load factor becomes:

$$\xi = [\ln(N_d)]\cdot[\Gamma(1+m)]^{-1/m} \qquad (18)$$

The load factor only depends on the design life, N_d, and the slope of the S-N curve for the detail. It would therefore be very easy to generate a table of the values to facilitate the designer. The stress found using the random load factors is the "once in a lifetime" stress given the expected load history. The use of the random load factor is illustrated in Figure 7.

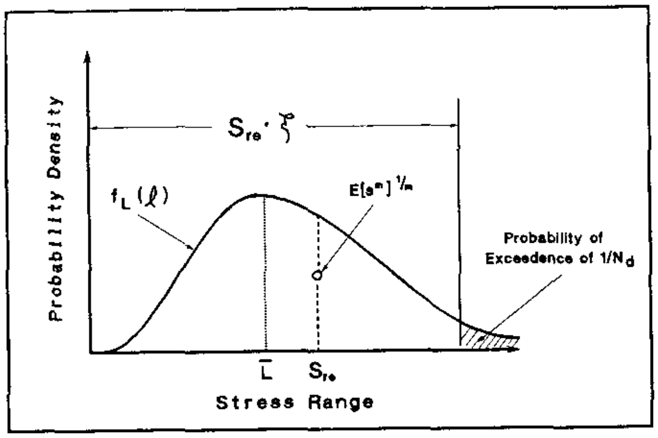

Figure 7. Random Load Factor

THE LOADING MODEL

In the previous sections a methodology for designing structural details to achieve a certain level of reliability over a vessels life has been described. One of the key elements of that approach is a reasonably accurate estimate of the expected loading distribution which the ship will experience in its lifetime. Knowledge of the statistics of the lifetime load distribution - its coefficient of variation and distribution type - is needed in order to perform the Reliability-Conditioned analysis.

It is always worthwhile when venturing into an area which is new to a particular field of engineering to take the time to see what other engineering disciplines have done to handle similar problems. The fatigue of steel structural details is not a problem isolated to the field of Naval Architecture. Civil Engineers have been faced with a similar problem in the construction of highway bridges. The loading experienced by these bridges is somewhat random in nature (variable traffic patterns and vehicle weights) and the structures are exposed to a corrosive environment.

AASHTO Specifications and Proposed Revisions

The American Association of State Highway and Transportation Officials (AASHTO) acts as the cognizant design authority for the design of highway bridges in the United States. The current AASHTO fatigue specifications [16] were first introduced in the early 1970's. They represented a significant improvement over the previous editions by the introduction of two new concepts. First, stress range alone is used to define the fatigue strength of a structural detail. Secondly, all structural details are assigned to one of five categories, A through E. Each of these categories has a separate allowable S-N line for design as shown in Figure 8.

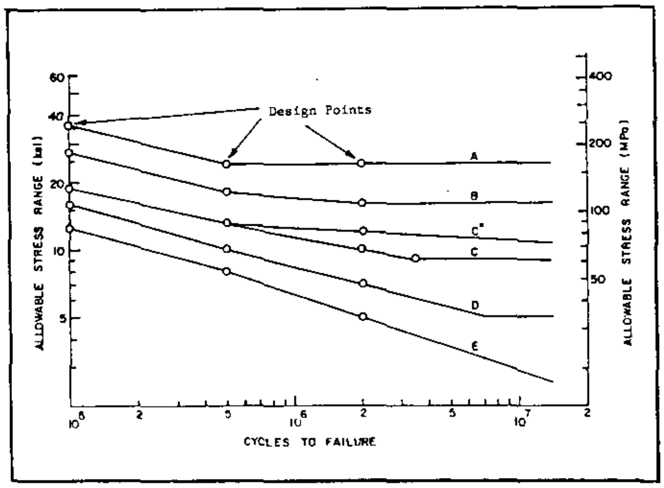

Figure 8. AASHTD Design S-N Lines for Non-Redundant Load-Path Structure [17]

All highway bridges are designed for the same load history [7]. That history was derived from a nationwide survey of loadmeters on a variety of bridges in a number of geographic regions. Based on the loadmeter survey, the AASHTO specifications recognize three separate loading cases depending upon which type of road the bridge is located on (major highway, state highway, county, etc.). These cases correspond to a specified design number of load cycles with which the S-N curves are entered to establish the maximum allowable stress range. The stress induced by the passage of one "design truck" over the bridge is then compared to the allowable stress. The detail is accepted if this stress is less than the allowable stress range.

Though much better than its predecessors, there are a number of oversimplifications and inconsistencies in the AASHTO fatigue specifications. Principle among these is the modeling of the loading with only three design points. Since real bridge loadings will vary from these three design points, it is possible to have some details with overly large factors of safety. In the time since the last revision of the specifications, a number of investigators have proposed improvements to correct some of these problems. Several of those works have dealt with a developing a better means for determining the fatigue design loading so that a uniform level of safety could be achieved for all details on all bridges. Albrecht [17] has provided a comprehensive review of these works.

Fundamental to all investigations into fatigue design is the need for loading data. Yamada and Albrecht [18] collected 104 stress range histograms from 29 bridges in 8 states which had been instrumented as part of a National Cooperative Highway Research Program (NCHRP) study. Because the histograms were from different details on different bridges, they were normalized with respect to the maximum measured stress range. This data set has been used in a number of investigations and has recently been extended by Shaaban [19]. Of interest to the ship structural designer is the manner in which Albrecht [7] utilized these stress range histograms to construct the load curve in a LRFD format fatigue design method.

The construction begins by finding the equivalent stress range, S_{re}, of a single normalized stress range histogram. Once the equivalent stress range is calculated for the histogram, it replaces the histogram in subsequent calculations and becomes one point on the load curve. The process is repeated for each stress range histogram available. The resulting distribution of the normalized equivalent stress ranges are plotted and tested in order to establish an estimated load distribution type and standard deviation. For the case of the highway bridges investigated by Albrecht, a log-normal distribution with a coefficient of variation of 0.12 was found to satisfactorily represent the distribution of the normalized equivalent stress ranges. A more complete discussion of Albrecht's method can be found in reference [4].

Approach for Ship Structures

Unfortunately, at this time there is no large body of stress histograms available for a variety of ship types, structures, and trading patterns. What is available however, is the collection of excellent data from the SL-7 project sponsored by the Ship Structure Committee [20 - 22]. That data was primarily a collection of scratch gauge recordings for eight SL-7 class vessels over a period of 7 data years.

The scratch gauges do not directly record stress, rather they record the longitudinal change in length over a known distance as a result of hull loadings. Knowledge of the mechanical properties of the material to which the scratch gauges are attached allows for the determination of stress level. On all eight ships the scratch gauges were installed on the starboard side, 2nd longitudinal shell girder, just forward of frame 186. The Sea Land McLean had an additional gauge installed in approximately the same location on the port side. The gauges were set up to record the maximum deflection (stress range) experience in a four hour period. In all, a total of better than 53,000 readings were recorded over the life of the project.

Admittedly this data is not without its problems. The scratch gauges were not able to distinguish between the contributions of torsional, lateral, and vertical bending moments to the total loading. In addition, for a fatigue analysis one is usually interested in the total number of stress cycles experienced, not the peak stress range in a given four hour period. The difference here is graphically illustrated in Figure 9. Assuming a nominal 7.5 second period for the loading, there would be 1920 cycles in the four hour period. The scratch gauge only recorded the maximum peak to trough stress, which may not even have occurred in the same loading cycle.

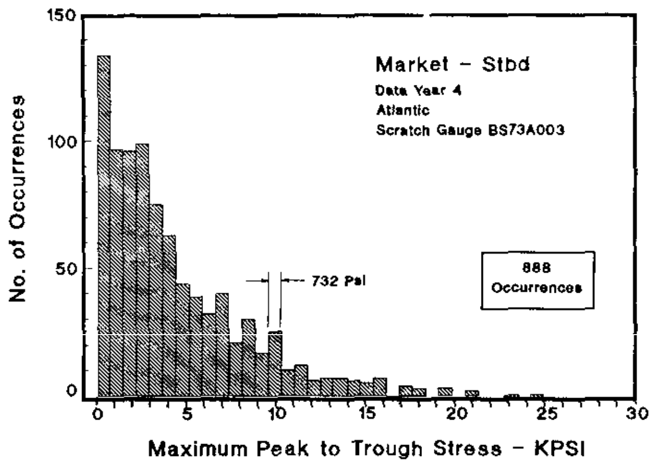

Figure 9. Example Stress Range Histogram for One Ship Data Year [21]

Despite the limitations of the data, some useful insights and practical applications can be garnered from it. The data was reported in Reference [21] as a series of 63 histograms, each representing one gauge for one "ship data" year, a sample of which is given in Figure 10. In addition, histograms for the cumulative totals for the Atlantic and Pacific oceans and a total for both oceans were given for each data year. Finally, a seven year summary histogram for the Atlantic, Pacific and grand total for both oceans were also provided. Munse [9] showed that if one assumes a 7.5 second period, the 53,000 reading represent approximately 10^8 stress cycles - the same number of

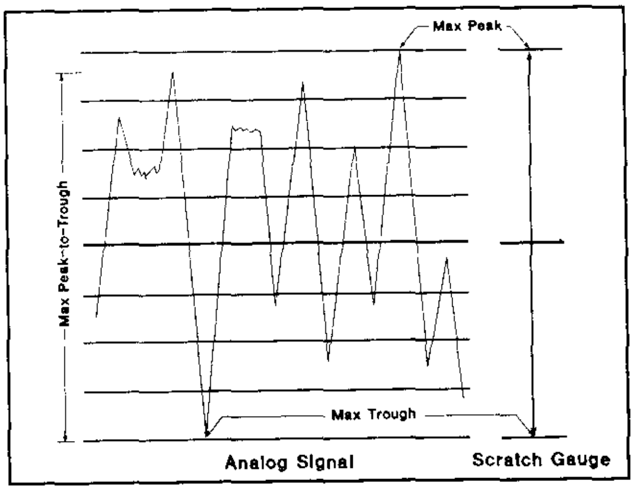

Figure 10. Example Comparison of Scratch Gauge with Analog Signal [21]

cycles which is typically used to represent the number of cycles in a ship's "design life". Also, Mansour [15] showed that the distribution of the cumulative histogram could be reasonably well approximated with a exponential distribution with the parameter λ = 3.89. This is consistent with the generally accepted notion that the long term stress histories can be approximated with an exponential distribution and the short term histories by a Rayleigh distribution. Both the exponential and the Rayleigh distributions are special cases of the more general Weibull distribution.

The equivalent stress range concept allows one to find a single valued equivalent constant amplitude stress to replace the entire random stress distribution. If the equivalent stress range is found for the seven year total histogram (representing the lifetime random stress range distributions) it is single valued and no variability is implied in its use. This is the approach taken by Munse in reference [11]. The authors have shown [4] that using a single histogram, and thus a single equivalent stress range to represent the loading is non-conservative. A better approach would be to use all of the available histograms and develop a load curve in the manner described by Albrecht. The major difference is that since the histograms were all from gauges attached in the same relative location on nearly identical ships, the histograms need not be normalized. However, since the histograms are based on variable sample sizes, the statistical characteristics of the equivalent stress ranges needs to be based on weighted histograms. The weight factors are the sample sizes.

Making the assumption that the stress range histograms for each ship data year represent a stationary, ergodic process, a load curve can be developed. A computer program was written which would find the equivalent constant amplitude stress range for each of the 63 individual ship data year histograms. Each of these stress ranges were then considered to be one point on the load curve. Using the Chi-squared and Kolmogorov-Smirnov goodness of fit tests it was determined that a normal distribution provided the best fit (at the significance level α = 5%) for the load curve data. The mean equivalent constant amplitude stress range S_{re} is given by the following weighted average:

$$\bar{S}_{re} = \frac{1}{h} \sum_{i=1}^{h} S_{re}|_i \, n_i \qquad (19)$$

where $S_{re}|_i$ is the equivalent stress range for the individual ship data year histograms, n_i is the corresponding sample size, and h is the total number of histograms. The variance of S_{re} is given by

$$\mathrm{Var}(S_{re}) = \frac{\sum n_i \, S_{re}^2|_i}{\sum n_i} - \bar{S}_{re}^2 \qquad (20)$$

The equivalent constant amplitude stress S_{re}, as seen in equation (9), depends on the value of m for the detail of interest. Thus the mean value and standard deviation of the load curve depend on the value of m. However, the distribution type and the coefficient of variation of the load curve should be independent of the value of m. In order to determine if the independence assumption was true, a number of load curves were developed and tested using different values for m. In all cases the difference in the COV's was negligible and distribution types were the same.

The statistics of the load curves are shown in Table 1 for the Atlantic, Pacific, and both oceans based on the individual ship data year histograms.

Table 1. Load Curve Development

	Atlantic	Pacific	Totals
Number of Histograms	20	39	63
Distribution Type	Normal	Normal	Normal
COV	0.259	0.208	0.25

The difference in the COV's for each ocean can be partially attributed to the fact that larger parts of the Pacific voyages were done at lower latitudes than the typical US East Coast-Northern Europe run in the Atlantic. Thus there were typically longer periods of calmer seas. This tended to lower the S_{re} for the Pacific voyages and contributed to the lower COV's.

For the design of fatigue details on new ships the authors recommend using a normal distribution with a COV of 0.25 to provide the statistics of the load curve in the RC method. These values are useful for the general case, but could be varied by the individual designer based on more detailed knowledge of the ship's intended loading and voyage patterns.

DESIGN EXAMPLES

In order to more clearly understand the RC fatigue design method, and to demonstrate how the RC method could be used to rate fatigue details, some common structural details will be examined. Munse [11] provided a list of 72 of the most common structural fatigue details. This list includes the values for m, C, and v_N for each detail. Figure 11 shows some of the fatigue details found most often in ships and Table 2 gives the information available for each.

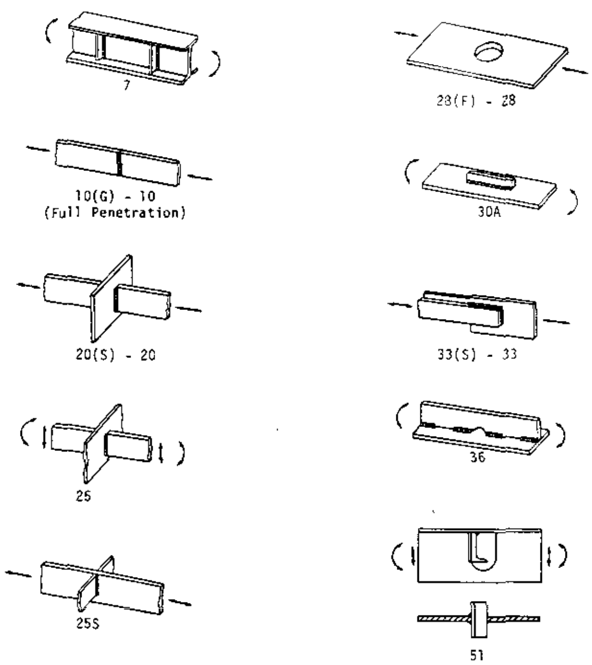

Figure 11. Common Fatigue Details in Ship Structures [11]

Table 2. Fatigue Detail Data

Detail #	m	Log C	S_N (ksi)	v_R	v_R'	v_L'
10	7.589	16.63	13.71	1.24	0.128	0.25
25	7.090	15.79	12.55	1.14	0.129	0.25
25A	8.518	19.47	22.21	1.32	0.118	0.25
28	7.746	17.41	16.40	1.20	0.122	0.25
33	3.660	9.86	3.22	0.75	0.184	0.25
33S	10.368	19.59	13.12	1.38	0.100	0.25
36	6.966	15.15	10.63	1.03	0.123	0.25
51	3.818	10.93	5.85	0.58	0.142	0.25

Note: S_N found at $N_d = 10^8$ cycles

Demonstration of the RC Method

The problem chosen is that of designing the non-tight collar shown in Figure 12. This is a typical structural detail in tank vessels, and one which has been known to experience problems. It has been given the reference number 3A11 by Jordan and Knight [23]. For this problem, it is desired that the level of reliability be 99.9% for a design life, N_d, of 10^8 cycles. As can be seen from Figure 12, the non-tight collar contains two weld details which need to be examined. The weld detail which will be examined here is #25 (as shown in Figure 11).

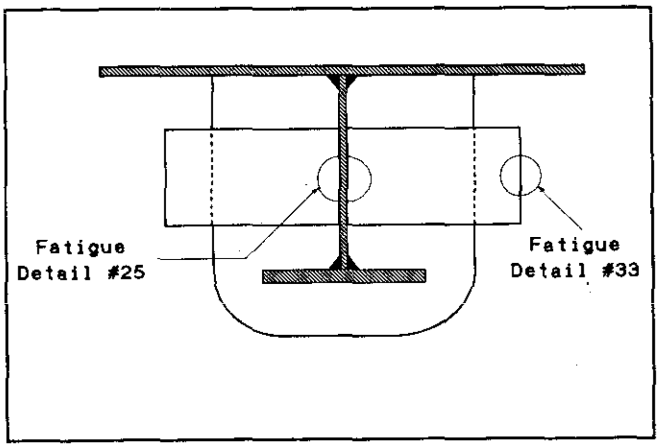

Figure 12. Non-tight Collar Detail No. 3A11 [23]

For this example, the fatigue life data is assumed to be Log-Normally distributed. The distribution of the equivalent constant amplitude stress ranges from the vessel load histories is assumed to be Normal with a COV of 0.25. With the desired level of reliability equal to 99.9%, the probability of failure is 10^{-3}. From Table 2, for detail #25, m = 7.090 and Log C = 15.79. With this information the following calculations can be made:

S_N	= 12.55 ksi	(equation 3)
v_R	= 1.137	(equation 4)
v_R'	= 0.129	(equation 6)
v_L'	= 0.25	(given)

Both of the steps of the RC method are required. From the first step, using the above information and simulation with VRTs, the required value of L' for a $P_f = 10^{-3}$ is

$L' = S_{re} = 6.073$ ksi (using 4 iterations of 1000 cycles)

To relate this equivalent stress to a design stress, the random load factor is required. For this example, the long term stress distribution is assumed to follow an exponential shape (Weibull with k =1) and the random load factor is found from equation (18).

$\xi = 18.42 \cdot .2928 = 5.394$ (equation 18)

Using equation (16), the random load factor from equation (18) is applied to find S_{rd}.

$$S_{rd} = S_{re} \cdot \xi = 6.073 \text{ ksi} \cdot 5.394 = \underline{32.76 \text{ ksi}}$$

To develop partial safety factors for design, the second step of the RC method is carried out. The "most likely failure point", which is found by iteratively solving equations (14) and (15), is given by

$R^* = L^* = 9.76$ ksi

The partial safety factors are then

$\gamma = 1.607$; $\phi = 0.778$

Rating of Fatigue Details

One of the real benefits of having a consistent method of evaluating structural details in fatigue would be the ability to chose the best structural detail for a particular application. The RC method is well suited for this because it provides a realistic and consistent measure of the level of reliability inherent to a particular detail.

To demonstrate how the RC method could potentially be used to rate structural details in fatigue, three common details will be investigated.

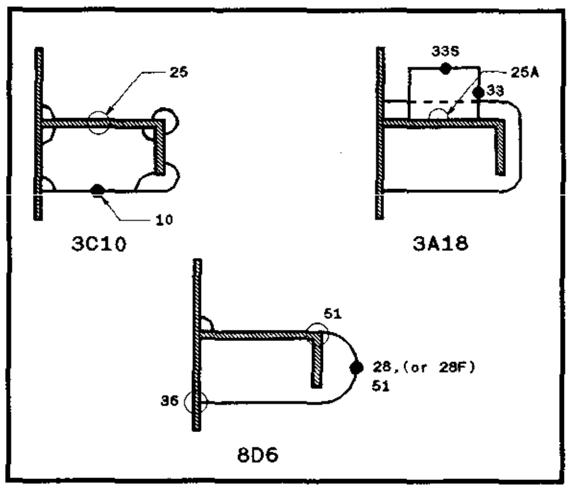

Figure 13. Details for Fatigue Rating [23]

These details were regularly used at the intersection of transverse web frames and side longitudinals in the tank sections of VLCCs. In reference [23] these details are identified as detail numbers 3A18, 3C10, and 8D6, and are illustrated as shown in Figure 13. It is important to note the welding details associated with each structural detail.

The procedure demonstrated on the fatigue detail number 25 in the previous example is used on each of the details from Figure 13. A summary of the resulting design stresses, S_{rd}, and partial safety factors are given in Table 3. The results in Table 3 indicate that the controlling factor in detail 3A18 is the effect of vertical relative movement between the longitudinal and the web. The fillet welds of the collar plate are not the problem area as it might seem, rather it is cracks growing across the collar plate or into the web from the corners where the plate overlaps the web. Detail 8D6 seems to be able to withstand a higher loading. The controlling factor here is crack growth from the edge of the top fillet weld or from the arc of the cutout. It should be noted that fatigue detail #28 was used in the calculation because data for #28(F) was not available. However, the difference between #28 and 28(F) is that the former cutout is machined and the later left flame cut. It should be apparent that if the cutout is left flame cut it would have shorter life expectancy. From the results in Table 3 detail 3C10 would appear to be the detail of choice. Both weld details have very high design stresses and they are very close to one another. This would indicate that there is little wasted strength in this design. It should be noted however, that detail 3C10 would probably be the most expensive to fabricate of the three structural details investigated.

Discussion of Results

In light of the results of the calculations summarized in Table 4 it is interesting to qualitatively assess the three structural details investigated. Detail 3A18 is a detail that was used on many VLCC's built in the northern european region in the early 1970's. These proved to be fairly poor details in that cracking of the collar and web occurred quite often, and unusually early in the vessel's lives. In some cases ship owners had repair yards backfit a collar plate on the underside of the longitudinals to help alleviate the problem.

Detail 3C10 is representative of the type which was commonly used in the VLCC and ULCC's built in a variety of Japanese yards in the mid to late 1970's. Dubbed the "crab-eye" type slot, these details were a result of investigations by the Japanese classification societies and others [24] after word of the problems on the early VLCCs spread. Experience with this type of detail has been generally very good over the relatively short life that these vessels with these details have seen.

The last of the three details is more typical of the type found on U.S. built tankers and bulk carriers. The detail was identified by Jordan and Cochran [25] as one of the family of similar details which experience the highest number of observed failures. This is apparently due to crack growth from the rough flame-cut edges of the cutout typically found in many ships. The results of the analysis in the previous section indicates that if those edges were machined there would be considerably more resistance to fatigue.

Table 3. Fatigue Detail Rating Results

Structural Detail	Weld Detail	ϕ	γ	ξ	S_{re} (ksi)	S_{rd} (ksi)
3A8	10	.781	1.608	5.108	6.66	34.00
	25	.778	1.607	5.395	6.07	32.76
3C10	25A	.806	1.629	4.648	10.99	51.08
	33	.647	1.506	8.859	1.39	12.27
	33S	.852	1.668	3.945	6.71	26.45
8D5	28	.796	1.621	5.024	8.06	40.47
	36	.794	1.619	5.471	5.21	28.50
	51	.746	1.579	8.601	2.77	23.79

CONCLUSIONS

There exists a definite need in the ship structural design community to begin to implement probabilistic methods. Implementation in the form of a Load and Resistance Factor Design (LRFD) format offers the advantages of an easy to understand and use approach which can be applied to almost any design situation. The most difficult potential failure mode in which to attempt to use probabilistic methods is the fatigue design of structural details. It is however, the potential failure mode where the most stands to be gained by using probabilistic methods.

The Reliability-Conditioned approach discussed here utilizes the the LRFD format and is flexible enough to be able to handle just about any distribution type and a variety of limit state equations. It is relatively easy to use and would be well suited for generating partial safety factors for design code implementation of a LRFD format probabilistic design method. A suitable implementation of this approach would be for the design authority to provide a table of load amplification factors, γ, and resistance reduction factors, ϕ, for each fatigue detail. Another table, containing values of the random load factor, ξ, for one design life ($N_d = 10^8$), should also be provided. A simple equation would allow the designer to find ξ if the design life was other than 10^8 cycles. Such a table was developed by Munse and provide in reference [11]. The designer would then only have to:

1) Look up values of C and m for the detail, then use equation (2) to find S_N for the desired N_d
2) Look up the appropriate random load factor for the values of m and N_d.
3) Look up the partial safety factors ϕ and γ for the detail.
4) Solve the following simple equation:

$$S_{rd} = \frac{\phi \cdot S_N}{\gamma} \xi \qquad (22)$$

The scratch gauge data from the SL-7 program has been used to provide a realistic, yet practical means for furnishing information concerning the statistics of the loading distribution for fatigue design. The investigation into using the SL-7 data showed that for this analysis the data tends to confirm some commonly accepted ideas regarding ship loading in a seaway. While not perfect, the data provides a tremendous boost to those attempting to use probabilistic methods in ship design.

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REFERENCES

1. Shinozuka, M., "Development of Reliability-Based Aircraft Safety Criteria: An Impact Analysis," Technical Report AFFDC-TR-76-36, Air Force Systems Command, Wright-Patterson Air Force Base, Ohio, 1976.

2. Harris, D.O., E.Y. Lim and D.D. Dedhia, "Probability of Pipe Fracture in the Primary Coolant Loop of a PWR Plant," Science Applications Inc., Prepared for the U.S. Nuclear Regulatory Commission, Washington, D.C., August, 1981.

3. Thayamballi, A., Y-K. Chen and D. Liu, "Fracture Mechanics Based Assessment of Fatigue Reliability in Ship Structures," Ship Structures Symposium '84, Arlington, Va., Oct. 15-16, 1984, pp. 111-138.

4. White, G.J. and B.M. Ayyub, "Reliability-Based Fatigue Design for Ship Structures," Naval Engineers Journal, Vol. 99, No. 4, May 1987.

5. White, G.J. and B.M. Ayyub, "Reliability-Based Design Format for Marine Structures," Journal of Ship Research, SNAME, Volume , No. 1, March, 1987.

6. Ellingwood, B.R., T.V. Galambos, J.C. MacGregor and A.C. Cornell, "Development of a Probability Based Load Criterion for American Nation Standard A58," National Bureau of Standards, Special Publication No. 577, June, 1980.

7. Albrecht, P., "S-N Fatigue Reliability Analysis of Highway Bridges," Probabilistic Fracture Mechanics and Fatigue Methods: Applications for Structural Design and Maintenance, ASTM STP 798, J.M. Bloom and J.C. Ekvall, Eds., American Society of Testing and Materials, 1983, pp. 184-204.

8. Albrecht. P., "Fatigue Reliability Analysis of Highway Bridges," Civil Engineering Report, University of

Maryland, College Park, Maryland, May 1982.

9. Wirsching, P.H., "Probability-Based Fatigue Deign Criteria for Offshore Structures," Final Project Report, API-PRAC Project No. 80-15, Department of Aerospace and Mechanical Engineering, University of Arizona, Tucson, Arizona, January, 1983.

10. Ang, A.H-S. and W.H. Munse, "Practical Reliability Basis for Structural Fatigue," Meeting Reprint 2492, ASCE National Structural Engineering Conference, April 14-18, 1975.

11. Munse, W.H., T.W. Wilbur, M.L. Tellalian, K. Nicoll and K. Wilson, "Fatigue Characterization of Fabricated Ship Details for Design," Ship Structure Committee, SSC-318, 1983.

12. Benjamin, J.R. and C.A. Cornell, Probability, Statistics, and Decision for Civil Engineers, McGraw-Hill Book Company, New York, 1970.

13. White, G.J., and B.M. Ayyub, "Reliability Methods for Ship Structures", Naval Engineers Journal, Vol. 97, No. 4, May, 1985, pp. 86-96.

14. Ayyub, B.M. and G.J. White, "Reliability-Conditioned Partial Safety Factors," Journal of Structural Engineering, ASCE, Vol. 113, No. 2, Paper No. 21241, February, 1987, pp. 279-294.

15. Mansour, A.E., H.Y. Jan, C.I. Ziegelman, Y.N. Chen and S.J. Harding, "Implementation of Reliability Methods to Marine Structures," Trans. Society of Naval Architects and Marine Engineers (SNAME), Vol. 92, 1984.

16. "Specifications for Highway Bridges," American Association of State Highway and Transportation Officials, Washington, D.C., 1983.

17. Albrecht, P. "Review of Fatigue Design Methods for Highway Bridges," Civil Engineering Report, University of Maryland, College Park, Maryland, May 1986.

18. Yamada, K. and P. Albrecht, "Collection of Live Load Stress Histograms of U.S. Highway Bridges," Civil Engineering Report, University of Maryland, College Park, Maryland, 1975.

19. Shaaban, H.A., "Fatigue Rating of Steel Highway Bridges," Unpublished Ph.D. dissertation, Department of Civil Engineering, University of Maryland, College Park, Maryland, May 1985.

20. Fain, R.A. and E.T. Booth, "Results of the First Five Years of Extreme Stress Scratch Gauge Data Collected Aboard Sea-Land's SL-7's," Ship Structure Committee, SSC-286 (SL-7-25), 1979.

21. Booth, E.T., "SL-7 Extreme Stress Data Collection and Reduction," Ship Structure Committee, SSC-304 (SL-7-26), 1981.

22. Oliver, J.C. and M.K. Ochi, "Evaluation of SL-7 Scratch Gauge Data," Ship Structure Committee, SSC-311 (SL-7-27), 1981.

23. Jordan, C.R. and L.T. Knight, "Further Survey of In-Service Performance of Structural Details," Ship Structure Committee, SSC-294, 1980.

24. Kawahara, M., H. Satoh, T. Iwasaki, M. Toriumi, and Y. Kawashima, "Fatigue Analysis of Ship Structure - Part II: Crab-eye Type Slot," Nippon Kokan Technical Report - Overseas, April 1977.

25 Jordan, C.R. and C.S. Cochran, "In-Service Performance of Structural Details," Ship Structure Committee, SSC-272, 1978.