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# Structural Safety Evaluation of Steel Jacket Platforms

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**ABSTRACT**

This paper describes an attempt to bring a state of the art methodology to the state of practice. The methodology aims at the evaluation of the failure probability of a steel-jacket platform under extreme environmental loading conditions. Of course, this probability of failure should not be considered as the "true" probability of failure of the structure over a particular period of time, but simply as an overall safety measure to be used as a decision tool. In short, it is based on a search for the most probable component failure sequences leading to the structure collapse. Componental failures include brace buckling, plastification of a section, and punching of a chord by a brace.

After each componental failure, the structural stiffness is locally modified and component residual strength is accounted for by applying equivalent nodal forces on the structure. The probability of occurrence of a failure sequence is a joint probability whose computation requires special attention (in particular the depen- dency between the individual component failures involved must be accounted for). Once the most likely failure sequences have been identified, their probabilities of occurrence are combined in order to estimate the overall probability of failure.

Bringing this state of the art methodology to the state of practice, i.e. developping a practical tool that can be applied to real situations, is not an easy task. Among the various problems that have to be solved, the following must be mentioned :

- choice of the random variables (and of the corresponding distributions) among the many parameters that can be identified in a realistic situation (in particular, the extreme environmental condition parameters),
- choice of an appropriate structural analysis method in order to perform many reanalyses at the lowest possible cost (each time a component fails, a new analysis must be performed),
- accurate computation of joint probabilities of failure events,
- development of a simple and realistic post-failure behaviour model for each type of component failure.

The complete or partial solution to each of the aforementioned problems is described and discussed in the paper.

Finally, some application examples are presented in order to show the capabilities and limitations of the methodology.

**INTRODUCTION**

In recent years, an important amount of research work has been devoted to the development of methodologies for the reliability analysis of redundant structures (10, 14). To our knowledge however, very few practical applications of these methodologies have been presented if not attempted at all. By practical applications, we mean applications to real structures, significantly larger than the 2-D frames on which the proposed methods are usually applied to in the publications.

The purpose of this paper is not to present a new methodology, but "simply" to describe an attempt to bring "state of the art" methodologies to the "state of practice". It is hence mostly a list of problems with proposed solutions, or with still to be found solutions. It also includes general reflexions and suggestions.

## General considerations

Steel-jacket platforms are the most common type of fixed offshore platforms, ranging in height from a dozen of meters to several hundreds. Our target for the application of the existing methodologies is neither the first type nor the second type of structure but an intermediate type of about hundred meters.

It is well known that the various loads applied to an offshore structure (waves, wind, current) are random in nature. Moreover, uncertainty and randomness are present in the structure itself : material resistance, geometric parameters, initial defects,... Also, some uncertainty is introduced by all the physical models used to predict the load effects and the structural response. The combination of these random or uncertain parameters results in a non-zero probability of something going wrong, the something ranging from a single component failure to the total collapse of the structure.

The structural component failure modes considered in this paper are the following :

- tubular member buckling,
- plastification of a section,
- punching of a chord by a brace.

Foundation failures are not discussed here but they could be considered as well.

In general, the failure of one component does not mean the failure of the structure. However, because of the load redistribution that necessarily follows a first failure, other member failures can be triggered, eventually leading to the collapse of the structure. Even before the complete collapse is reached, the structure can become unfit for serviceability reasons (e.g. displacements too large for normal operation of the platform).

Needless to say that a complete reliability analysis of such a system is a formidable task. Even with the analytical tools (Advanced First Order and Second Order Reliability Methods) now available (14) , some simplifying assumptions must be made. Some of them will be presented in a subsequent section. As a consequence, any probability of failure obtained in that context should not be considered as a "true" probability of failure, but merely as a safety measure to be used for decision making.

## Brief description of the general methodology

Among the various methods for structural system reliability analysis proposed in the litterature (4, 10, 14), one class seems more popular that the others. It may be called the beta-unzipping, the progressive collapse, or the member replacement method, but it is more or less always the same methodology. This is the methodology that was chosen here for practical applications because it is basically the probabilistic extension of the classical deterministic approach called progressive collapse or static push-over (7).

This is probably the reason why it is the most popular one, but it doesn't mean that the other methodologies (stable configurations, ideal plasticity) are not valid. The following is a brief review of the major features of this methodology.

A major assumption is that all the member failures, and hence the structure failure, occur at one instant, for example when the lateral wave load is maximum. More precisely, failures are assumed to occur over a short period of time during which the load is being applied proportionally. For a given period such as the life time of the structure, the distribution of this extreme load can be obtained from available oceanographical data. If the other time varying sources of loading are included (e.g. wind and current), the definition of the extreme loading conditions is more complex, in particular if the various sources of loading are correlated (which is generally the case).

The advantage of this assumption is that it makes the reliability problem time-independent. Of course, fatigue failures cannot be treated in that context. However, structural failure due to the existence of fatigue cracks can be accounted for, just like any other static failure mode.

Once a given member failure has occurred, its stiffness, and hence the overall structural stiffness is modified. A residual strength is modelled, for example by applying appropriate forces at the nodes of the failed member, and a new stress calculation is performed.

After a sufficient number of successive member failures have occurred, the structural failure criterion (collapse, large displacements) is met and a failure path or failure sequence identified.

In a reliability analysis context, a failure path is nothing else that a cut-set. The probability of failure following a particular failure path is therefore a joint probability, whose computation requires special attention. Generally, many different failure paths may lead to the structure failure. Hence, the probability of structural failure, i.e. the probability of occurrence of any possible failure path, is simply the probability of the union of all possible failure paths.

Unfortunately, even for a simple structure with only a few members, the number of failure paths is enormous and a complete listing of them is practically impossible. It is therefore usually suggested to look only for the most critical failure paths. Taking the union of those failure paths and computing the probability of the resulting event provides a lower bound to the probability of structural failure. Generally, because those failure paths are the most likely, it is a close bound.

Moreover, as will be seen in a subsequent section, the information gathered during the search for the most likely failure paths can be used in order to obtain an upper bound to the probability of structural failure.

## From the state of the art to the state of practice

When trying to apply the above methodology to practical situations, one faces several problems which can be classified as follows :

**Loading aspects**

- selection of random variables,
- choice of a hydrodynamic force model.

**Mechanical and structural aspects**

- choice of the component failure criteria,
- modelisation of component post--failure behaviour,
- minimization of the cost of repeated stress analyses.

**Probabilistic redundancy analysis**

- choice of probability density functions,
- computation of joint probabilities of failures,
- search for most likely failure paths.

In the subsequent parts of this paper, the above mentioned aspects are addressed successively. In each case the problems are listed, and some solutions are proposed, illustrated by several examples.

## PROBABILISTIC MODEL OF THE ENVIRONMENTAL LOADING

### Preliminary comments

As stated in the introduction, the reliability analysis is performed under a single extreme event, assuming that the corresponding load is applied proportionally from an initial value (e.g. zero or dead load only) to a final value.

In other words, the load acting on the structure is described by a random vector and not by a multidimensional stochastic process. This is why this type of analysis is often called "time-independent".

Let $\underline{L}$ of dimension N be this random vector, where N can vary from 1 to the total number of degrees of freedom of the platform structural model.As well be seen in a subsequent section, the reliability analysis requires as input the random internal forces and moments in every structural member (e.g. beam element).

The member replacement method major advantage is that the behavior of the structure in any damaged state is obtained by the superposition of several linear responses. In the intact state, only the response to the external action is considered. In the damaged states, the responses to additional self-equilibrated loads accounting for residual strength of failed members are also included. In this part, only the external actions are considered.

Let $\underline{S}^i$ be the vector of internal forces and moments in member i. It is linearly related to the vector $\underline{L}$ by an equation such as

$$\underline{S}^i = \underline{\underline{C}}^i \, \underline{L} \quad (1)$$

where $\underline{\underline{C}}^i$ is the assumed deterministic influence matrix associated to member i. Practically, the probabilistic characterisation of $\underline{S}^i$ is not easy to obtain from the probability distribution of $\underline{L}$, except in special cases. Those special cases include the case where the dimension of $\underline{L}$ is small and the case where $\underline{L}$ is gaussian.

Most of the applications published so far in the litterature correspond to the first case or to a mixture of both cases. Typically, the dimension of $\underline{L}$ is equal to two,with a component $L_1$ representing the dead load and a component $L_2$ representing the environmental load (deterministic constant load pattern x random amplitude). An important consequence of such a reduced random load space dimension is a large correlation between all the internal forces in the structure. As will be seen later in an example, this may have a significant influence on the reliability estimates.

The following is the brief description of a method corresponding to the second case : no limitation on the dimension of $\underline{L}$ but equivalent gaussian assumption. Under this assumption, only the expected value of $\underline{L}$, $E(\underline{L})$, and its covariance matrix, $\Sigma_{\underline{L}}$, are required. In order to obtain $E(\underline{L})$ and $\Sigma_{\underline{L}}$ , the following steps are successively performed by the program CHARGE (3) :

- stochastic modelling of the marine environment,
- probabilisation of Morison's equation,
- calculation of the second order characterisation of $\underline{L}$.

**Stochastic modelling of the marine environment**

In the extreme loading situation, the structure is subjected to the combined effects of wind, wind current, tide current and waves. In order to estimate the hydrodynamic forces acting on the structure, the random kinematic profiles $\underline{V}$ (x, y, z) corresponding to each of these external actions must be described. A possible way to describe them involves splitting the profile in a random intensity parameter $\lambda$ and a random field $\underline{W}(x, y, z)$ assumed independent from $\lambda$ and such that :

$$\underline{V}(x,y,z,) = \lambda . \underline{W}(x,y,z) \qquad (2)$$

For example, in the case of the linear wave action (random height H and random period T), $\lambda = H/T$ if $\underline{V}$ is the velocity profile and $\lambda = H/T^2$ if $\underline{V}$ is the acceleration profile.

The randomness in $\lambda$ represents the physical uncertainy associated to a given action while the randomness in $\underline{W}$ represents the model uncertainty associated to a particular wave theory.

With this model, the interaction between the various sources of loading are easily accounted for by superposition of the corresponding profiles.

**Probabilisation of Morison's equation**

The Morison's equation is a hydrodynamic model that transforms kinematic profiles into drag or inertia forces through the use of the force coefficients $C_d$ and $C_M$. These coefficients systematically vary with parameters such as the Keulegan-Carpenter number and the relative roughness. Random variations around the systematic variation can be observed on experimental data. Therefore, $C_D$ and $C_M$ should be treated as random variables. What are their distribution functions, are they independent from each other, from one member to the other ? Those questions are difficult to answer at present.

**Second order characterisation of the loading**

Assuming a linear variation of the profile between the two nodes of an element, the equivalent nodal forces are easily related to the action intensity variables (the $\lambda$'s), to the nodal values of the kinematic profiles (the $\underline{W}$'s), and to the force coefficients (the C's). Because this relation is not linear, the second order characterisation of $\underline{L}$ requires a higher order characterisation of each of the above mentioned families of random variables (each family is assumed weakly correlated to the others).

Considering for example the family of action intensity variables, it is necessary to define :

- a 4th order characterisation of each variable i.e. $E(\lambda_i^m)$ for m = 1,4
- various moments of the type $E(\lambda_i^{mi}\ \lambda_j^{mj}\ \lambda_k^{m})$ for mi, mj, mk $\in \{0,1,2,3\}$ with mi+mj+mk = 3 or 4.

These moments can be obtained from the knowledge of the joint distribution of the various intensity variables.

In some cases, the necessary information is totally lacking (kinematic profiles, force coefficients) and additional research is required. In the meantime simplifying assumptions (perfect correlation of decorrelation) can be made.

**Model reduction**

The method described above leads to the probabilistic characterisation of a load vector having a dimension equal to the number of degrees of freedom of the structural model. For realistic medium size structures this number ranges from one hundred to one thousand, thus implying a huge correlation matrix ($\Sigma$L) and lengthy computation to obtain $E(\underline{S}^i)$ and $\Sigma_S i$ from $E(\underline{L})$ and $\Sigma_L$.

The possibility of reducing the dimension of the model was therefore investigated. A possible method consists in a diagonalisation of the correlation matrix followed by a reduction of the number of eigen-values (all eigen-values below a given threshold are assumed equal to zero).

The table 1 compares component reliability indices obtained for different levels of reduction. It is still difficult to draw general conclusions from this example but it shows that such a reduction method is promising.

**Conclusion**

The two extreme cases presented in this part illustrate well the type of dilemna one has to face when trying to apply a model to practical situations.

Do we choose a simple model for which all the required information is available but with a risk of being too crude, or do we choose a sophisticated model for which some of the required information is lacking and therefore some assumptions have to be made ? There are no easy answers to this kind of questions.

May be the reduction method mentioned in the last paragraph will help us solve the dilemna in the case of the probabilistic description of the environment but as will be seen later on, other similar dilemna will show up in the choice of the failure criteria.

## MECHANICAL AND STRUCTURAL ASPECTS

As explained in the introduction, the methodology requires the definition of component failure criteria, the modelisation of component post-failure behaviour, and the repeated use of a stress analysis program. As will be seen, these three aspects of the methodology may lead to practical problems.

The following is a description of the problems involved and of possible ways to tackle them.

### Definition of component failure criteria

#### General considerations

There are no major constraint imposed by the probabilistic analysis on the expression of the failure criteria. In the case of the First Order Reliability Methods mentioned earlier, the only requirements are continuity and differentiability with respect to all the random variables.

A failure criterion can generally be expressed as an interaction equation between internal actions in the member and resistance variables, which, for tubular members, are functions of several parameters such as :

- yield stress,
- stain hardening,
- Young's modulus,
- residual stresses,
- section parameters (diameter and thickness),
- out-of-roundness of the section,
- out-of-straightness of the member.

When performing a structural reliability analysis the first problems to be dealt with are the choices of interaction equations and the choice of a set of random variables for each type of equation. Those choices are important ones because they affect quite significantly the results of the reliability analysis.

Regarding the first choice, any one of the interaction equations given by the codes (e.g. AISC, API, DnV) is a priori adequate, provided that explicit safety factors are removed and that model uncertainty (see below) is properly accounted for.

As far as the second choice is concerned, the brutal probabilisation of all the parameters involved in a particular failure equation is rarely the best solution, and this for two main reasons. First, it requires the knowledge of the joint density function (or the appropriate set of conditional distribution functions) of all the variables and this information is usually not available. Second, it does not account for the so-called model uncertainty that is reflected by the dispersion observed when experimental data are compared to predicted data.

A better approach is to identify the most significant (deterministically as well as probabilistacally) factors of the interaction equation and account for all the other uncertainties as well as the model uncertainty with an additional random variable. Concerning the parameters listed above, it is usually recognized that the uncertainties associated to Young's modulus and to the section parameters (diameter and thickness) are negligible compared to those associated to the other parameters (e.g. yield stress).

In the following, possible choices of interaction equation and significant parameters for various failure criteria are presented. The question of model uncertainty is discussed separately later on.

#### Plastification of a tubular section

The following failure criterion is the one proposed by Toma and Chen (11) on the basis of a mechanical model of imperfect tubular section :

$$1.0-(M/Mp)-1.18(P/Py)^2 = 0 \quad \text{for } 0 < P/Py < 0.65 \qquad (3)$$

$$1.0-0.70(M/Mp)-(P/PY) = 0 \quad \text{for } 0.65 < P/Py < 1$$

where

Py = plastic axial capacity = A. $\sigma$y
Mp = plastic moment capacity = Z. $\sigma$y
$\sigma$y= yield stress
A = section area
Z = plastic modulus
P = axial load
M = bending moment at section

The authors have found the effect of out-of-roundness to be negligible. Moreover, residual stresses do not affect the interaction curve as it corresponds to a totally yielded section. Thus, the only remaining resistance parameter to be considered is the yield stress. It can be found in various publications (1, 11) that its dispersion is well represented by a positively skewed Gumbel distribution.

**Buckling of a tubular member**

Most of the proposed buckling interaction equations are of the following form (AISC) :

$$1.0-(P/P_U)-(C_M.M_A)/(Mp.(1-P/P_E))=0 \quad (4)$$

where

$P_U$ = buckling strength = $k_o$Py
$P_E$ = euler buckling load
$C_M$ = reduction factor = $0.6-0.4(M_B/M_A)$
$M_A$, $M_B$ = end moments ($M_A > M_B$)
$k_o$ = function of the reduced slenderness ratio

This type of formula is obtained by assuming a linear interaction of thrust and moment at the most highly loaded section of the column.

In order to be consistent with the choices made for the previous failure criterion, the yield stress has to be treated as a random variable. Another good candidate is the axial strength reduction coefficient $k_o$.

Given the yield stress and the effective length of a column the uncertainty observed on $k_o$ is due to parameters such as out-of-roundness of the section, out-of straightness of the column, and residual stresses. A large number of compressive tests have been performed to statistically describe $k_o$ for different values of slenderness ratio. As could be expected, the uncertainty in $k_o$ depends on the value of the slenderness ratio. Except for those of Chen and Ross (2), most of the tests involved columns different from the tubular members used in offshore structures. The use of these results for the reliability analysis of jacket structures may therefore be difficult to justify. This is anyway the kind of information that is required.

The strength of tubular columns have been studied by Toma and Chen (11) using a non-linear finite element model of tubular columns. The effects of the parameters listed above were investigated. For example they found that an increase of out-of-straightness from 0.1% to 0.2% leads to a decrease of 15 to 20% of the compressive strength for the range of slenderness ratios typical of jacket structures.

This parametric study was not conducted with the objective of performing a statistical analysis of the results however and it is difficult to draw from it any practical conclusion regarding the distribution of $k_o$. Most of the statistical data that such an analysis would require (joint statistics on the influencial parameters) are lacking anyway and this is an area were additional investigations are urgently needed.

**Punching of a chord by a bracing**

Most of the test data available to attempt a statistical description of this failure mode are limited to simple planar joints under simple loading conditions (i.e. pure axial or pure bending load). Before additional information is obtained there is no reason to select a more complex interaction equation than the simple linear interaction formula proposed for example by ARSEM (13) :

$$1.0-(P/P_{\mathring{u}})-(M_i/M_{iu})-(M_o/M_{ou}) = 0 \quad (5)$$

where

$P_{\mathring{u}}$ = ultimate axial load capacity of the joint
$M_i$ = in-plane bending moment in the brace
$M_o$ = out-of-plane bending moment in the brace
$M_{iu}$ = ultimate in-plane bending capacity of the joint
$M_{ou}$ = ultimate out-of-plane bending capacity of the joint

Ochi, Makino and Kurobane (8) have evaluated the ultimate capacity of unstiffened tubular joints under axial brace loading (e.g. $P_{\mathring{u}}$) for several types of joints (X, Y, T and K). Their method consists in adjusting semi--empirical models to experimental results by multiple regression techniques. The ultimate strength is deduced from the load deformation curves by chosing the first peak load. Using this method, the ultimate axial capacity $P_{\mathring{u}}$ can generally be expressed as follows :

$$P_{\mathring{u}} = f.T^2 \sigma y.\varepsilon \quad (6)$$

where f is a function of the joint geometrical parameters, the axial chord stress, the yield stress, and the ultimate strength of the chord, T is the chord diameter and $\varepsilon$ is an error term. The form of the function f varies with the type of load (tension or compression) and the type of joint.

Using a different method and very few test results, Wardenier (12) has derived similar expressions for the in-plane and out-of-plane capacities $M_{iu}$ and $M_{ou}$.

Hence it seems that a minimum of four random variables (the yield stress and three error terms) are necessary if this failure criterion is to be included in a reliability analysis.

**Model uncertainty**

The three interaction equations presented above can all be writtenin the following way :

$$1 - F = 0 \qquad (7)$$

where F is a function of random and deterministic parameters.

A simple way of accounting for the model uncertainty associated to a particular interaction equation consists in replacing the 1 in the above equation by a random variable Z, with its mean, hopefully close to 1, representing the biais of the model and its probability density function representing the model uncertainty.

The only way of obtaining the required information is to compare predicted results to measured results. This type of investigation is difficult to perform because it requires careful statistical treatment. In particular, one must be sure that the estimated model uncertainty does not reflect imperfect knowledge of the interaction equation variable (e.g. yield stress).

This type of investigation has recently been performed by Kotoguchi et al. (6) on steel beam-columns, but in general the information required for offshore structure tubular members is far from complete and additional work is required.

**Remark**

Because the above mentioned failure criteria are to be used in a system reliability analysis, the problem of statistical correlation between the various resistance parameters accross the structure has to be addressed. It is probably reasonnable to assume that all variables (including model uncertainty variables) are decorrelated except the yield stress but this assumption has to be confirmed. Without additional information, an objective attitude is to perform the reliability analysis with the two extreme assumptions of perfect correlation or perfect decorrelation. By the way, the yield stress is a priori a random field accrosss the structure but also within a member along its length and accross the section. Therefore, the yield stress appearing in the expression of the plastic moment capacity is more or less an averaged value accross a particular section. It may therefore have less variability than shown by experimental tests performed on individual steel samples.

**MODELLING OF COMPONENT POST-FAILURE BEHAVIOUR**

Component post-failure behaviour is one of the key factors that determine the effective redundancy of a structure. It is hence necessary to properly model it in a probabilistic analysis that includes structural redundancy. The challenging problem here is to compromise between the accuracy of the deterministic model and its ability to be used in a probabilistic context.

One class of models that can be easily incorporated in a probabilistic analysis includes all bi-linear, two-states component models. In the unfailed state, the component is linear elastic, e.g. a standard beam or truss element. In the failed state, the component still behaves linearly, but with a modified stiffness matrix. Moreover additionnal nodal forces and/or moments related to the strength (e.g. yield stress) of the component are applied at its nodes.

The resulting formulation is simple if axial forces and bending moments interaction is not accounted for. It is more involved if interaction is accounted for (10), but still tractable.

With this type of models, various component behaviours, ranging from brittle to perfectly plastic, can be described (see fig. 1). It only requires linear stress analyses, which makes the probabilistic formulation rather simple. More precisely, in any state of the structure, the stresses in a non-failed member result from the superposition of the actual random loads applied to the structure and the residual strengths of the failed members. They can therefore be explicitely written in terms of the random variables in the following way :

$$\underline{S}^i = \underline{\underline{C}}^i . (\underline{L} + \underline{R}) \qquad (8)$$

where

$\underline{S}^i$ = internal forces and moments in member i
$C^i$ = influence matrix at member i
$\underline{L}$ = external random load vector
$\underline{R}$ = self-equilibrated random vecteur of equivalent forces and moments due to residual strength of failed members

Because of this explicit formulation, the structural analyses can be performed separately from the reliability estimations, and the computation is greatly reduced.

This class of models is therefore very attractive. However, the important question to be answered is the following : how well does it describe the post-failure behaviour of a buckled brace, a plastified section, or a punched member ?

One model of this type has been tested in the case of buckling of 3.D truss structures (9) and compared statisfactorily with experimental results. Only axial forces were accounted for and the failed state was described by a zero stiffness and a *residual strength equal to a fraction* of the buckling strength.

Another class of models includes all the non-linear models which can of course be made as accurate as desired. The drawbacks of such models are obviously a higher computation cost but also a much more complicated probabilistic formulation.

This is mostly due to the fact that the stresses in the members become implicit functions of the random variables. Therefore structural analyses and reliability estimation cannot be separated.

## REPEATED STRESS ANALYSES

Whether the structural component models (including post-failure behaviour) are linear or not, the methodology generally requires many successive stress analyses in order to identify the critical failure sequences of a redundant structure. For a structure of reasonnable size (several hundreds of nodes with 6 degrees of freedom each), the cost of those repeated stress analyses rapidly becomes prohibitive.

In the following, we present two methods aimed at reducing this cost as much as possible. The first one is based on a flexibility matrix approach (Sherman-Morison algorithm) and the second one on a stiffness matrix approach (substructuration technique).

### First method (Sherman-Morison algorithm)

Most of the structural analysis computer codes are based on the so--called "stiffness method" which consists first in assembling the individual element stiffness matrices into a global structure stiffness matrix K and second, in solving the resulting linear system of equations (K.U = F with U the vector of nodal displacements and F the vector of nodal forces) by a a factorisation procedure (e.g. Cholesky method).

By this method the complete inversion of the stiffness matrix is avoided. If the dimension of the load vector is N, the influence matrix of equation (1) is obtained by solving N successive linear systems of equations corresponding to N different force vectors. This will be referred to as the direct method.

The Sherman-Morison (SM) algorithm (5) provides a quick way of modifying the inverse of a matrix if one column of the original matrix is modified. The modified inverse is obtained directly from the previous inverse without having to inverse the modified original matrix. In the context of structural analysis the inverse of the stiffness matrix is the flexibility matrix which is costly to obtain for a realistic structure. However, if the S-M algorithm can be used, which is the case, this initial investment might be worthwile.

Indeed, when the stiffness matrix of a failed beam is modified, twelve columns of the global structure stiffness matrix are modified. Therefore by applying twelve times the S-M algorithm, the modified flexibility matrix is quickly obtained. In order to save even more computer time, the M-S algorithm can be improved to perform *the twelve modifications at once.*

Some results are presented on table 2. They show the reduced computer time corresponding to the computation of the modified flexibility matrix according to the three methods (direct, M-S, modified M-S), and this for different numbers of degrees of freedom.

As can be seen, the computer time reduction becomes more significant as the number of degrees of freedom increases. It is therefore expected that for realistic structural models (several hundreds of degrees of freedom), the initial investment of computing the flexibility matrix will be rapidly compensated by the saving in subsequent structural reanalyses.

### Second method (substructuration)

The substructuration technique is not new. It is the base of the so-called *super-elements* methods available on some finite-element codes. Originally, it was developped in order to analyse large structures when computer storage capabilities were limited.

In short, it works as follows. The original structure is divided into substructures of super-elements which do not overlap (each element, e.g. beam, belongs to only one substructure). Those substructures are connected by some nodes, common to at least two substructures, and called primary nodes. All other nodes, called secondary nodes, belong to alonly one substructure. Instead of building a single stiffness matrix for the *entire* structure, only substructure stiffness matrices are built and stored.

First, each submatrix is condensed at the primary nodes of its substructure and assembled to a global stiffness matrix. This global stiffness matrix, as well as each individual substructures matrix, is much smaller that the stiffness matrix that would have been obtained following a standard procedure.

The global stiffness equation is then solved at the primary node level. Finally, each substructures stiffness equation is solved at the secondary node level, using the displacements obtained at the primary nodes.

In our case, the major advantage of the method is not the reduction of the central memory space required, but the fact that substructures stiffness matrices, and other matrices necessary to solve the secondary level equations, can be stored individually. Recall that each time a new component fails, its own stiffness matrix is modified, thus affecting the whole structure stiffness matrix. Without substructuration, an entirely new large system of equations has to be solved. With substructuration, only the stiffness matrix and the other required matrices of the substructure to which the failed element belongs have to be modified. The primary level system of equation is also modified but it is much smaller than the system obtained without substructuration.

The method has been tested on two different structures : a small one (50 nodes, i.e. 300 DOF) and a medium one (300 nodes, i.e. 1800 DOF). In both cases, the structure has been divided in three substructures. The results, expressed in terms of CPU time ratios, the reference being the case without substructuration, are given in table 3. In this table, the expression "new analysis" means that one element of one substructure has been modified. Without substructuration, the cost of a new analysis is obviously equal to the cost of the first analysis.

About these results, three comments must be made :

. the cost of the first analysis is roughly the same in both cases,
. no significant CPU time is saved in the new analysis of a small structure,
. for the medium size structure, the cost of a new analysis is reduced by more that 50%.

The preceeding results should only be considered as indicators. A different time reduction would probably be obtained if the structures were divided differently into substructures. It is very likely that an optimum substructuration scheme can be found in each case. Hence some additionnal work still needs to be done.

### Conclusion

Two methods aimed at reducing the cost of repeated stress analyses have been presented. Both methods seem promising but it is presently not possible to determine which one is the most efficient. The comparison is presently under investigation.

From a probabilistic point of view this type of investigation is not very motivating but recall that the objective here is to apply structural system reliability techniques to practical situations and our experience shows that in such situations, the repeated stress analyses account for most of the cost of a reliability analysis.

## PROBABILISTIC REDUNDANCY ANALYSIS

As explained in the introduction, the structural failure event can be described as a union of intersections of individual component failure events, each intersection corresponding to a particular failure path. Hence, the estimation of the probability of structural failure requires the computation of individual component failure probabilities as well as joint failure probabilities.

These two aspects are briefly discussed in the following sections. Subsequently, a method for obtaining the critical failure sequences and bounds on the system probability of failure is presented. Practical aspects are discussed in view of the results of application examples.

### COMPUTATION OF COMPONENT AND JOINED FAILURE PROBABILITIES

With the recent developments of First and Second Order Reliability Methods the computation of individual components as well as joint probabilities of failures has ceased to be a major obstacle in structural reliability analysis.

The next major problem to tackle is providing these efficient methods with the appropriate input data !

Indeed, in order to compute probabilities of failure, some choices must be made regarding the probability distribution type of each random variable. In most of the theoritical work published in the litterature, distribution types are assumed more or less arbitrarily because examples of application are only given for demonstration purposes. In practical applications however, this becomes an important issue since the computed probabilities are significantly influenced by the choice of distribution laws. It was seen in previous sections that the information required to properly characterise all the random variables (load, resistance, and model uncertainties) is still incomplete. Except in some cases, the best information available reduces to a second order characterisation.

Should we then assume some distributions based on some "best engineering jugment" or should we restrict the reliability analysis to a first and second moment formulation (which is equivalent to assume that all variables are normally distributed) ? This is an other dilemna that becomes crucial in practical applications. This problem will be considered more generally in the conclusion.

We now consider another practical problem : the search for critical failure paths.

## SEARCH FOR CRITICAL FAILURE PATHS

Let us first consider a particular failure path Fi where each Fi is an individual component failure event. Obviously, the following inequalities hold :

$$P(F1) > P(F1 \cap F2) > \ldots > P(\bigcap_i Fi) \quad (9)$$

As we proceed along the failure path, the probability of reaching the successive steps decreases. This remark suggests to generate the most likely failure paths as follows :

Starting in the intact state of the structure, the probabilities of failure of each component according to each possible failure mode are computed. The state of the structure is then changed according to the most probable initial failure. If this is a failed state of the structure (according to a predefined criteria), the algorithm stops. If not, probabilities of subsequent failures following the previous one are computed. These are joint probabilities of two failure events.

The most probable failure sequence is then identified among all the two-steps sequences just generated and all the one-step sequences previously generated, except the one just chosen. Again, after checking that the corresponding state is not a failed state new sequences are generated and the associated probabilities computed.

More generally, at any step of the algorithm, the state of the structure is changed according to the most probable failure sequence among all the sequences generated up to that point and not yet chosen. Eventually, a failure state is reached. The sequence leading to that state is the most likely one.

Once the most likely failure path (or sequence) has been obtained, the next most likely one can be found by pursuing the search further on.

As already mentioned, after the first most likely failure paths have been found, the probability of their union provides a lower bound to the probability of failure of the system.

As the branch and bound algorithm is searching for the most likely failure paths, it generates incomplete failure paths, from which many complete failure paths could have been generated. Because failure paths are intersections of failure events, the event corresponding to an incomplete failure path contains all the events corresponding to all the complete failure paths that could have been generated from this incomplete path.

As a consequence, the union of all the incomplete failure paths, and of the generated most likely complete paths, contains the true structural failure event. Hence, the probability of this union is an upper bound to the probability of failure of the structure.

The situation is best illustrated by a simple example such as the one shown on figure 2. On this figure, the tree generated by the algorithm (each branch corresponds to a member failure) is represented as a subtree of the complete (very small) failure tree. The three minimal cut-set representations corresponding to the lower bound, the true value and the upper bound of the system probability of failure are also shown.

Actually, even before having identified the most likely failure path, an upper bound on the probability of failure can be obtained at any step of the analysis by incomplete failure paths generated up to that step. In fact, each time a new damaged state is explored (one new structural analysis) the number of incomplete failure paths increases and this upper bound decreases.

This is shown on fig. 3 in the case of a small jacket structure. Note that because the results are shown in terms of the reliability index . Therefore the upper bound on the probability of failure becomes a lower bound on the reliaiblity index.

## EXAMPLES

The following examples are performed on the simplified model of a small jacket structure standing in 30 meters of water (see fig. 4). The model is made of 52 nodes and 147 beam elements.

### Example 1

In this example, only first and second moments of the load variables are considered. Their statistical characteristics are the following ($\nu$ denotes the coefficient of variation) :

**Intensity variables**

- $\lambda_1$ = wind speed
  $E(\lambda_1) = 20$ m/s, $\nu_{\lambda_1} = 0.08$
- $\lambda_2$ = tidal current velocity
  $E(\lambda_2) = 0.95$ m/s, $\nu_{\lambda_2} = 0.14$
- $\lambda_3$ = wave action (velocity term = H/T)
  $E(\lambda_3) = 0.415$ m/s, $\nu_{\lambda_3} = 0.08$
- $\lambda_4$ = wave action (acceleration term = $H/T^2$)
  $E(\lambda_4) = 0.02$ m/s, $\nu_{\lambda_4} = 0.08$
- $\rho_{\lambda_1\lambda_4} = \rho_{\lambda_1\lambda_3} = \rho_{\lambda_3\lambda_4} = 1$, others = 0
- All actions are in the same direction (positive x - direction).

**Profile variables**

- $E(\underline{W}^i)$ are given by deterministic models of wave, current and wind kinematics
- The coefficients of variations are assumed constant accross the structure with the following values :
  $\nu_{W^1} = 0.17$, $\nu_{W^2} = 0.30$,
  $\nu_{W^3} = \nu_{W^4} = 0.10$
- The $\underline{W}$'s are assumed fully correlated accross the structure for each type of action and uncorrelated from one type to another.

**Force coefficients**

- mean values and coefficients of variation are constant accross the structure
  wind : $E(C_D) = 1.00$ $\nu_{C_D} = 0.12$
  water: $E(C_D) = 0.60$ $\nu_{CD} = 0.35$
  $E(C_M) = 2.00$ $\nu_{CM} = 025$
- All coefficients are decorrelated accross the structure.

The methodology described earlier is used to obtained $E(\underline{L})$ and $\Sigma_{\underline{L}}$ where $\underline{L}$ is the random nodal force vector (dimension = 6x52 = 312).

The coefficients of correlation of the horizontal forces (wave direction) at node 201 (see figure 4) and other nodes are given below in table 4.

**Failure criteria**

Only the plastification and buckling criteria are considered. Failures are assumed perfectly brittle (e.g. no residual strength).

Structural failure is defined by a 50% reduction in global stiffness.

**Results**

They are given on figure 5. If the algorithm described in the previous section is used and stopped after the first most likely failure mode has been identified (4306 - 4301) the bounds on the structure reliability are
$\beta_{LOW} = 5.07$ and $\beta_{UP} = 5.27$

The failure tree generated to obtain these bounds is quite small (branches outside the dotted lines) and this is due mainly to the brittle failure mode assumption. Given that a member has failed, the probability of having a second failure is almost equal to one. Hence no increase in the reliability index can be noticed when going from the first to the second failure.

This is true for all the additional failures shown on the failure tree (branches inside the dotted-lines).

### Example 2

In this example, the loading has been simplified : only two horizontal nodal forces are applied at nodes 201, 202, 203 and 204. The direction of application makes a 30 degrees angle with the positive x - direction. A constant coefficient of correlation of 50% is assumed between the individual forces. The failure modes are assumed plastic (e.g. full residual strength after failure).

The results are shown in figure 6. Because of te plastic behaviour assumption, their is a significant increase in the reliability index when going from a first failure to a second one.

By applying the algorithm until the first most likely failure mode is identified, the bounds on the structure reliability are $\beta_{low} = 4.16$ and $\beta_{up} = 4.32$. Also shown on the figure below the failure tree is the evolution of the bounds (essentially the lower bounds) as the number of structural reanalyses increases.

The effect on the results of the correlation between the nodal forces is demonstrated on figure 7. Only a subtree of the previous failure tree is shown but it is sufficient to show some important effects.

As expected, going from zero correlation to full correlation has a strong influence on the reliability values. Less expected is the change in the ordering (in terms of reliability) of the failure sequences.

This clearly demonstrates the necessity of well describing the correlation between nodal forces on the structure.

**CONCLUSIONS**

As already stated in the introduction, the purpose of this paper was not to present a new methodology but rather to present some of the problems one has to face when trying to apply an existing methodology to practical situations.

In summary, two types of problems arise. The first type is linked to the size of the structure encountered in practical situations (large dimension of the load vector, cost of successive structural analyses, large number of potential failure sequences). For this type of problems, some solution have been proposed (load model reduction, Sherman Morison algorithm or substructuration, search for most likely failure sequences). Other solution certainly exist and there is no doubt that this type of technical problems can be handled if they are given enough attention.

The second type of problems is of a different nature and more fundamental. Generally speaking it is linked to the modelling assumptions of any reliability analysis.

It has been seen several times in the above discussions that the statistical information available to perform reliability analysis of steel-jacket platforms is far from being complete.

Ideally, the solution is to perform more tests and collect more data, with the objective of using them to obtain statistics. Those tests may be real, or simulated (by refined non-linear finite-element analyses for example) but in any case there are time consuming and costly. And one cannot always rely on hypothetical futur information. It is to be expected anyway that our extraordinary appetite for statistical data will never be satisfied !

In the meantime, given the available information, the problem is to choose a probabilistic model that is simple enough (so that most of the necessary input data is available) and still accounts for the influential factors such as correlation (in the loads and in the resistances as well) and post-failure behaviour.

In cases where there is not enough information to favor a particular assumption, different assumptions have to be considered in the reliability analysis.

Because the purpose of the reliability analysis is eventually to help make a decision (e.g. repair or no repair) it has to be checked if the decision arrived at is affected or not by the choice of a particular assumption.

If it is not affected the problem is solved but if it is affected, there is no easy solution to the problem, except to use our best engineering judgement or to use another model.

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| Element Number | Exact $\beta$ N=312 | Approximate $\beta$ | | |
|---|---|---|---|---|
| | | N'=10 | N'=20 | N'=30 |
| 1 | 3.66 | 3.75 | 3.69 | 3.66 |
| 2 | 3.83 | 3.97 | 3.88 | 3.83 |
| 3 | 5.96 | 6.10 | 6.01 | 5.96 |

Table 1 - Effect of Load Model Reduction (N'<N) on component reliability indices

| Number of DOF | Direct Method | Sherman Morison | Modified S-M |
|---|---|---|---|
| 228 | 1.00 | 0.41 | 0.34 |
| 300 | 1.00 | 0.36 | 0.30 |
| 348 | 1.00 | 0.31 | 0.25 |

Table 2 - Influence of the structural reanalysis method (Sherman-Morison) on CPU time

| Number of DOF | without substructuration | with substructuration | |
|---|---|---|---|
| | | 1st anal. | new anal |
| 300 | 1.0 | 0.8 | 0.7 |
| 1800 | 1.0 | 1.3 | 0.4 |

Table 3 - Influence of the structural reanalysis method (substructuration) on CPU time

| Pairs of Nodes | Correlation Coefficient |
|---|---|
| 201 - 208 | 0.70 |
| 201 - 202 | 0.65 |
| 201 - 301 | 0.81 |
| 201 - 401 | 0.65 |
| 201 - 501 | 0.62 |

Table 4 - Examples of Nodal Forces Correlations

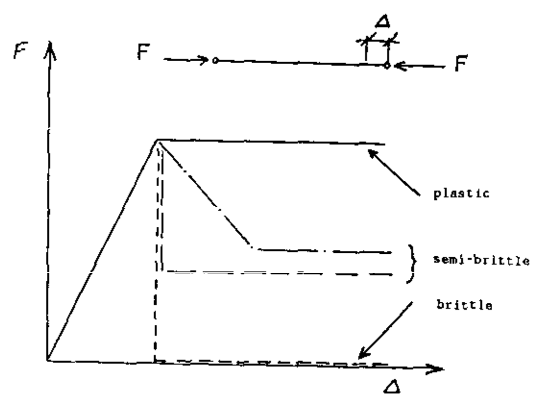


Figure 1 - Possible models of component post failure behaviour

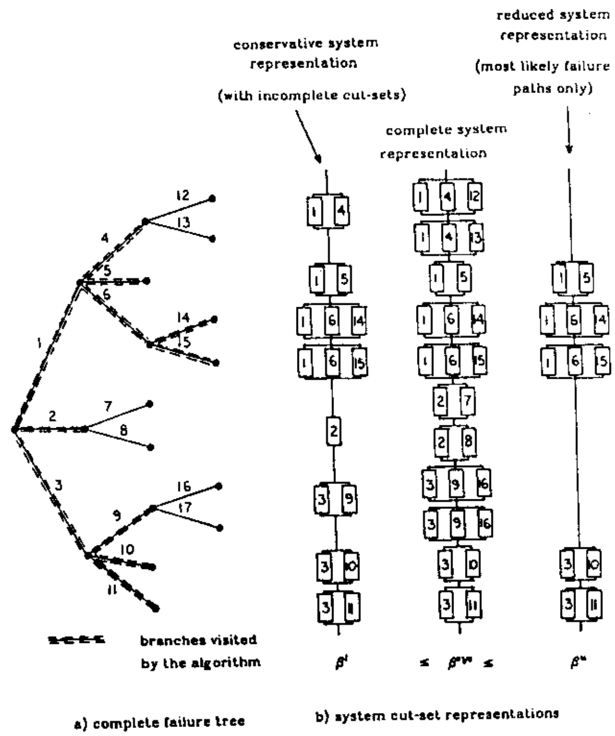


Figure 2 - Lower and upper bounds on system reliability

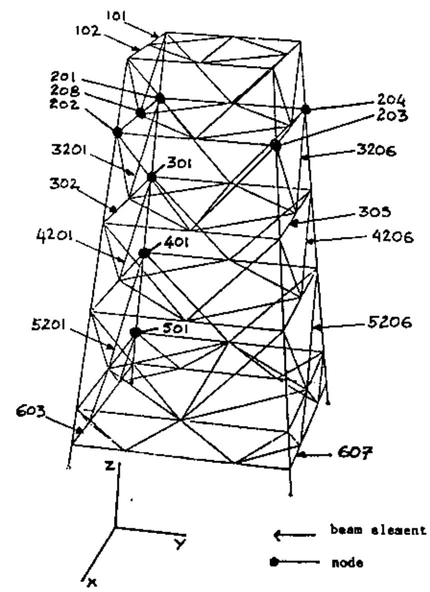


Figure 4 - Simplified finite element model of a small steel-jacket platform

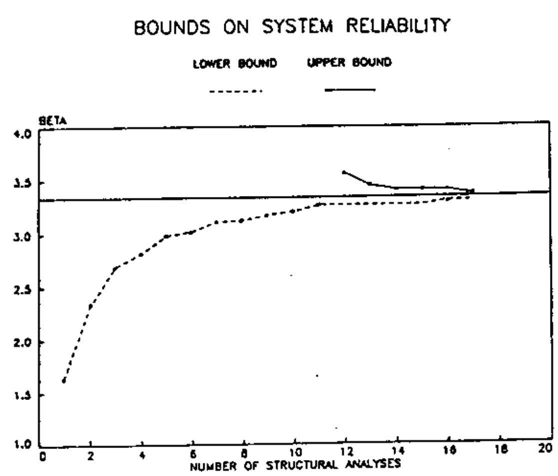


Figure 3 - System reliability bounds versus number of structural reanalyses

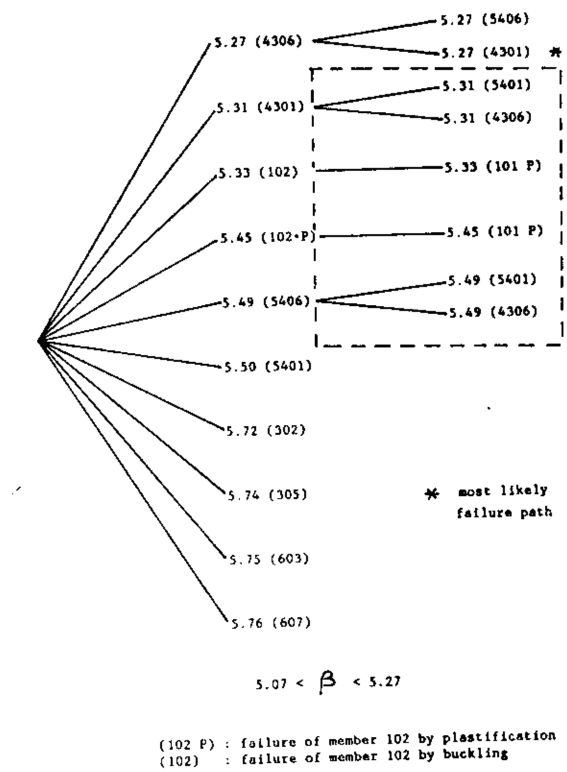


Figure 5 - Failure tree for example 1

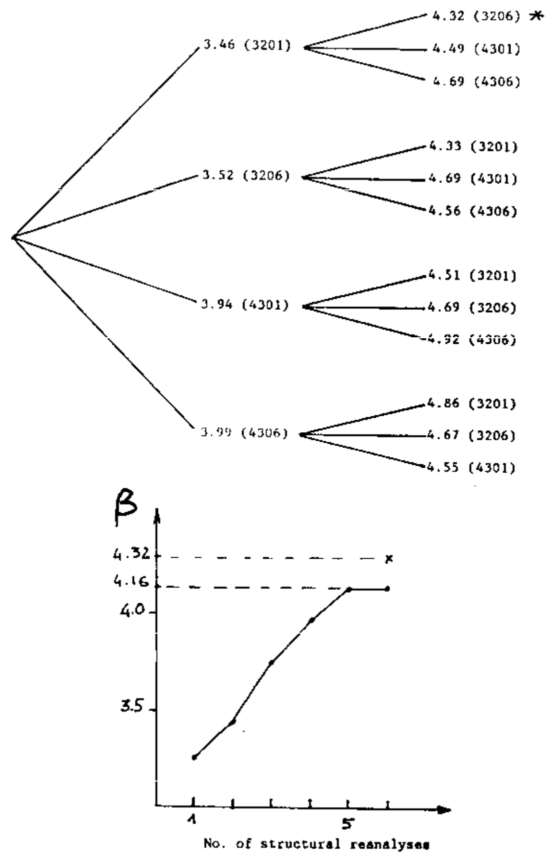


Figure 6 - Failure tree for example 2

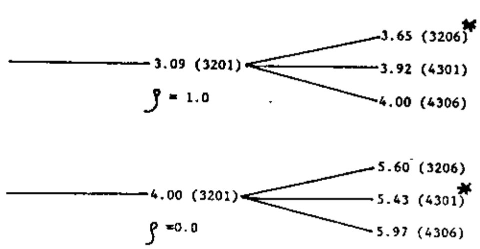


Figure 7 - Influence of nodal forces correlation