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# Reliability of Plates Under Combined Loading

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## ABSTRACT

Buckling of plates is an important design consideration both for ships and fixed and floating offshore structures. A probabilistic procedure for elastic buckling and collapse analysis of unstiffened plates in marine structures is presented. The procedure is developed for plates under combined biaxial stress, shear stress and lateral pressure, due to still water loading and wave induced loading. Uncertainties due to loading variables (static and time varying), geometric variables (thickness and imperfections), as well as material variables (Youngs modulus and yield stress) are included. The reliability analysis is based on first- or second-order reliability methods combined with methods for outcrossing analysis. The procedure has been developed specifically for this problem, but has general applicability in structural reliability analyses with a mixture of time-dependent and time-independent basic variables.

## INTRODUCTION

Plate elements in ship hulls, other submersible and semi-submersible marine structures, and fixed offshore structures are loaded by a combination of stresses and lateral pressure. The stresses and lateral pressure are modeled as time-dependent stochastic processes, whereas the material properties and geometry parameters are modeled as random variables. The behavior of plate elements with respect to various failure criteria is formulated in terms of limit state functions which include time dependent as well as time independent basic variables.

A strength model for an unstiffened plate under combined loading is presented and load models for still water loading and wave induced loading as well as structural behavior are presented. A reliability analysis for a problem with time independent as well as time dependent basic variables is developed, where special emphasis is put upon a coupling between a first-order reliability method and an outcrossing analysis for a Gaussian vector process. The reliability method is applied to the load and strength models. Finally, results from an example are presented and general conclusions are offered.

## STRENGTH MODEL

The elastic buckling and plasticity failure of a plate subjected to combined stresses and lateral pressure, Fig. 1, can be formulated as

$$\eta(\sigma_x, \sigma_y, \tau_{xy}, p, A, B, T, \delta, E, \nu, \sigma_F) \geqslant 1 \qquad (1)$$

where $\eta(\ )$= degree of utilization or usage factor, $\sigma_x$ = stress in the x-direction, $\sigma_y$ = stress in the y-direction, $\tau_{xy}$ = shear stress, $p$ = lateral pressure, $A$ = plate length, $B$ = plate width, $T$ = plate thickness, $\delta$= represents geometrical imperfection, $E$= Young's modulus, $\nu$= Poisson's ratio, and $\sigma_F$ = yield stress. In general, the variables $A$, $B$, $T$, $\delta$, $E$, $\nu$, and $\sigma_F$ are time independent and the variables $\sigma_x$, $\sigma_y$, $\tau_{xy}$, and $p$ are time dependent. For a stiffened plate also properties of the stiffeners enter the limit state function.

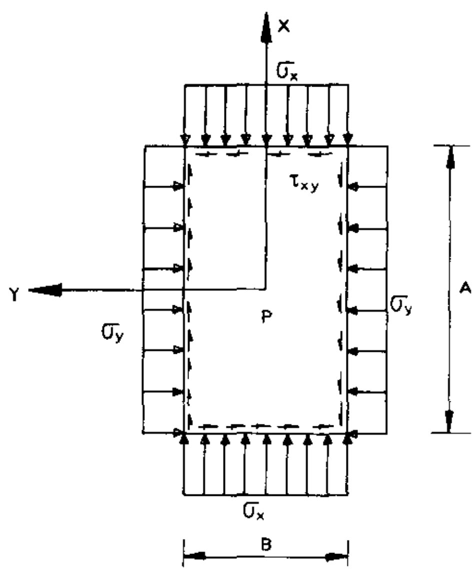


Fig. 1 A plate under biaxial stress, shear stress and lateral pressure.

In general, no closed form solutions are available for the usage factor which is obtained numerically by iterative procedures. Plate elements in marine structures are generally integrated parts of larger orthogonally stiffened structures. In [1] a procedure has been presented to compute the usage factor for an unstiffened plate subjected to biaxial stress and lateral pressure. The boundary conditions prescribe the plate as simply supported out-of-plane along all edges, with edges to remain straight during deformation but free to move in-plane. Imperfections from welding and the erection procedure are assumed to be small and in the shape of the natural buckling mode. The imperfection size is described by the dimension-free parameter $\delta$. The procedure has later been extended to include shear stresses and is here used with this extension.

A further generalization to a complete hull section is suggested in [2], based on a simplified mechanical description of the hull behavior. From a reliability analysis point of view this generalization is straightforward and the limit state function uses cross sectional forces and external pressure rather than local stresses and external pressure.

## LOAD MODEL

Two types of loads are considered. Loads due to ship/platform weight, cargo/topside weight, buoyancy, and water pressure are modeled as a renewal square wave process, see Fig.2. These loads are denoted still water loads. The amplitudes are random variables which, depending on the operational pattern, are modeled as independent or dependent. The duration of a pulse for a ship corresponds to one voyage in a loaded condition and the duration between ballast changes in a ballast condition. For simplicity, the duration of a pulse is taken as deterministic and constant. The still water loading induces sectional forces in a cross section, which again induce stresses. A linear structural analysis is applied in the transformation from global forces to local stresses. Local stresses within each pulse are described by correlated random variables with statistics derived from statistics for sectional forces. In [3] such statistics have been presented for ships based on extensive measurements.

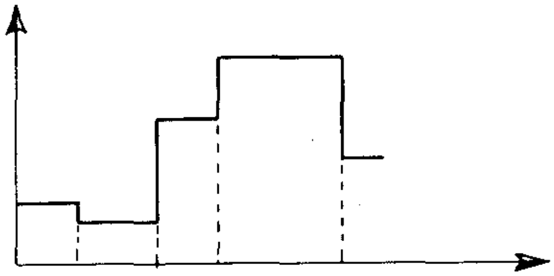

Fig. 2 Load model for still water load.

To model the wave induced loading, the sea condition is divided into stationary sea states. Here a sea state is defined by the most important wave characteristics, i.e., the main wave direction $\Theta$, the significant wave height $H_S$ and the mean wave period $T_Z$. Short crested seas are modeled by combining long crested waves from different directions weighted according to a wave energy spreading function.

Long term probabilities for various main wave directions are determined from wind measurements, wave measurements on stationary buoys, or better from sailing ships thereby accounting for operational patterns. A wave scatter diagram is used which gives the fraction of time with different combinations of $H_S$ and $T_Z$. The one-dimensional Pierson-Moskowitz wave spectrum, which is uniquely defined for each specific combination of $H_S$ and $T_Z$, is used within each sea state to describe the wave energy on different frequencies. Within each sea state stationary conditions are assumed. Each pulse in the still water load process contains a number of sea states.

Linear wave theory is applied and a Gaussian wave loading is assumed. A linear structural behavior is assumed and wave induced load effects are thus Gaussian processes. Transfer functions from the sea elevation to stresses and pressure are denoted by $H_{\sigma_x}(\omega)$, $H_{\sigma_y}(\omega)$, $H_{\tau_{xy}}(\omega)$, and $H_p(\omega)$, respectively. The vector $(\sigma_x(t), \sigma_y(t), \tau_{xy}(t), p(t))$ is a Gaussian vector process with zero mean value. Covariances for the components and time derivatives are

$$Var[\sigma_x] = \int_0^\infty |H_{\sigma_x}(\omega)|^2 S_w(\omega) d\omega \qquad (2)$$

$$Var[\dot{\sigma}_x] = \int_0^\infty \omega^2 |H_{\sigma_x}(\omega)|^2 S_w(\omega) d\omega \qquad (3)$$

$$Cov[\sigma_x, \sigma_y] = \mathrm{Re}[\int_0^\infty H_{\sigma_x}(\omega)\overline{H_{\sigma_y}(\omega)} S_w(\omega) d\omega] \qquad (4)$$

$$Cov[\dot{\sigma}_x, \dot{\sigma}_y] = \mathrm{Re}[\int_0^\infty \omega^2 H_{\sigma_x}(\omega)\overline{H_{\sigma_y}(\omega)} S_w(\omega) d\omega] \qquad (5)$$

and similar equations. An overbar denotes a complex conjugate, and $S_w(\omega)$ is the wave spectrum.

## RELIABILITY METHOD

The reliability of a structural element is generally analyzed with respect to one or more failure criteria. For one criterion the performance is described through the limit state function $g(\,)$

$$g(z) \begin{cases} < 0 & \text{for z } \textit{in failure set} \\ = 0 & \text{for z } \textit{on limit state surface} \\ > 0 & \text{for z } \textit{in safe set} \end{cases} \qquad (6)$$

The vector Z is a vector of basic variables describing uncertainties in loading, material properties, geometry, statistical estimates and analysis models. In addition, the limit state function can depend on a set of deterministic design parameters and time. The failure probability for the considered failure mode, $P_F$, is the probability that the vector Z has a value for which $g(z)<0$.

## Outcrossing Analysis

Let the vector of basic variables be $Z = Z(t)$, where $Z(t)$ is an ergodic vector process. The failure probability for a failure criterion with limit state function $g$ is in a time period $[0,D]$

$$P_F = P(g(Z(t)) \leq 0 \text{ for } some\ t \in [0,D]) \quad (18)$$

or simply as

$$P_F = P(\min_{0 \leq t \leq D} g(Z(t)) \leq 0) \quad (19)$$

The safe set is $S = \{z \mid g(z) > 0\}$ and an upper bound on the conditional failure probability is, [5]

$$P_F \leq P(g(Z(0)) \leq 0) + \nu(S)D = P_0 + \nu(S)D \quad (20)$$

The first term on the right hand side $P_0$ is the instantaneous failure probability and the second term is the expected number of outcrossings of the safe set. As an alternative the conditional failure probability can be approximated by

$$P_F \approx 1 - (1-P_0)\exp\left(-\frac{\nu(S)D}{1-P_0}\right) \quad (21)$$

The mean outcrossing rate of the safe region, $\nu(S)$, may be obtained from a generalization of Rice's formula, [5],

$$\nu(S) = \int_{\partial S} f_Z(z) \int_{\dot{z}_N=0}^{\infty} \dot{z}_N f_{\dot{Z}_N \mid Z}(\dot{z}_N, z) d\dot{z}_N \, d(\partial S) \quad (22)$$

where $\dot{z}_N$ is the projection of the time derivative $\dot{Z}$ on the out-bound normal vector at a point on the limit state surface $\partial S$. With $k$ time dependent basic variables the calculation of $\nu(S)$ requires an $k-1$-fold surface integration. An additional problem is that often the limit state surface is only given in an implicit way.

For stationary Gaussian processes an asymptotic result is available, which may be used as an approximation to (22). The mean value vector and covariance matrix for $(Z(t), \dot{Z}(t))$ are

$$\mu_{(Z(t),\dot{Z}(t))} = \begin{bmatrix} \mu_Z \\ 0 \end{bmatrix}, \quad C_{(Z(t),\dot{Z}(t))} = \begin{bmatrix} C_{ZZ} & C_{Z\dot{Z}} \\ C_{\dot{Z}Z} & C_{\dot{Z}\dot{Z}} \end{bmatrix} \quad (23)$$

Consider first a transformation corresponding to the Rosenblatt transformation (10)

$$U(t) = L(Z(t) - \mu_Z) \quad (24)$$

where U is a set of uncorrelated and standardized normal variables. L satisfies

$$L^{-1}(L^{-1})^T = C_{ZZ} \quad (25)$$

L is a lower triangular matrix and may be determined by a Cholesky triangularization procedure for positive definite matrices. The asymptotic value of the mean outcrossing rate of the safe set $S_u$ in $u$-space corresponding to $S$ in $z$-space is, [8],

$$\nu(S_u) \sim \nu(S'_u) \quad (26)$$

$\partial S'_u$ is the tangent hyperplane at the design point in $u$-space computed for the case where Z is independent of time, see Fig. 3.

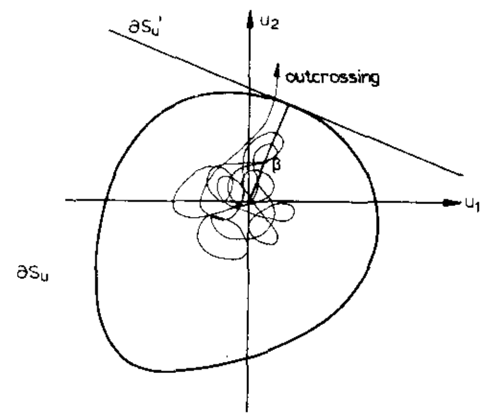


Fig. 3 Outcrossing of safe set and safe set with linearized boundary.

The approximating hyperplane has the equation

$$\partial S'_u: \ \beta - \alpha^T u = 0 \quad (27)$$

An outcrossing of the safe set $S_u$ by the vector process is then replaced by an up-crossing of level $\beta$ by the scalar process $\alpha^T U(t)$. This process has zero mean and unit variance and the mean up-crossing rate of level $\beta$ is

$$\nu(\beta) = \frac{1}{\sqrt{2\pi}} \sqrt{Var[\alpha^T \dot{U}]}\, \phi(\beta) \quad (28)$$

where $\phi(.)$ denotes the standardized normal probability density. The variance of the derivative process is

$$Var[\alpha^T \dot{U}] = \alpha^T C_{\dot{U}\dot{U}} \alpha = \alpha^T L C_{\dot{Z}\dot{Z}} L^T \alpha \quad (29)$$

The approximation to the failure probability in (21) is thus taken as

$$P_F \approx 1 - \Phi(\beta) \exp\left[-\frac{\sqrt{\alpha^T L C_{\dot{Z}\dot{Z}} L^T \alpha}\, \phi(\beta) D}{\sqrt{2\pi}\Phi(\beta)}\right] \quad (30)$$

It is noted that the approximation is independent of elements in the cross covariance matrix $C_{Z\dot{Z}}$. An improved asymptotic formula for the outcrossing rate is available in [9]. This formula is slightly more involved and includes $C_{Z\dot{Z}}$ as well as curvature information in the point $\beta\alpha$. When the process $\alpha^T U$ is narrow banded a better result may be obtained by replacing the mean up-crossing rate (28) by an effective rate determined by an interpolation between the up-crossing rate for the envelope process for small levels and the up-crossing rate of the process itself for high levels, [5,10].

## Combined Reliability Analysis Procedure

In the general case the vector of basic variables consists of the sets

When all basic variables are random variables the computation of the failure probability can be efficiently done by first- or second-order reliability methods. When one basic variable is a random function of time, i.e. a random process, the interest is generally only on the maximum or minimum value of this function over some specified time interval. This maximum or minimum value is a random variable for which the distribution may be determined, and first- or second-order reliability methods can then be applied directly. When two or more of the basic variables are random functions in time a load combination problem exists and modifications of first-order techniques are needed. In the past, approximate rules such as Turkstra's rule, [4,5], have been used to replace the random processes by one or more combinations of random variables. Turkstra's rule has proved to provide sufficient accuracy for many linear combinations of independent processes. The use of Turkstra's rule or a similar rule for dependent variables and for nonlinear combinations has, however, not been verified.

Nonlinear load combinations are here treated as first-passage problems for a vector process out of a safe set. Emphasis is put upon Gaussian vector processes describing response of a linear structure to wave loading. Based on the first-passage analysis a failure probability conditioned on the basic random variables is computed. The overall failure probability is then obtained by integrating the conditional failure probability over all values of the time independent basic variables.

The first-order reliability method, the outcrossing analysis for Gaussian vector processes, and the combined reliability analysis procedure are described in the following.

### First-Order Reliability Method

The failure probability for a given failure mode is denoted by $P_F$ and may be computed by

$$P_F = P(g(\mathrm{Z}) \leqslant 0) = \int_{g(\mathbf{z})\leqslant 0} f_{\mathrm{Z}}(\mathbf{z}) d\mathbf{z} \qquad (7)$$

where $f_{\mathrm{Z}}(\mathbf{z})$ is the joint probability density function of Z. An exact evaluation of $P_F$ is rarely feasible and first-order reliability methods (FORM) have therefore evolved as practical methods to evaluate good approximations in an efficient way, [5]. A further improvement is achieved by use of a second-order reliability method (SORM), [5].

In the evaluation of (7) a FORM uses a variable transformation of Z into a set of uncorrelated and standardized normal variables U,

$$\mathrm{U} = \mathrm{T}(\mathrm{Z}) \qquad (8)$$

For independent variables one possible choice for the transformation T is

$$\mathrm{T}: U_i = \Phi^{-1}(F_i(Z_i)),\ i=1,2,\ldots,n \qquad (9)$$

where $\Phi(\ )$ denotes the standardized normal distribution function and $F_{Z_i}(\ )$ is the distribution function for $Z_i$. For dependent variables the Rosenblatt transformation, [6], has been suggested in [7],

$$\mathrm{T}: U_i = \Phi^{-1}(F_i(Z_i \mid Z_1,\ldots,Z_{i-1})),\ i=1,..,n \qquad (10)$$

where $F_i(\ \mid\ ,\ldots,\ )$ is the distribution function for $Z_i$ conditioned on $(Z_1,\ldots,Z_{i-1})$.

The equation for the limit state surface in $u$-space becomes

$$g_u(\mathbf{u}) = g(\mathrm{T}^{-1}(\mathbf{u})) = 0 \qquad (11)$$

In a FORM the limit state surface in $u$-space is approximated by its tangent hyperplane at the point on the surface closest to the origin, i.e. the point with the highest probability density. This point $\mathbf{u}^* = \mathrm{T}(\mathbf{z}^*)$ is called the design point and is found by a minimization procedure with one constraint. The design point is expressed as

$$\mathbf{u}^* = \beta\alpha \qquad (12)$$

where $\beta$ denotes the distance from the origin to the approximating tangent hyperplane in $u$-space. $\beta$ is called the first-order reliability index and the sign of $\beta$ is determined as the sign of $g_u(0)$. $\alpha$ is a unit vector normal to the limit state surface at the design point and is directed towards the failure set. The components in the vector are called sensitivity factors. The first-order approximation to the failure probability is

$$P_F \approx \Phi(-\beta) \qquad (13)$$

The design point $\mathbf{u}^*$ must generally be obtained by an iterative search algorithm. One algorithm consists of constructing a sequence $\mathbf{u}_1, \mathbf{u}_2, \ldots, \mathbf{u}_m, \ldots$, according to the rule

$$\mathbf{u}_{m+1} = \left\{ \mathbf{u}_m^T \alpha_m + \frac{g_u(\mathbf{u}_m)}{|\nabla g_u(\mathbf{u}_m)|} \right\} \alpha_m \qquad (14)$$

where $\alpha_m$ is a unit vector defined by

$$\alpha_m = -\frac{\nabla g_u(\mathbf{u}_m)}{|\nabla g_u(\mathbf{u}_m)|} \qquad (15)$$

The gradient of $g_u$ is related to the gradient of $g$ through

$$\nabla g_u(\mathbf{u}) = \mathbf{J}^T \nabla g(\mathbf{z}) \qquad (16)$$

where $\mathbf{J}$ is the Jacobian matrix,

$$\mathbf{J} = \left\{ \frac{\partial z_i}{\partial u_j} \right\} \qquad (17)$$

For independent variables the use of the transformation (9) leads to a diagonal Jacobian matrix, while the Rosenblatt transformation (10) leads to a lower triangular matrix.

$$Z = (Z_1, Z_2(t)) \qquad (31)$$

where $Z_1$ is a vector of random variables and $Z_2(t)$ is an ergodic random process. In a general formulation some distribution parameters describing $Z_2(t)$ may be modeled as random variables and thus contained in $Z_1$. In that case $Z_2(t) | Z_1 = z_1$ is an ergodic random process, and the procedure described below can equally well be applied.

With a limit state function $g$ the failure probability in a time period $[0,D]$ is

$$P_F = P(\min_{0 \le t \le D} g(Z_1, Z_2(t)) \le 0) = \qquad (32)$$

$$\int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} P(\min_{0 \le t \le D} g(z_1, Z_2(t)) \le 0) f_{Z_1}(z_1) d z_1$$

The conditional failure probability

$$P_F(z_1) = P(\min_{0 \le t \le D} g(z_1, Z_2(t)) \le 0) \qquad (33)$$

is given by (30) where $P_F$, $\beta$, $\alpha$ and possibly also L and $C_{\dot{Z}\dot{Z}}$ now depend on the value $z_1$. A direct $n$-fold integration to compute the failure probability by (32) is impractical for nontrivial cases. Therefore a first-order reliability method, as described earlier, is applied for this integration to approximate the failure probability. One possible approach is suggested in [11]. An alternative formulation, differing in the order in which the numerical calculations are performed, has been suggested in [12]. In this approach an auxiliary standard normal variable $U$ is introduced and a limit state function $h(u, z_1)$ is defined as

$$h(u, z_1) = u - \Phi^{-1}(P_F(z_1)) \qquad (34)$$

It can be easily proven that $P(h(U, Z_1) \le 0)$ is equal to the unconditional failure probability $P_F$ in (32). Based on the formulation in (34) and the expression for $P_F(z_1)$ in (30) a first-order or second-order reliability method can be directly applied. When the iteration procedure in (14) is used, derivatives of $P_F(z_1)$ in (30) are needed. These derivatives may be computed numerically, but analytical formulas can be developed based on parametric sensitivity factors in first-order reliability methods, [5].

## RELIABILITY OF AN UNSTIFFENED PLATE UNDER COMBINED LOADING

The elastic buckling and plasticity failure of a plate subjected to combined stresses and lateral pressure may be formulated by a limit state function given by, see (1)

$$g(z) = 1 - \eta(\sigma_x, \sigma_y, \tau_{xy}, p, A, B, T, \delta, E, \nu, \sigma_F) \qquad (35)$$

Within each sea state, the stresses and lateral pressure are induced by still water loading and wave loading, i.e.,

$$\begin{aligned} \sigma_x(t) &= \sigma_{xs} + \sigma_{xw}(t) \\ \sigma_y(t) &= \sigma_{ys} + \sigma_{yw}(t) \\ \tau_{xy}(t) &= \tau_{xys} + \tau_{xyw}(t) \\ p(t) &= p_s + p_w(t) \end{aligned} \qquad (36)$$

where $\sigma_{xs}, \sigma_{ys}, \tau_{xys}$ and $p_s$ are still water induced stresses and lateral pressure which are time independent random variables within each still water load pulse. $\sigma_{xw}(t)$, $\sigma_{yw}(t)$, $\tau_{xyw}(t)$ and $p_w(t)$ are wave induced stresses and lateral pressure. It is assumed that still water loading is independent of wave induced loading. The set of basic variables is hence divided into $Z_1 = (\sigma_{sx}, \sigma_{sy}, \tau_{sxy}, p_s, A, B, T, \delta, E, \nu, \sigma_F)$, and $Z_2(t) = (\sigma_{xw}(t), \sigma_{yw}(t), \tau_{xyw}(t), p_w(t))$.

To perform the combined reliability analysis a time interval of constant amplitude still water loading is first considered. Conditioning on $Z_1 = z_1$, the conditional failure probability is obtained from (30), where the mean outcrossing rate of the safe set is obtained from

$$\nu(S) \approx \sum_{i=1}^{m} p_i \sum_{j=1}^{n} q_j \, \nu(\beta_{ij}) \qquad (37)$$

in which $p_i$ is the probability of the $i$th wave direction, $q_j$ is the probability of the $j$th sea state, and $\nu(\beta_{ij})$ is the mean up-crossing rate for the given wave direction and sea state, which is computed by (28) and (29). The output from this combined reliability analysis is a reliability index and design point

$$u^* = \beta\alpha = \beta(\alpha_u, \alpha_S, \alpha_{gm}) \qquad (38)$$

where $\alpha_u$ is the sensitivity factor for the auxiliary variable $U$ in (34), $\alpha_S$ is the vector of sensitivity factors for the stresses and pressure induced by the still water loading, and $\alpha_{gm}$ is the vector of sensitivity factors for the geometry and material parameters.

The reliability for a period containing one pulse of the still water load has been determined. In practice the interest is on a larger reference period, e.g., 1 year or the design life time. Let the number of pulses in the still water load model within the reference period be $N$. With independent still water loading from pulse to pulse, the failure probability in the reference period is approximated by

$$P_F \approx 1 - \Phi_N(\boldsymbol{\beta}, \boldsymbol{\rho}) \qquad (39)$$

where $\boldsymbol{\beta}$ is a vector with identical elements $\beta$ from (38), and $\boldsymbol{\rho}$ is a correlation matrix with elements

$$\rho_{ij} = \begin{cases} 1 & i = j \\ \alpha_{gm}^T \alpha_{gm} & i \ne j \end{cases} \qquad (40)$$

The theory for series system reliability has been applied and the probability in (39) can be evaluated by a simple one-dimensional integral, see [5]. The above procedure can be modified if a different model for the still water loading is adopted. Along the same lines, a procedure can also be developed which uses the duration of a sea state rather than the pulse duration in the still water loading as a basic time interval.

## EXAMPLE

The procedure is applied for a reliability analysis of a plate element located at the bottom of a ship hull at midship. Fixed deterministic values are assumed for Poisson's ratio, and for the length and width of the plate. A fixed deterministic value is also assumed for the normal stress in the y-direction due to still water loading. Components of a 3-dimensional multinormal distribution are assumed for the distributions of still water induced stress in x-direction, shear stress, and the lateral pressure, [3]. A lognormal distribution is assumed for the plate thickness. Normal distributions are assumed for Young's modulus and yield stress. These distributions and their parameters are given in Table 1. The units in Table 1 are *m* for lengths and *MPa* for Young's modulus, stresses and lateral pressure.

**Table 1: List of plate parameters**

| Variable | Distribution | Mean | Stand. Dev. |
|---|---|---|---|
| $\sigma_{xs}$ | Multinormal 1 | 54.29 | 5.5 |
| $\sigma_{ys}$ | Fixed | 41.24 | - |
| $\tau_{xys}$ | Multinormal 2 | 0.00 | 0.5 |
| $p_s$ | Multinormal 3 | 0.166 | 0.02 |
| $A$ | Fixed | 5.30 | - |
| $B$ | Fixed | 1.20 | - |
| $T$ | Lognormal | 0.015-0.025 | 0.0015-0.0025 |
| $\delta$ | Normal | 0.01 | 0.001 |
| $E$ | Normal | $2.1 \times 10^5$ | $1.5 \times 10^4$ |
| $\nu$ | Fixed | 0.3 | - |
| $\sigma_F$ | Normal | 385 | 24 |

A time period of one year is used. Five main wave directions are considered corresponding to head sea, quarter forward seas, and beam sea, with fractions of occurrence time 25%, 2x25%, and 2x12.5%, respectively. These numbers have been selected somewhat arbitrarily. The sea scatter diagram used is based on the sea scatter diagram for the North Atlantic, Station India, but it is somewhat simplified. For all sea states and wave directions the one-dimensional Pierson-Moskowitz wave spectrum, [5], is used.

The probability of failure for various plate thicknesses is given in Table 2. Column 2 in this table is related to an analysis with uncertainties in environmental loading only, whereas column 3 is related to an analysis with all sources of uncertainty included. Table 2 indicates that the uncertainities in material properties and geometry parameters are important and should be included along with uncertainties in environmental loading when computing the failure probability. The most important source of uncertainty is, however, due to uncertainty in the environmental loading with a squared sensitivity factor around $\alpha^2 = \alpha_u^2 = 0.40$–$0.55$, meaning that 40-55% of the total uncertainty is due to this source. The second and third most important uncertainties are those related to thickness and yield stress with sensitivity factors of $\alpha^2 = 0.18$–$0.30$ and $\alpha^2 = 0.23$–$0.27$, respectively. The fourth most important uncertainty is the still water induced lateral pressure with sensitivity factor around $\alpha^2 = 0.03$. The remaining uncertainties are less important and have sensitivity factors smaller than $\alpha^2 = 0.01$.

The computed probabilities are for one year with the assumption of continuous voyage of the ship at a speed of 7 *Knots* in an environment described by the assumed wave directions and sea scatter diagram. In a real situation a ship has several voyages and stops within each year, is traveling at different speeds and in different environments. These factors should be included in the analysis before deciding that a thickness of, e.g., 25 *mm*, provides enough safety or is very conservative.

**Table 2: Failure probabilities for example plate**

| Thickness | Loading uncertainty only | All uncertainties |
|---|---|---|
| 0.0150 | 0.959E-01 | 0.152E+00 |
| 0.0175 | 0.213E-01 | 0.523E-01 |
| 0.0225 | 0.669E-03 | 0.170E-01 |
| 0.0250 | 0.340E-05 | 0.811E-03 |

## SUMMARY AND CONCLUSIONS

A probabilistic method for elastic buckling and collapse analysis for unstiffened plates under combined biaxial stress, shear stress, and lateral pressure is developed. The procedure is applied to evaluate the failure probability for a plate under still water loading and wave induced loading. The uncertainties in the wave loading and the still water induced stresses and lateral pressure as well as the uncertainties in the the geometry parameters and material properties of the plate are considered.

A formulation as a first-passage problem for a stress vector process outcrossing a safe set is first applied, accounting for uncertainties in the wave loading only and conditioning on all the other variables. This leads to the conditional failure probability. A fast integration technique based on a first-order reliability method is then applied to compute the overall failure probability.

An illustrative example analysis is performed to compute the probability of failure of a bottom plate of a ship hull. The results indicate the importance of considering the material and geometry parameter uncertainties. The most important sources of uncertainty are those related to the wave loading, plate thickness and yield stress.

The procedure may be extended to include other sources of uncertainty and can be used to develop new design formats for unstiffened as well as stiffened plates in ships and fixed and floating offshore structures. A calibration of partial safety factors and load combination factors for sectional forces and material properties may be derived by applying the method.

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