THE SOCIETY OF NAVAL ARCHITECTS AND MARINE ENGINEERS
601 Pavonia Avenue, Suite 400, Jersey City, New Jersey 07306
Paper presented at the Marine Structural Reliability Symposium, Sheraton National Hotel, Arlington, Virginia, October 5-6, 1987

A Markovian Approach to Compliant Offshore Platforms

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ABSTRACT

A stochastic method is presented for dynamic and fatigue analysis of compliant platforms with nonlinearity in the stiffness. The wave loading is idealized as a component of a multidimensional Markov process. The differential equation of motion is expressed in terms o a set of first order stochastic differential equations. Ito's rule for stochastic differential is applied to obtain differential equations for moments after performing an averaging operation. The equations are closed using Gaussian and non-Gaussian closure techniques. The differential equations for moments are solved in the time domain using numerical methods. The response is modeled as a mixture distribution. It is observed that the response is non-Gaussian and the probability density is significantly underestimated at the tails by the Gaussian assumption. The probability distribution for peak guyline tensions is estimated by mapping a Gaussian process into the non-Gaussian process of guyline tensions and estimating the level crossings. The tension fatigue damage is estimated for a guyline using Palmgren-Miner's rule. It is shown that fatigue damage estimation under a non-Gaussian tension distribution is higher compared to the Gaussian.

INTRODUCTION

A deep water compliant platform is designed to move with the load. The sway periods for these structures are kept well above the design wave periods, thereby reducing the dynamic loads. Various structural configurations such as articulated platforms, tension-leg platforms, and guyed tower platforms have been used. The response behavior of such systems can be realistically estimated if the wave-structure system is modeled properly. For structural system behavior with large displacements, the geometrically nonlinear behavior of the components providing stiffness requires that the system be idealized as nonlinear. The wave loading is essentially random and needs to be modeled as a stochastic process. Analysis of compliant platforms thus is the analysis of nonlinear system with stochastic input.

A method is developed here for an offshore guyed tower platform subjected to random waves. The platform is a slender tower resting on a flexible foundation held vertical by a number of guylines attached near the top of the tower (1). Each guyline is a multicomponent mooring line consisting of a lead line, clump weight and trailing line. Under operating conditions the tower does not move appreciably, but during design storms the clump weights lift off, making the system "soft" and thus absorbing the wave energy.

Time-domain solutions with various degrees of complexity are available for dynamic analyses when the ocean-structure system is assumed to be deterministic. For random wave loading, some of the approximate methods that have been developed include stochastic equivalent linearization of the structure and waveloading, perturbation and Monte Carlo simulation. Smith and Sigbjornsson (2) presented a method to estimate the second order statistics of compliant platforms using an iterative method similar to perturbation. However, phenomena which are typical for nonlinear structures - such as jump - can be observed only when higher order statistics are considered. Monte Carlo simulation (3) has shown that linearized frequency domain methods are inadequate to represent the response of nonlinear compliant systems. Moreover, this method requires a significant amount of computer time and is expensive.

A stochastic dynamic analysis of such a softening nonlinear system such as a guyed tower under a generalized load is not yet available. With some approximation for loading, the system behavior can be estimated. Markov process theory can be used here. The Markov property of a random process refers to the independence of the future behavior of the process from its past behavior given the knowledge of its present state. It is known that every stationary Gaussian process with a rational spectral density (spectral density with numerator and denominator containing polynomials in square of the frequency) can be represented as a component of a multidimensional Markov process. The spectral density of the load process can always be approximated to any desired accuracy by a rational function. Thus, if the load process is stationary, Gaussian and non-Markovian, it can be approximated by a component of a multidimensional Markov process of desired accuracy by varying the dimensions of the Markov process

(4). This multidimensional Markov process can be represented by a set of stochastic differential equations involving the components of the process and parameters of the rational spectral density (5,6). Alternatively, this defines a filter which converts a Gaussian white noise into the loading (7). The expanded phase-space including the filter components will have Markov characteristics and will be more amenable mathematically. Markov process theory can be used to estimate the probability distribution of the response, which spectral methods are incapable of providing. This approach is used here to obtain the response for a nonlinear guyed tower.

The equivalent horizontal stiffness of the platform provided by the guylines is estimated using catenary equations (8,9). This nonlinear stiffness is expressed as a sum of the linear and cubic terms. The platform is assumed to be subjected to wave loading which is stationary, ergodic and zero-mean Gaussian. The spectral density of the moment of the wave load about the base is fitted to a rational spectrum using the Lavenberg-Marquardt algorithm for least square estimation of the nonlinear parameters (10). As described earlier, this loading is then expressed as a component of a two-dimensional Markov process through two stochastic differential equations describing the filter. Extending the phase-space of the system by adding the coordinates describing the processes in the filter, and using Ito's rule for stochastic differentials (11), differential equations for all the moments including the joint moments up to the fourth order are derived for this system. The effect of the cubic nonlinearity in the stiffness is that the equations for the second order moments contain terms of the fourth order moments on the right hand side. Similarly, the equations for the third order moments and the equations for the fourth order moments have sixth order moments on the right hand side. To solve the moment equation up to the fourth order, fifth and higher order cumulants are considered to be equal to zero. This gives the relationship between moments of the fifth and sixth order in terms of lower order moments. These differential equations for moments are then solved in the time domain using the Runge-Kutta-Verner method to obtain the moments of the response (10). The probability distribution is then modeled as a mixture distribution, which is the weighted sum of two known distributions with the same mean and variance. The optimal weighting factors are estimated using the third and fourth central moments.

Using elementary probability laws, the probability distribution for the guyline tensions is estimated from the probability distribution of the displacement. To obtain the probability density of the peaks, a Gaussian process is mapped into this process by the double inversion technique and level crossings are obtained (12). For a narrow band process, the peak distribution is considered to be one minus the ratio of the expected rate of crossings at a certain level to that at the mean. The probability density is then obtained by numerical differentiation. Knowing the tension fatigue curves of the guylines in terms of percentage of breaking load range and cycles of failure (13), the cumulative fatigue damage is estimated by a method similar to the Palmgren-Miner hypothesis.

It is shown that the probability density function of the displacement of the tower deviates significantly from the Gaussian at the tails for high sea states. The non-Gaussian transient response provides jerk load for the guylines and the estimated guyline fatigue will be unconservative if the tower displacements are assumed to be Gaussian.

MATHEMATICAL MODEL OF THE PLATFORM SYSTEM

The tower of the platform is modeled as a rigid column with hinge support at the bottom. The guylines are assumed to be circumferentially symmetric. The local dynamics of the cables are assumed to have no effect on the tower, and the tower is considered to be moving in one plane only.

The horizontal restraint to the tower is provided by guylines and buoyancy tanks. Each guyline is a multicomponent mooring line comprising a lead line, a clump weight and a trailing line attached to an anchor pile. The horizontal restoring force and vertical reaction at the point of attachment of the mooring line to the tower develop when the tower moves from its original position and can be estimated using catenary equations for the cable (8,9). Since the initial geometric configuration based on initial tension is known, the angle of rotation of the tower is estimated by varying the tension in the mooring line. Knowing the tower rotation, θ, and the corresponding horizontal restoring force, R_X, a least square regression analysis is performed to yield regression constants c_1 and c_2 such that

$$R_X = c_1\theta + c_2\theta^3 \tag{1}$$

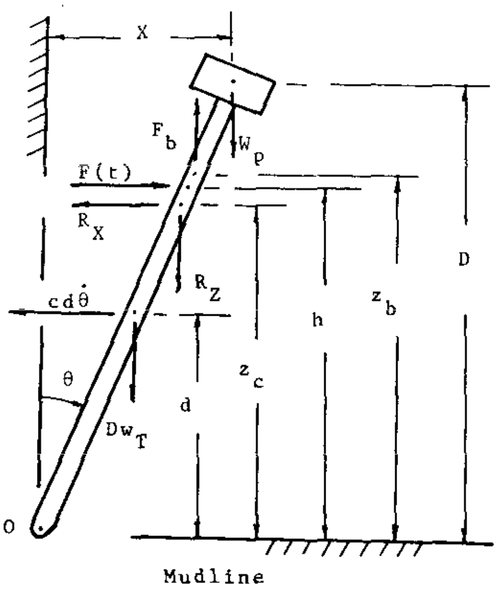
The vertical reaction at the point of attachment of the guyline and tower, R_Z, is given by

$$R_Z = c_1' + c_2'\,\theta^2 \tag{2}$$

The reason for this particular form of Eqs. 1 and 2 will be given in the following section.

DYNAMICS OF THE TOWER

An idealized tower with all the necessary geometric and loading parameters is shown schematically in Fig. 1. The equations of motion, under the assumptions described earlier, are obtained by taking the moments of the load and restoring forces about the base. The governing equation is given by



$D = 1500$ ft
$z_b = 1000$ ft
$z_c = 1300$ ft
$d = 750$ ft
$W_p = 20 \times 10^6$ lb
$w_T = 4250$ lb/ft
$F_b = 12 \times 10^6$ lb
$J_0 = 2.72 \times 10^{12}$ lb-sec^2-ft
Nos. of Guylines = 16
Equivalent dia. of Tower = 60 ft
$\rho = 1.988$ lb-sec^2/ft^4
Water Depth = 1425 ft

Fig. 1 Idealized tower

$$J\,\ddot{\theta} + cd^2\dot{\theta} + z_c R_x - (DW_p + Ddw_T + z_c R_z - F_b z_b)\theta = F(t)\,h \qquad (3)$$

in which θ, $\dot{\theta}, \ddot{\theta}$ = angular displacement, velocity and acceleration of the tower, respectively; J = structural inertia about the base; c = damping constant; d = half the tower depth; D = distance between the center of the platform and the base; W_p = platform weight; w_T = weight per unit length of the tower, F_b = buoyancy force; z_b = moment arm for F_b; F(t) = wave load; and h = moment arm for F(t).

Substituting Eqs. 1 and 2 in Eq. 3, the governing equation of motion with cubic nonlinearity in the stiffness is obtained as

$$J\,\ddot{\theta} + cd^2\dot{\theta} + (z_c c_1 - DW_p - Ddw_T - z_c R_z + F_b z_b)\theta + z_c(c_2 - c_2')\,\theta^3 = F(t)\,h \qquad (4)$$

MARKOV PROCESS IDEALIZATION OF LOAD

Realistic modeling of waves as a random process makes the differential equation given by Eq. 4 a stochastic differential equation with random right hand side. Waves can be modeled as a stationary, ergodic, zero mean Gaussian process characterized by an empirical wave height spectrum e.g. a Pierson-Moskowitz spectrum given by

$$S_{hh}(\omega) = \frac{8.1 \times 10^{-3} g^2}{\omega^5} \exp\left[-0.74(9/v\omega)^4\right] \qquad (5)$$

in which g = acceleration due to gravity; v = wind velocity; and ω = wave frequency. Using Airy's Linear wave theory, and Morrison's equation for wave load estimation, the spectral density for the total moment of the linearized wave load about the base is given by

$$S_{MM}(\omega) = \sum_{i=1}^{N}\sum_{j=1}^{N} h_i h_j [CM_i CM_j S_{\ddot{u}_i \ddot{u}_j}(\omega) + CD_i CD_j S_{\dot{u}_i \dot{u}_j}(\omega) + CM_i CD_j S_{\ddot{u}_i \dot{u}_j}(\omega) + CM_j CD_i S_{\dot{u}_i \ddot{u}_j}(\omega)] \qquad (6)$$

in which N = number of sections into which the tower is divided to estimate the total moment; h_i = distance between the base of the tower and the center of the section; $CM_i = C_m \rho V_i$; C_m = inertia coefficient, ρ = mass density of water; V_i = effective volume of the ith section; $CD_i = 1/2\ \rho\ C_d A_i\ \sqrt{8/\pi}\ \sigma(\dot{u} - \dot{x})$; C_d = drag coefficient, A_i = effective area of the ith section; $S_{X_i X_j'}(\omega)$ = spectral density of X at the ith location and X' at the jth location; X and X' are the acceleration and velocities of water particles denoted by $\ddot{u}$ and $\dot{u}$, respectively; $\dot{x}$ = velocity of the structure; and $\sigma(\dot{u} - \dot{x})$ = standard deviation of relative velocity.

Knowing the spectral density function of the moment, a rational spectral density function can be fitted to it by adjusting its parameters (14). The algebraic form of the rational spectral density function used here is given by

$$\hat{S}_{MM}(\omega) = \frac{\hat{G}\,\omega^2}{(\omega^2 - \omega_o^2)^2 + (C_o\omega)^2} \qquad (7)$$

The parameters $\hat{G}$, ω_o and C_o can be obtained by

minimizing $\sum_{i=1}^{N} [\hat{S}_{MM}(\omega_i) - S_{MM}(\omega_i)]^2$ using

standard algorithms (10).

Eq. 7 can be written as

$$S_{MM}(\omega) \doteq G \frac{|P(i\omega)|^2}{|Q(i\omega)|^2} \qquad (8)$$

in which $Q(x) = z^p - [a_1 z^{p-1} + \ldots + a_p]$ having roots with negative real parts. $P(z) = b_0 z^q + b_1 z^{q-1} + \ldots + b_q$ and $z<p$ with both the polynomials having real coefficients. The random process $M(t)$ represented by Eq. 8 is an ARMA (AutoRegressive Moving Average) Gaussian process. It can be shown that the stationary Gaussian ARMA process $M(t)$ is the first component of the p-dimensional stationary elementary Gaussian process $M^*(t) = [M_1(t), M_2(t), \ldots, M_p(t)]$ satisfying the linear stochastic equations (5,6)

$$dM_j(t) = M_{j+1}(t)\, dt + \beta_j dw(t),\ j=1,\ldots,p-1 \qquad (9)$$

and

$$dM_p = \sum_{k=0}^{p-1} a_{p-k} M_{k+1}(t)\, dt + \beta_p dw(t) \qquad (10)$$

in which $w(t)$ is a standard Wiener process (a normal process with independent increments and mathematical expectation equal to zero), and coefficients $\beta_1, \beta_2, \ldots \beta_p$ are given by

$$\beta_p = [a_1 \beta_{p-1} + \ldots + a_{p-1}\beta_1] \qquad (11)$$

$$\beta_{p-j} = \sum_{i=1}^{p-j-1} \beta_{p-j-1} a_i + b_{q-j},\ j=1,1,\ldots p-1 \qquad (12)$$

and $b_{-1} = \ldots = b_{-(p-q-1)} = 0$; $q\ p$, $\beta_1 = b_{q-(p-1)}$

Comparing Eqs. 7 and 8, the moment of waveload about the base can be expressed by two linear stochastic differential equations given by

$$d\theta_3 = \theta_4\, dt + \sqrt{\hat{G}}\, dw(t) \qquad (13)$$

$$d\theta_4 = -C_o\theta_4 dt - \omega_o^2\theta_3 dt - C_o\sqrt{\hat{G}}\, dw(t) \qquad (14)$$

in which θ_3 = moment of wave load about the base, and θ_4 relates to the derivative of θ_3.

APPLICATION OF THE METHOD OF MOMENT FUNCTIONS

Ito stochastic equations for extended phase-space (i.e. including the filter coordinates θ_3 and θ_4) of the system (Eq. 4) are given by

$$d\theta_1 = \theta_2 dt \qquad (15)$$

$$d\theta_2 = (-a\theta_2 - b\theta_1 - \varepsilon\theta_1^3)\, dt + \theta_3 dt \qquad (16)$$

$$d\theta_3 = \theta_4 dt + \sqrt{G_o}\, dW(t) \qquad (17)$$

$$d\theta_4 = (-C_o\theta_4 - \omega_o^2\theta_3)dt - C_o\sqrt{G_o}dw(t) \qquad (18)$$

θ_1, θ_2 = angular displacement and velocity of the tower, respectively, $a = (cd^2 + c_{hd} h_{eff})/J_o$; $b = (z_c c_1 - Dw_p - Ddw_T - z_c c_1 + F_b z_b)/J_o$; $\varepsilon = z_c(c_2 - c_2)/J_o$, $J_o = J + C_m \rho V h^2_{eff}$; h_{eff} = height above the base where the effective force is acting; $c_{hd} = \sum_{j=1}^{\text{all sections}} 0.5\, C_d \rho A_j \sigma_{(\dot{u}_j - \dot{x}_j)}$; and $\sqrt{G_o} = \sqrt{\hat{G}/J_o}$.

Eqs. 15 to 18 can be expressed as

$$d\underset{\sim}{\theta} = f(\underset{\sim}{\theta},t) + F(\underset{\sim}{\theta},t)\, dw(t) \qquad (19)$$

where $\underset{\sim}{\theta} = \{\theta_1\ \theta_2\ \theta_3\ \theta_4\}^T$

If $<\psi(\theta)>$ is the Sth ordered mixed moment of θ, where $\psi(\theta) = \prod_{i=1}^{n} \theta^{s_i}$ with $s_i = 0,1,\ldots p$, such $\sum_{i=1}^{n} s_i = S$, then using Ito's rule for stochastic differentials, the differential equation for the moments can be obtained by averaging

$$\frac{d}{dt}\ \psi(\theta) = \sum_{i=1}^{n} \frac{\partial\psi}{\partial\theta_i} f_i + \frac{1}{2} \sum_{i,j=1}^{n} \frac{\partial^2\psi}{\partial\theta_i \partial\theta_j} (F\ \underline{D}\ F^T)_{ij} \qquad (20)$$

f and F are known from Eq. 19, and $\underline{D}$ = an nxn diagonal matrix with diagonal elements $<(dw(t))^2>$ and < > denotes expectation.

SOLUTION OF MOMENT EQUATIONS

From Eq. 20, the first through fourth order moment equations can be developed. The total number of equations that can be developed for a particular order of moment depends on the dimension of the idealized Markov process. When the phase-space is described through four first order differential equations; as in the present case (Eqs. 15 through 18), then ten second order, twenty third order and thirty-five fourth order moment equations are developed. If the probability distribution of the response is assumed to be Gaussian, only the first and second order equations need to be solved. However, due to nonlinearity, the first order moment equations contain third order moments, and the second order moment equations contain fourth order moments on the right hand side. Assumption of a Gaussian distribution means that cumulant functions of order three and higher are zero. With this property, the moments of third and fourth order can be expressed in terms of the first two order moments. This is called Gaussian closure, which is equivalent to the equivalent linearization technique. When the nonlinearity is significant, the distribution of the response may deviate significantly from the Gaussian. The non-Gaussian distribution can be characterized if the higher order moments are known. For this purpose, it is assumed that the cumulant functions of order five and higher are zero. With this assumption, the fifth and sixth order moments can be written in terms of the first four moments. This system of differential equations is thus closed at the fourth moment level and can be solved using numerical techniques in the time domain. After a sufficient length of time, it can be seen that the moments obtained remain almost constant and can be considered as stationary moments. Another way to obtain stationary moments is as follows. The left hand side of Eq. 20 gives the derivatives of the moments and thus can be considered to be zero. Consequently, these equations now become nonlinear algebraic equations and can be solved numerically.

However, these nonlinear algebraic equations can be sensitive to the initial guess, and criteria based on moment inequalities must be used to check whether the correct solution has been obtained. In this study, the differential equations have been solved in the time domain and the response is obtained over a sufficient length of time so that the moments remain constant. This procedure also allows examination of the transient response. Once the moments are known, the next step is to establish a probability distribution for the response.

RESPONSE PROBABILITY DISTRIBUTION

Knowing the first four moments of the response, the probability distribution can be modeled as a weighted sum of Gaussian and non-Gaussian distribution as

$$F_r = \sum_{\ell=0}^{L} p_\ell F_\ell \tag{21}$$

with

$$p_\ell > 0 \text{ and } \sum_{\ell=0}^{L} p_\ell = 1.0 \tag{22}$$

With the F_ℓ's having the same mean and variance, the weighting factors are obtained using the information on third and fourth central moments by minimizing η under the constraints given by Eq. 22, where (12)

$$\eta = \sum_{k=3}^{4} (\phi_k - \hat{\phi}_k)^2 \tag{23}$$

in which ϕ, $\hat{\phi}_k$ = dimensionless central moments of the kth order of the response and the mixture distribution, respectively.

FATIGUE OF GUYLINES

The guylines are subjected to a large number of tension fluctuations and the possibility of tension fatigue must be considered. Cyclic bending of the cables also occurs at the fairleads. This is of small amplitude and relatively low frequency. Only tension fatigue is considered here. Platform response to the waves influences the amplitude and mean load experienced by the guylines. Similar to Eqs. 1 and 2, additional tension in the maximally stressed guyline can be related to the tower displacement by

$$\left.\begin{aligned} T_g &= \alpha\theta + \beta\theta^3 \qquad && \theta > 0. \\ &= 0 \qquad ; && \theta < 0. \end{aligned}\right\} \tag{24}$$

From the probability distribution of the tower rotation θ, the probability density of the guyline tension, f_T, and mean and variance of the tension range in the guyline can be easily calculated using elementary probability laws (15). Since the tower rotation is non-Gaussian and a nonlinear relationship exists between tension and tower rotation, the probability density function of guyline tensions is non-Gaussian. To obtain the peak density of the tension peaks, level crossings for the non-Gaussian guyline tension process must be obtained. This can be done by mapping a Gaussian process $T_g'(t)$ into $T_g(t)$ such that (10)

$$T_g'(t) = \Phi^{-1}\{\hat{F}_{\tilde{T}_g}[\tilde{T}_g(t)]\} \tag{25}$$

$$\tilde{T}_g(t) = \hat{F}_{\tilde{T}_g}^{-1}\{\Phi[T_g'(t)]\} \tag{26}$$

in which $\tilde{T}_g(t) = (T_g(t) - m_T)/\sigma_T$ and $\hat{F}_{\tilde{T}_g}(\tilde{T}_g(t)$ = probability distribution of $\tilde{T}_g(t)$; Φ, Φ^{-1} = Gaussian and inverse Gaussian; and m_T, σ_T = mean and standard deviation of tension, respectively.

It can be shown that the mean rate at which $T_g(t)$ crosses any threshold level t, ν_t, with positive slope can be obtained from the mean rate at which $T_g'(t)$ upcrosses the level t'

$= \Phi^{-1}\{\hat{F}_{\tilde{T}_g}[\tilde{T}_g(t) = \frac{t - m_T}{\sigma_T}]\}$ and can be approximated

by (10)

$$\nu_T = \frac{\sigma_{\dot{T}}}{\sqrt{2\pi}} \phi(t') \tag{27}$$

in which $\sigma_{\dot{T}}$ = root mean square of $\dot{T}$ and can be obtained in a similar way as σ_T.

The probability distribution of the tension peaks is calculated using the heuristic assumption that for a narrow band process, the required distribution, F_{pT}, is given by (16)

$$F_{pT}(t) = 1 - \frac{\nu_T}{\nu_{m_T}} \tag{28}$$

in which ν_T, ν_{mT} = expected rate of crossings at level t and mean, respectively. Knowing the distribution, the density function can be obtained by numerical differentiation.

The total number of tension cycles per year for a particular sea state, N can be estimated as

$$N_{c_i} = P_{osi}\ \nu \times 3.1536 \times 10^5 \tag{29}$$

in which P_{osi} = percentage of occurrence of sea state i;
ν = mean rateof crossing of the platform

Data on the fatigue behavior of large diameter ropes is scant. A modified version of the tension fatigue curve for cable diameter of 8.5 cm presented by Waters, Eggar and Plant (13) is used here. The tension fatigue curve is presented in terms of load range breaking load percentage vs. cycles to failure. The curve is a straight line on semi-log paper. Knowing the nominal diameter of the guyline, the breaking load of the cable can be predicted using the curves given by Ronson (17). The fatigue curves presented by Waters, Eggar and Plant can then be used to estimate the fatigue damage of the cable using Palmgren-Miner's rule. The fatigue curve is represented by

$$\log_{10} N_f = c + mP_{bt} \tag{30}$$

in which N_f = numher of cycles to failure; c = constant; m = slope of the curve; and P_{bt} = percentage of breaking tension.

The total damage for a sea state is given by

$$\Delta = \int_0^{100} \frac{P_{osi}\ \nu\ 3.1536 \times 10^5 p(P_{bt}) dP_{bt}}{N_f(P_{bt})} \tag{31}$$

in which Δ = accumulated damage;
$p(P_{bt})$ = probability density of the tension in terms of percentage of breaking tension; and $N_f(P_{bt})$ = number of cycles to failure at the percentage breaking tension P_{bt}.

Thus, the fatigue damage can be estimated for all the sea states and summed to obtain the annual fatigue damage. The reciprocal of this will give the fatigue life of the guyline.

RESULTS AND DISCUSSION

An idealized guyed tower platform shown in Fig. 1 is analyzed for four different sea states under wind velocity of 30, 40, 70 and 80 ft/sec. The details of the platform and the guying system are shown in Tables 1 and 2. Ask explained earlier, the platform is subjected to waves characterized by a Pierson-Moskowitz wave spectrum. The drag and inertia coefficients are assumed to be 1.0 and 1.5, respectively. Structural damping is assumed to be 5% of the critical.

Table 1. Platform Data

Equivalent diameter of tower = 60 ft
Force from buoyancy tanks = F_b = 12×10^6 lbs.
Moment arm for F_b = z_b = 1000 ft
Platform weight = W_D = 20×10^6 lbs
Mass moment of inertia including added mass about the base = J_o = 2.72×10^{12} lbs-sec^2-ft
Distance of center of the platform to the base = D = 1500 ft
Water depth = 1425 ft
Number of cables = 16
Distance of cable attachment point from base = z_c = 1300 ft
Weight per unit length of tower = w_T = 4250 lbs/ft
Mass density of sea water = 1.988 lbs/sec^2/ft^4
(1 ft = 0.3048 m, 1 lb = 0.4536 kg)

Table 2. Guyline Data

Angle of the cable to horizontal at point of attachment = θ_e = 28°
Length of lead line = 3560 ft
Weight of lead line = 40 lb/ft
Length of clump weight = 140 ft
Weight of the clum = 1920 lb/ft
Length of trailing line = 6900 ft
Weight of trailing line = 40 lb/ft
Individual guyline breaking load = 2.2×10^6 lbs

The spectrum of the moment of wave load about the base is obtained by dividing the tower into segments 25 ft high and estimating the wave load at the center of each section. The idealized rational spectral density along with actual spectral density are shown in Fig. 2 for wind velocity of 80 ft/sec.

Both transient and stationary response statistics are obtained using numerical solution of the differential equations of moments generated by Eq. 20. Runge-Kutta-Verner's fifth and sixth order method is used (10). The initial conditions for platform motion are assumed to start from rest. Fig. 3 shows the transient and stationary standard deviation of the platform displacement. The

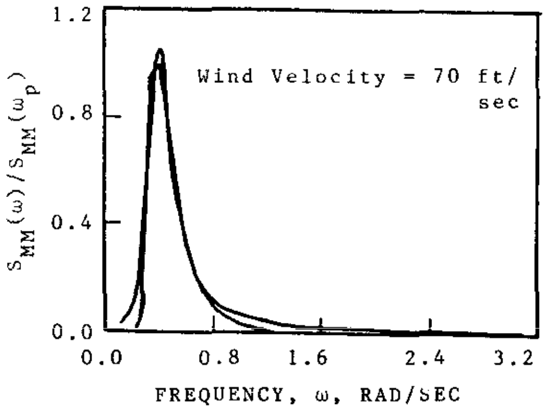


Fig. 2 Rational spectral density

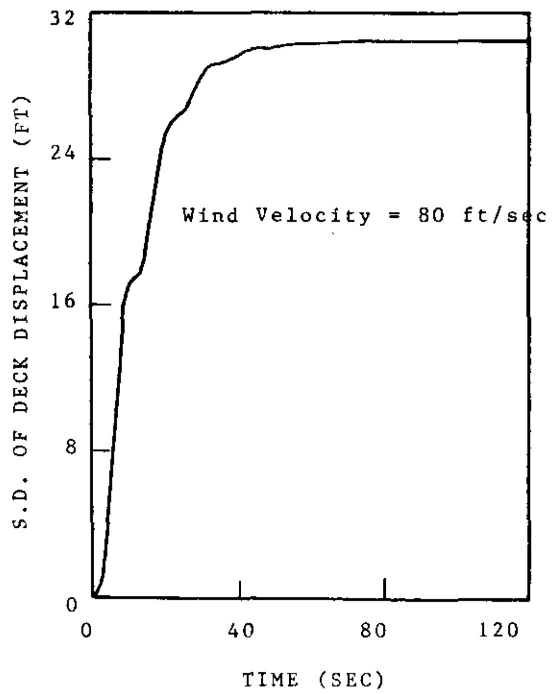


Fig. 3 S.D. of deck displacement for wind velocity of 80 ft/sec (Non-Gaussian closure)

root mean square of the deck displacement remains almost constant after 80 sec. Figs. 4 and 5 show the variation of kurtosis of tower displacement and velocity with respect to time, respectively. The stationary kurtosis of the displacement is leptokurtic, whereas the kurtosis of the velocity is platykurtic. In the transient state, the standard deviation of the velocity increases monotonically while the kurtosis oscillates and then reaches a steady state. Translating this behavior in terms of

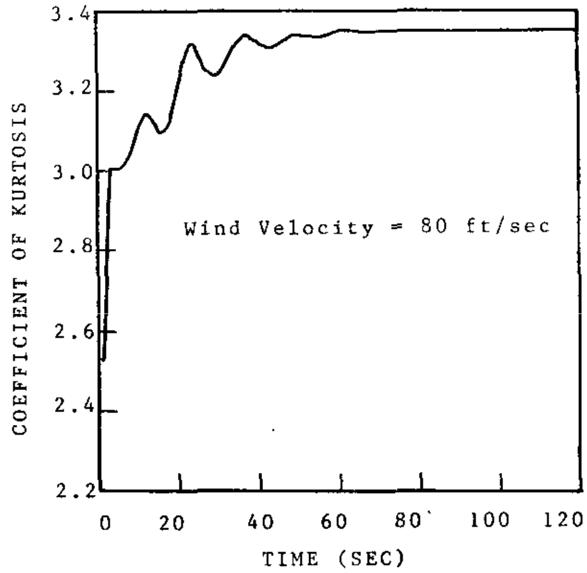


Fig. 4 Kurtosis of tower displacement

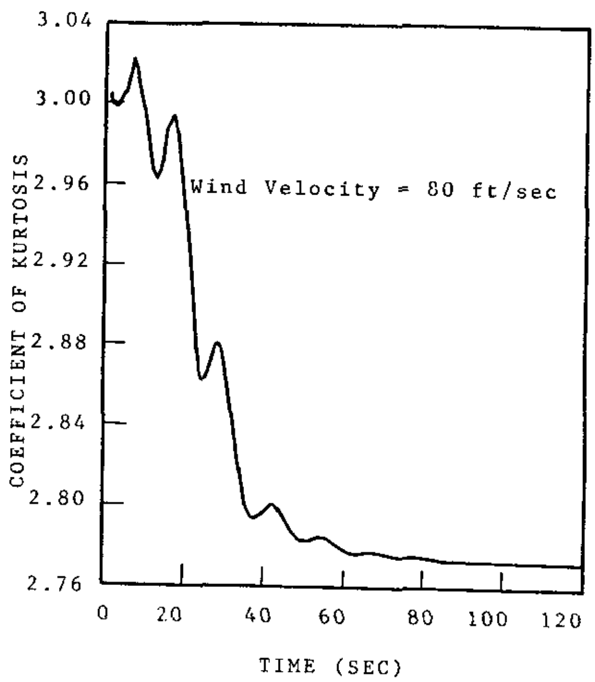


Fig. 5 Kurtosis of tower velocity

probability density function of the guyline tensions shows that sudden changes or jerk load on the guylines may be expected in this range. Table 3 shows the stationary response statistics of the deck displacement with Gaussian and non-Gaussian closure techniques. Even though both methods seem to provide almost the same second order statistics, at higher sea states the kurtosis of the response is seen to

Table 3 Stationary Statistics of Deck Displacement

Wind Velocity ft/sec	Significant Wave Height, ft	Standard Deviation ft G	NG	Coeff. of Excess G	NG	p_1	p_2
30	5.81	1.47	1.47	0.	0.00	1.00	0.00
40	10.38	4.35	4.35	0.	0.03	0.99	0.01
70	31.84	26.32	26.41	0.	0.24	0.92	0.08
80	41.58	30.35	30.65	0.	0.36	0.88	0.12

deviate from the Gaussian. The non-Gaussian response distribution is modeled as a weighted sum of Gaussian and Laplace distributions. The Laplace density function is given by (18)

$$f_L(x) = \frac{1}{2\beta} \exp(-|x-\alpha|/\beta) \tag{32}$$

and the mean, variance, coefficient of skewness and kurtosis are given by α, $2\beta^2$, 0. and 6.0, respectively. Figs. 6 and 7 show how the non-Gaussian distribution affects the probability density of the response near the mean and at the tails, respectively. At the tails beyond a

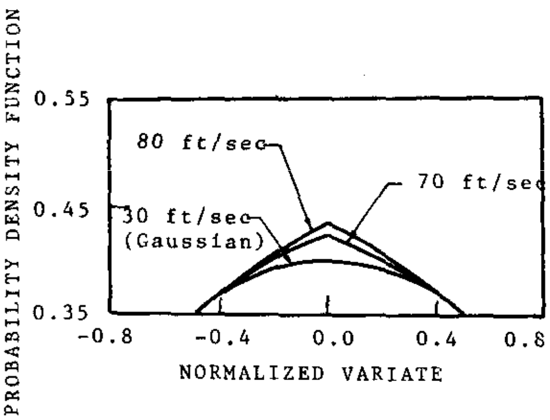


Fig. 6 Probability density of deck displacement of normalized variate

normalized variate equal to 3.0, the Gaussian assumption is seen to underestimate the probability by several orders of magnitude. This means that for the same level of probability density, non-Gaussian density will predict a higher value of tower displacement and a Gaussian assumption will underestimate the guyline tension.

The tension fatigue curve, represented by Eq. 30 and used in this study, is shown in Fig. 8. Table 4 shows the fatigue of a guyline under maximum tension. Both Gaussian closure and non-Gaussian closure methods are employed to obtain the fatigue damage. It is seen that the fatigue damage with Gaussian closure is unconservative at higher sea states compared to the fatigue damage using a non-Gaussian distribution for the tower displacement. Since the probability distribution is leptokurtic, this is expected (16,19).

Table 4 Fatigue Damage/Year of Maximum Stressed Guyline

Sea State Significant Wave Height, ft	% Occurrence	Fatigue Damage/Year Gaussian Closure	Non-Gaussian Closure
5.81	49.0	0.4644×10^{-1}	0.4644×10^{-1}
10.38	21.0	0.6067×10^{-1}	0.6087×10^{-1}
31.84	2.5	0.1626×10^{-1}	0.2155×10^{-1}
41.58	1.0	0.2873×10^{-2}	0.3704×10^{-2}

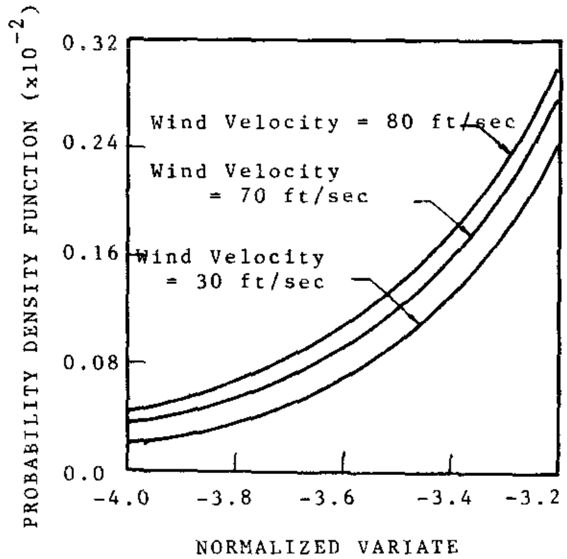


Fig. 7 Probability density of deck displacement of normalized variate

CONCLUSIONS

It is now well known that probabilistic methods of analysis are very useful tools in designing safe and reliable systems based on realistic assumptions. The experience with

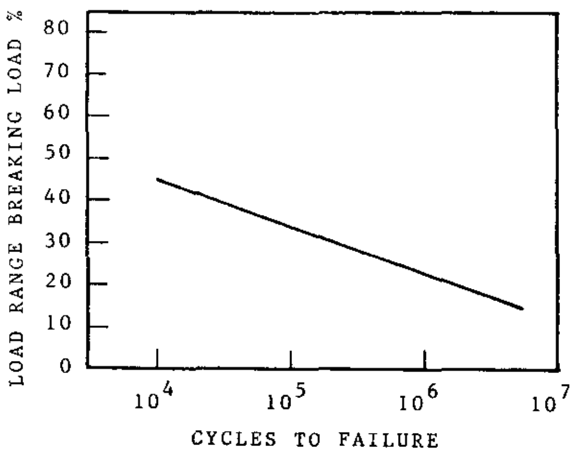


Fig. 8 Tension fatigue curve

jacket platforms confirms this. Deepwater compliant structures, however, cannot be analyzed in the same way as jacket platforms since they behave differently. Nonlinearity is of fundamental importance in the development of procedures for probabilistic analysis. The work presented here attempts to capture the essential effects of nonlinear behavior on the response, and is in a sense a modest beginning. The problem of probabilistic fatigue analysis of the mooring lines under nonlinear tension characteristics has also been addressed. Based on the work presented here, the following main conclusions can be drawn:

(i) A stochastic method is developed to estimate the transient and stationary responses of a nonlinear compliant platform based on Markov idealization of the load.

(ii) Transient response in terms of kurtosis shows the possibility of jerk loading of the guylines.

(iii) Stationary response is non-Gaussian at higher sea states. Displacement is leptokurtic while velocity is platykurtic.

(iv) The probability density function of displacement is underestimated by a Gaussian assumption, leading to underestimation of guyline tension.

(v) The non-Gaussian probability distribution of the tower displacement is seen to affect the prediction of the fatigue behavior of the guylines. Fatigue of the guylines is seen to be unconservative using Gaussian closure.

ACKNOWLEDGEMENTS

This material is based upon work partly supported by the National Science Foundation under Grants No. MSM-8352396, MSM-8544166 and MSM-8644348. Any opinions, findings and conclusions or recommendations expressed in this publication are those of writers and do not necessarily reflect the views of the National Science Foundation.

REFERENCES

1. L.D. Finn, "A New Deepwater Offshore Platform - The Guyed Tower," Proceedings of the Eighth Annual Offshore Technology Conference, OTC 2688, Vol. III, Houston, 1976, pp 819-830.

2. E. Smith and R. Sigbjornsson, "Nonlinear Stochastic Analysis of Compliant Platforms," Proceedings of Twelfth Annual Offshore Technology Conference, Houston, OTC 3801, 1980, 3, 69-76.

3. O. Mo and T. Moan, "Environmental Load Effect Analysis of Guyed Towers," Proceedings of Third International Offshore Mechanics and Arctic Engineering Symposium, Houston, 1984, 1, 68-79.

4. R.L. Stratonovich, "Topics in the Theory of Random Noise Vol. 1,. Gordon and Breach, 1963.

5. R.S. Lipster and A.N. Shiryayev, Statistics of Random Processes Applications, Springer-Verlag, 1978.

6. M. Arato, "On Sufficient Statistics of Gaussian Processes with Rational Spectral Density Function," Theory of Probability and Its Applications, Vol. 30, No. 1, 1986, pp 103-116.

7. V.V. Bolotin, Random Vibrations of Elastic Systems, Martinus Nijhoff Publishers, The Hague, The Netherlands, 1984.

8. G. Orgill, "Optimum Design of the Cable Array Mooring System for an Offshore Guyed Tower," Dissertation submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy, Dept. of Civil and Environmental Engineering, Duke University, 1983.

9. K.A. Ansari, "How to Design a Multi-component Mooring System," Ocean Industry, March, 1979.

10. International Mathematics and Statistics Library, Inc., IMSL. Library Reference Manual Volume 4, Edition 9, Houston, Texas, 1982.

11. K. Ito, "On a Formula Concerning Stochastic Differentials," Nagoya Mathematical Journal, Vol. 3, 1951, pp 55-65.

12. M. Grigoriu, "Contribution to Approximate Reliability Analysis," Report 81-15, Department of Civil Engineering, Cornell University, N.Y., 1981.

13. D. Waters, D. Eggar, and H. Plant, "Developments in Fatigue Assessment of Large-Diameter Wire Ropes Used in Offshore Moorings," Proceedings of the Seventeenth Annual Offshore Technology Conference, OTC 5000, Houston, 1985, pp 361-365.

14. P.T.D. Spanos, "ARMA Algorithms for Ocean Wave Modeling," Journal of Energy Resources Technology, ASME, Vol. 105, 1983, pp 300-309.

15. A. Papoulis, "Probability, Random Variables and Stochastic Processes," McGraw Hill Book Co., New York, 1985.

16. A. Haldar and H.B. Kanegaonkar, "Stochastic Fatigue Response of Jackets Under Intermittent Wave Loading," Proceedings of Eighteenth Offshore Technology Conference, Houston, OTC 5332, Vol. 4, 1986, pp 377-386.

17. K.T. Ronson, "Ropes for Deepwater Mooring," Proceedings of Twelfth Offshore Technology Conference, Houston, OTC 3850, 1980, pp 485-496.

18. M. Abromowitz, and I.A. Stegun, "Handbook of Mathematical Functions," National Bureau of Standards, 1964.

19. L.D. Lutes, M. Corazao, S.J. Hu, and J. Zimmerman, "Stochastic Fatigue Damage Accumulation," Journal of Structural Engineering, ASCE, Vol. 110, No. 11, 1986, pp 2585-2601.