Application of Reliability Assessment Methods to Marine Frame Structures Based on Ultimate Strength Analysis

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ABSTRACT

This paper presents recent development in the reliability assessment of marine frame structures which are modeled as relatively stiff frames and subjected to quasi-static extreme loads, based on ultimate collapse analysis. At first, a linearized failure condition of the section is introduced which takes into account combined load effects of bending moment, axial force and shearing force on the various failure modes. The failure criterion greatly facilitates generation of the safety margins and calculation of the failure probabilities. Structural failure is defined as production of large deflection due to collapse. Second, the so-called branch-and-bound method combined with the heuristic operations is applied to select the probabilistically dominant failure modes, which save the computation efforts to perform the reliability analysis of large-scale structures. Finally, the proposed methods are applied to the following marine structures: (1) an offshore jacket platform with brittle elements in which the bending moment and axial force dominate the failure criterion. (2) a transverse structure of ships in which the combined effect of bending moment, shearing force and axial force determines the plasticity condition and to which some notional load conditions are applied.

INTRODUCTION

Various types of marine structures including drilling rigs, platforms, etc. have been constructed. They are required to have better operating performance in the severe state of sea and weather, and as a recent trend, they are becoming larger in size and more complex. For these marine structures, which have little experience of service, a relative measure of their safety for comparison with the notional safety of existing structures can only be found by using reliability analysis methods. Many studies have been made of reliability analysis of marine structures, as reviewed from the viewpoint of design philosophy(1). However, there remain many works to be done for large structures which have too many failure modes to identify all of them for estimating system reliability based on ultimate collapse analysis(2-11).

This paper presents recent development in the reliability assessment of marine structures which are modeled as relatively stiff frames and subjected to quasi-static extreme loads, based on ultimate collapse analysis. Ultimate collapse is evaluated by using a linearized failure condition of the section under the combined effect of bending moment, shearing force and axial force to generate the safety margins, using a matrix method. Probabilistically dominant collapse modes are selected by applying the so-called branch-and-bound method combined with the heuristic operations. These methods are applied to the following marine structures: (1) an offshore jacket platform with brittle elements in which the bending moment and axial force dominate the failure criterion. (2) a transverse structure of ships in which the combined effect of the bending moment, shearing force and axial force determines the plasticity condition and to which some notional load conditions are applied.

Through the numerical examples, the effects of brittle members, combined loads and loading conditions on the probabilistic properties of ultimate collapse of marine frame structures are investigated.

GENERATION OF STRUCTURAL FAILURE MODES FOR PLANE FRAME STRUCTURE UNDER COMBINED EFFECT OF BENDING MOMENT, SHEARING FORCE AND AXIAL FORCE

Consider a frame structure whose elements are uniform and homogeneous and to which only concentrated loads and moments are applied. In such a frame structure, critical sections where plastic modes may form are the joints of the elements and the places at which the concentrated loads are applied. The following description is concerned with the case when various failures occur
under combined load effects of bending moment, shearing force and axial force. In the case of plastic collapse, behaviour of members is approximated and structural analysis is performed by combining a plastic node method and a matrix method based on the displacement method (11-19).

Derivation of Reduced Stiffness Matrices and Equivalent Nodal Forces

Let \( \mathbf{x}_t = (X_{x1}, X_{y1}, X_{z1}, X_{x2}, X_{y2}, X_{z2})^T \)
and \( \mathbf{\delta}_t = (\delta_{x1}, \delta_{y1}, \delta_{z1}, \delta_{x2}, \delta_{y2}, \delta_{z2})^T \)
denote the nodal force and displacement vectors of the unit element \( i, j \), e.g., the element number \( t \) in the local coordinate system shown in Fig. 1(a).

When the interaction of bending moment, shearing force and axial force is considered, the yielding condition of the deep girder consisting of the transverse ring of a tanker is usually given by a nonlinear and asymmetric surface with regard to internal forces, as shown by thin lines in Fig. 2. However, in order to facilitate the treatment of yield condition, the yield surface is approximated by a linearized function resulting in underestimation of the strength of the member, as shown by thick lines in Fig. 2. Then, plasticity condition of a cross section is given in the following form:

\[
F_k = R_k - C_k^T \mathbf{x}_t = 0 \quad (k=1, \ldots, n)
\]

In eq. (1), \( R_k \) is the reference strength of the element end \( k \), which is taken to be a fully plastic moment, i.e., \( R_k = C_ykAZ_{pk} \) (plastic section modulus of element end \( k \), \( C_yk \) : yield stress).

\( C_k^T \) is a factor determined by the dimension of element \( k \). Particularly, the expression for the effect of bending moment, shearing and axial force upon the plasticity condition is given as follows:

\[
C_k^T = \begin{pmatrix}
\frac{A_2}{Apk} \cdot \text{sign}(F_{x1}) & 0 & \frac{\sqrt{A_2}}{Apk} \cdot \text{sign}(F_{y1}) & 0 \\
0 & \frac{A_2}{Apk} \cdot \text{sign}(F_{x1}) & 0 & \frac{\sqrt{A_2}}{Apk} \cdot \text{sign}(F_{y1}) \\
0 & 0 & 0 & 0 \\
\end{pmatrix}
\]

\[
C_k^T = \begin{pmatrix}
0 & 0 & 0 & \frac{A_2}{Apk} \cdot \text{sign}(F_{x1}) & 0 & \frac{\sqrt{A_2}}{Apk} \cdot \text{sign}(F_{y1}) & 0 \\
0 & 0 & 0 & 0 & \frac{A_2}{Apk} \cdot \text{sign}(F_{x1}) & 0 & \frac{\sqrt{A_2}}{Apk} \cdot \text{sign}(F_{y1}) \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{pmatrix}
\]

where

- \( A_{pk} \) : cross-sectional area of the element end
- \( AP_{pk} \) : effective sectional area of the element end for shearing force
- \( \text{sign}(.) \) : sign of (.)
- \( a, b \) : coefficient of axial force and shearing force effect, respectively
- \( c \) : coefficient of bending moment effect
The plasticity condition (1) reduces to (i) in case of \( a=0, b=0 \) and \( c=1 \) : the well-known plasticity condition subjected solely to bending moment, (ii) in case of \( a=0, b=0 \) and \( c=1 \) : the plasticity condition considering the interaction of bending moment and axial force, and (iii) in case of \( a=0, b=0 \) and \( c=1 \) : the condition considering the interaction of bending moment, shearing force and axial force.

For the case of an offshore jacket platform in which the bending moment and axial force dominate the failure criterion, the following values are adopted (11):

\[
a = 1, \quad b = 0, \quad c = 1
\]

(2)

On the other hand, for the reliability assessment of a transverse ring of a ship in which the combined effect of bending moment, shearing force and axial force determines the plasticity condition, the following values are used:

\[
a = 1, \quad b = 0.5, \quad c = 1
\]

(3)

The failure condition of the element which behaves as a brittle material, such as buckling collapse of beam-columns with initial imperfection, punching shear failure of tubular joints and brittle fracture of welding joints with fatigue cracks, is also represented by Eq. (1), where the coefficients \( a, b \) and \( c \) are given in the following:

In case of the buckling collapse:

\[
a = \frac{\phi_k}{\Delta_{ek}}
\]

\[
\Delta_{ek} = \frac{1}{2} \left[ \sigma_k + \frac{\phi_k}{\Delta_{ek}} \left( \frac{\sigma_k}{\phi_k} - \frac{\Delta_{ek}}{\phi_k} \right) \right]
\]

\[
\phi_k = \text{initial imperfection}
\]

\[
\sigma_k = \text{Euler's buckling stress}
\]

\[
S = \text{core radius}
\]

\[
b = 0
\]

\[
c = \frac{\Delta_{ek}}{\Delta_{ek}}
\]

\[
f_k = \text{shape factor}
\]

(4)

In case of the punching shear failure:

\[
a = \sqrt{3} \left( \frac{t_{ek}}{T_k} \right) \sin \theta_k
\]

\[
t_{ek} \cdot T_k = \text{brace and chord thickness}
\]

\[
\theta_k = \text{brace angle (measured from chord)}
\]

\[
b = 0
\]

(5)

In case of the brittle fracture:

\[
a = \chi_r
\]

\[
K_r = \text{stress concentration factor}
\]

\[
b = 0
\]

\[
c = K_{sk} f_k
\]

(6)

Next, the behaviour of yielded section follows the plasticity theory because the perfectly elasto-plastic (or elasto-brittle) relationship has been employed into the plasticity condition. The relation between the nodal force vector \( \mathbf{X}_L \) and the displacement vector \( \delta_c \) of an element including plastic nodes is derived by using plasticity theory as follows (11-19):

\[
\mathbf{X}_L = k_i^p \mathbf{d}_i + \mathbf{X}_p
\]

where

\[
k_i^p = \text{reduced element stiffness matrix}
\]

\[
\mathbf{X}_p = \text{equivalent nodal force vector}
\]

The explicit forms of \( k_i^p \) and \( \mathbf{X}_p \) are given as follows:

(a) In case of an elastic element:

\[
k_i^p = k_i \quad (k_i = \text{elastic element})
\]

\[
\mathbf{X}_p = 0 \quad \text{stiffness matrix}
\]

(8a)

(b) In case of failure at left-hand end: (for ductile element)

\[
k_i^p(=k_i) = k_i - k_i C_i k_i (C_i k_i)
\]

\[
\mathbf{X}_p(=\mathbf{X}) = R k_i C_i (C_i k_i)
\]

(8b)

(c) In case of failure at right-hand end: (for ductile element)

\[
k_i^p(=k_i) = k_i - k_i C_i k_i (C_i k_i)
\]

\[
\mathbf{X}_p(=\mathbf{X}) = R k_i C_i (C_i k_i)
\]

(8c)

(d) In case of failure at both ends: (for ductile element)

\[
k_i^p = 0, \quad \mathbf{X}_p = 0
\]

(8c')

219
Consider an element with rigid bodies at both ends which is idealized for a transverse ring of a ship. Let $\mathbf{\Phi}_i$ and $(\mathbf{4}_i)_x$ respectively denote the nodal force and displacement vectors of the outside of unit element $i, j$ with rigid bodies whose lengths $\ell_i$ and $\ell_j$, as shown in Fig. 1(b). By using transformation matrix $\mathbf{\Phi}_i: \mathbf{\Phi}_i = \mathbf{\Phi}_i (\mathbf{4}_i)_x$, and the relation between $\mathbf{\Phi}_i$ and $\mathbf{4}_i$ for the elastic-plastic element, the following relation is obtained:

$$\mathbf{\Phi}_i = (\mathbf{\Phi}_i^{(o)})_i (\mathbf{\Phi}_i^{(o)})_i + (\mathbf{\Phi}_i^{(e)})_i$$

where

$$(\mathbf{\Phi}_i^{(o)})_i = \mathbf{\Phi}_i \mathbf{\Phi}_i (\mathbf{4}_i)_x$$

$$(\mathbf{\Phi}_i^{(e)})_i = \mathbf{\Phi}_i (\mathbf{4}_i)_x$$

Generation of Safety Margin and Structural Failure Criterion

Consider a plane frame structure with $n$ elements and at most $32$ loads applied to its $2n$ nodes. The failure criterion of the $i$-th elastic-plastic element is given by

$$Z_i = R_i - \mathbf{C}_i^T \mathbf{X}_i \leq 0$$

Structural failure of a frame structure is defined as occurrence of large nodal displacement due to plastic collapse. A criterion for structural failure is given as in the following manner. When any one element end yields, the internal forces are redistributed to the element ends. Similarly when some element ends $r_1, r_2, \ldots, r_{p-1}$ have failed, stress analysis is performed once again and the stiffness equation of the element is replaced by the corresponding reduced one, e.g., Eq. (11) or Eq. (10). The reduced element stiffness matrices are evaluated to all the failed elements, and they are assumed to have the total structure stiffness matrix:

$$(\mathbf{K}^{(o)}) (\mathbf{d}) = (\mathbf{L}) + (\mathbf{R}^{(o)})$$

where $(\mathbf{d})$: total nodal displacement vector referred to the global coordinate system

$$(\mathbf{K}^{(o)}) = \sum_{k=1}^n \mathbf{T}_k \mathbf{C}_k \mathbf{T}_k^T$$

$$(\mathbf{L}) = \mathbf{\Phi}_k$$

$$(\mathbf{R}^{(o)}) = \sum_{k=1}^n \mathbf{T}_k \mathbf{C}_k \mathbf{T}_k^T$$

$$(\mathbf{R}^{(o)}) = \sum_{k=1}^n \mathbf{T}_k \mathbf{C}_k \mathbf{T}_k^T$$

Finally, the nodal force vector $\mathbf{X}_i$ of the $i$-th element is given by

$$\mathbf{X}_i = \mathbf{F}_i (\mathbf{L}) + (\mathbf{R}^{(o)}) + \mathbf{X}_i^{(e)}$$

where

$$\mathbf{F}_i = \mathbf{\Phi}_i (\mathbf{4}_i)_x$$

Now that the element ends $r_1, r_2, \ldots, r_{p-1}$ have failed, the safety margin of the surviving element ends (element number $j$) is obtained by substituting Eq. (11) into Eq. (11):

$$Z_j = R_j - \mathbf{C}_j^T \mathbf{X}_j \leq 0$$

where $a_{jk}$ and $b_{ij}$ are the coefficients resulted from resolution of the vectors into their components. Occurrence of large nodal displacements due to plastic collapse is determined by investigating the property of the total structure stiffness matrix $(\mathbf{K}^{(o)})$. For example, when the element ends up to some specified number, e.g., element ends $r_1, r_2, \ldots, r_{p-1}$ have failed and the reduced total structure stiffness matrix $(\mathbf{K}^{(o)})$ satisfies the following condition, structural
failure results:
\[ \left| \frac{1}{(K_{pq})_{ij}} - \frac{1}{(K_0)_{ij}} \right| \leq \varepsilon \]  
(16)

where superscripts \((pq)\) and \((0)\) are used to denote the \(pq\)-th failure stage and the elastic condition, respectively. \(\varepsilon\) is the specified constant for determining the plastic collapse. The sequence of the failed element ends to produce structural failure, e.g., \(r_1, r_2, \ldots\), \(r_{pq}\) is called a complete failure path. By using the above equation, a criterion of structural failure is given by
\[ z_{pq}^{(a)} = 0 \quad (p = 1, 2, \ldots, pq) \]  
(17)

If there are any failed element ends \(r_p\), which have their coefficients \(z_{pq}^{(a)}\) equal to zero in the safety margin \(z_{pq}\) of the last yielded element end \(r_{pq}\), they are the redundant element ends which do not directly contribute to occurrence of the plastic collapse. Alternatively, those element ends are called essential without which no plastic collapses are formed. A minimum set of plastic nodes is a failure path including no redundant plastic nodes. A failure mode is a set of plastic nodes comprising the minimum set.

In summary, the plasticity condition of the element end under the combined loads has been approximated by a linear surface given by Eq. (1), and the safety margin of the element end has also been expressed as a linear combination of the strengths of the element ends and the applied loads. Consequently, reliability analysis is greatly facilitated when the strengths and the loads are normal random variables.

**AUTOMATIC SELECTION OF PROBABILISTICALLY DOMINANT FAILURE PATHS**

There are too many failure paths in a highly redundant structure (14,17,19) to generate all of them, which necessitates a procedure for selecting only the probabilistically significant failure paths. Efficient methods by using a branch-and-bound technique have been proposed (14,16,17,19) and this paper adopts the procedure given in the following.

**Branching Operations**

These operations are to select the plastic nodes such that stochastically dominant failure paths may be obtained. An element end (called here node for simplicity) is selected as a plastic node at the \(p\)-th failure stage based on two-dimensional joint probability (so-called two-dimensional branching). The node to be selected at the \(p\)-th failure stage is given by
\[ P(Z^{(1)}_{pq} \cap Z^{(0)}_{pq}) = \max_{i_p} P(Z^{(1)}_{pq} \cap Z^{(0)}_{pq}) \]  
(19)

where \(i_p\) : the set of nodes \(i_p\) to be selected at the \(p\)-th failure stage

\[ P(Z^{(1)}_{pq} \cap Z^{(0)}_{pq}) = \max_{i_p} P(Z^{(1)}_{pq} \cap Z^{(0)}_{pq}) \]  
(20)

The joint probability is calculated with Hermite polynomial expansion method (20). By repeating the selecting process, a sequence of plastic nodes to form a plastic collapse, e.g., a complete failure path \(r_1, r_2, \ldots, r_{pq}\) is found.

The lower and upper bounds, \(P_{p_{\text{L}}}(q)(p)\) and \(P_{p_{\text{R}}}(q)(p)\), of the probability \(P_{p}(q)(p)\) of a particular partial failure path \(q\) up to the \(p\)-th failure stage is evaluated by the following formulas:
\[ P_{p_{\text{L}}}(r)(p) = \max_{(Z^{(1)}_{pq} \cap Z^{(0)}_{pq})} P(Z^{(1)}_{pq} \cap Z^{(0)}_{pq}) \]  
(21)

\[ P_{p_{\text{R}}}(r)(p) = \min_{(Z^{(1)}_{pq} \cap Z^{(0)}_{pq})} P(Z^{(1)}_{pq} \cap Z^{(0)}_{pq}) \]  
(22)

In equation (22), \(S_i\)’s designate the non-failure events \(Z^{(0)}_{pq}(1) = 0 \quad (r = 1, 2, \ldots, p)\) rearranged in the decreasing order of probabilities (20):
\[ P(S_1)P(S_2) \cdots P(S_p) \]  
(24)

Further, the following bound (21) is also applicable when all the correlation coefficients are non-negative, i.e.
for the structure with brittle members.

Heuristic Operations

The number of branchings becomes enormous for a large scale structure with high degree of redundancy, even though the branch-and-bound method is applied. To reduce the computational effort, the following heuristic operations are applied. First, the reliability assessment is performed on some structural divisions which are presumed to be critical. The lower bounds of the resulting complete failure path probabilities are used as the reference value $P_{PM}$ for bounding operations. Second, the set of nodes for branching is restricted to the nodes which satisfy the monotony conditions of the failure probabilities. That is

$$l_i = \{ p | z_p^{(q)} \leq l_r \}$$ for $q \neq 1$.

Third, the contribution of the first plastic node is taken into account:

$$l_i = \{ p | p_i \geq a_1 \}$$

Fourth, the number of branchings from one failure stage is restricted to a specified number $a_1$.

APPLICATION TO MARINE FRAME STRUCTURES

The above method is applied to a jacket-type offshore platform with brittle members and a transverse structure of three types of ships. The former is given mainly to show the property on behaviour of the structure with brittle members and the latter is chosen to study the combined load effect and loading condition on the probabilistic collapse analysis. All the random variables are assumed to be normally distributed.

Jacket Structure

A jacket-type offshore platform shown in Fig. 3 is considered. The dimensions and strengths of members are shown in Table 1 and it is assumed that the strengths of the nodes in the same elements are completely dependent normal random variables. The moment mean values of the extreme wave loads are given in Fig. 3 and their coefficients of variation are 0.30. The brace elements are assumed to behave like brittle or ductile truss elements. The plasticity condition takes account of the combined load effect of bending moment and axial force ($c_1=1, c_2=1$ and $c_3=1$). The results are listed in Tables II and III.
Fig. 3 Jacket-type structure

Table 1 Numerical data of the jacket-type structure

<table>
<thead>
<tr>
<th>Element number</th>
<th>Cross sectional area ( A_{pt} )</th>
<th>Moment of inertia ( I_{p} )</th>
<th>Mean value of reference strength ( P_{r} ) kNm</th>
</tr>
</thead>
<tbody>
<tr>
<td>1, 2</td>
<td>( 5.37 \times 10^{-2} )</td>
<td>( 2.188 \times 10^{-3} )</td>
<td>2536.6</td>
</tr>
<tr>
<td>3, 4</td>
<td>( 2.5 \times 10^{-2} )</td>
<td>( 1.055 \times 10^{-3} )</td>
<td>1467.9</td>
</tr>
<tr>
<td>5, 6</td>
<td>( 3.73 \times 10^{-2} )</td>
<td>( 1.660 \times 10^{-3} )</td>
<td>2062.3</td>
</tr>
<tr>
<td>7, 8</td>
<td>( 4.68 \times 10^{-2} )</td>
<td>( 1.782 \times 10^{-3} )</td>
<td>2174.8</td>
</tr>
<tr>
<td>9, 10, 11, 12</td>
<td>( 1.0 \times 10^{-3} )</td>
<td>( 3.40 \times 10^{-3} )</td>
<td>30.63</td>
</tr>
<tr>
<td>13, 14, 19, 20</td>
<td>( 1.05 \times 10^{-3} )</td>
<td>( 1.05 \times 10^{-3} )</td>
<td>20.29</td>
</tr>
<tr>
<td>15, 16, 17, 18</td>
<td>( 9.30 \times 10^{-3} )</td>
<td>( 9.30 \times 10^{-3} )</td>
<td>183.5</td>
</tr>
</tbody>
</table>

Young's modulus \( E = 210 \) GPa
Mean value of yield stress \( \sigma_{yi} = 276 \) kPa
Correlation coeff. \( \rho_{Li,Lj} = 0.0 \)

The strengths of the element ends in the same elements are completely dependent normal random variables.
Table II Calculated results of the jacket-type structure with brittle braces

\[ \gamma_1 = \gamma_2 = \gamma = 3.0, \varepsilon = 0.001, CV_{R_c}/CV_{L_d} = 0.15/0.30 \]

<table>
<thead>
<tr>
<th>Type</th>
<th>Failure path (Brittle brace)</th>
<th>Failure probability **</th>
<th>Collapse type</th>
</tr>
</thead>
<tbody>
<tr>
<td>A-1</td>
<td>15+17+10+9+12+11</td>
<td>0.2132x10^-3</td>
<td></td>
</tr>
<tr>
<td>A-2</td>
<td>15+17+9+12+10+20+8</td>
<td>0.6740x10^-5</td>
<td></td>
</tr>
<tr>
<td>A-3</td>
<td>others [13]*</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B-1</td>
<td>15+17+10+9+12+20+7+23+11</td>
<td>0.4085x10^-6</td>
<td></td>
</tr>
<tr>
<td>B-2</td>
<td>15+17+9+12+10+20+7+23+8</td>
<td>0.6740x10^-5</td>
<td></td>
</tr>
<tr>
<td>B-3</td>
<td>others [4]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C-1</td>
<td>17+15+10+9+12+11</td>
<td>0.1881x10^-4</td>
<td></td>
</tr>
<tr>
<td>C-2</td>
<td>17+15+9+12+11+13</td>
<td>0.4085x10^-6</td>
<td></td>
</tr>
<tr>
<td>C-3</td>
<td>others [9]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>D-1</td>
<td>21+23+(5,6,7,8)**</td>
<td>0.1881x10^-4</td>
<td></td>
</tr>
</tbody>
</table>

Computation time (sec) 95.2

* Criterion of structural failure is based on singularity of reduced total structure stiffness matrix.

* The figure in brackets designates the number of selected failure paths.

** The figures in parenthesis designate the element end with \( \frac{Z_p}{\pi_p} \leq 0 \).

\( ^{5} \) The element end whose failure probability is the smallest.

\[ P_{f_q} = \min_{p \in \{I, 2, \ldots, P_q\}} P \left[ \frac{Z_{P_q}}{\pi_p} \leq 0 \right] \]

Heuristic parameter: \( (a_1, a_2, a_3) = (1.1, 0.0, 2) \)

Initial reference value: \( P_{f_{PM}} = 0.1825 \times 10^{-3} ; 15+17+10+9+12+11 \)
Table III Calculated results of the jacket-type structure with ductile braces

\[ \gamma_1 = \gamma_2 = \gamma = 3.0, \ c^* = 0.001, \ CV_{R_e}/CV_{E_j} = 0.15/0.30 \]

<table>
<thead>
<tr>
<th>Type</th>
<th>Failure mode (Ductile brace)</th>
<th>Failure probability #</th>
<th>Collapse type (Collapse mode)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A-1</td>
<td>(15,17,10,9,12,11) [5]</td>
<td>0.7861 \times 10^{-8}</td>
<td>A-1, A-2</td>
</tr>
<tr>
<td>A-2</td>
<td>(15,17,10,9,11,14) [4]</td>
<td>0.4751 \times 10^{-9}</td>
<td></td>
</tr>
<tr>
<td>A-3</td>
<td>(15,17,9,12,11,13) [2]</td>
<td>0.1879 \times 10^{-9}</td>
<td></td>
</tr>
<tr>
<td>A-4</td>
<td>(15,17,10,9,12,20,8) [1]</td>
<td>0.3030 \times 10^{-10}</td>
<td></td>
</tr>
<tr>
<td>A-5</td>
<td>(15,17,9,11,14,13) [2]</td>
<td>0.2564 \times 10^{-10}</td>
<td></td>
</tr>
<tr>
<td>B-1</td>
<td>(21,23,5,6,7,8) [15]</td>
<td>0.2347 \times 10^{-8}</td>
<td>A-3, A-4</td>
</tr>
<tr>
<td>C-1</td>
<td>(27,29,3,1,4,2) [1]</td>
<td>0.1214 \times 10^{-10}</td>
<td></td>
</tr>
</tbody>
</table>

Computation time (sec) 660.6

# Criterion of structural failure is based on singularity of reduced total structure stiffness matrix.

* The figure in brackets designates the number of selected failure paths.

## \[ P_{f_q} = \min_{p \in \{1,2,\ldots,p_q\}} P \left[ Z_{p,q}(p) \leq 0 \right] \]

\[ P_{f_p} = P \left[ Z_{p,q}(p) \leq 0 \right] \]

Heuristic parameter \( (a_1, a_2, a_3) = (1.1, 0.0, 2) \)

Initial reference value \( P_{f_p} = 0.6017 \times 10^{-8} \); 15+17+10+9+12+11

For the case of brittle and ductile braces, respectively.

It is seen that the dominant failure modes of both cases are essentially similar, which is formed by failure of the columns and braces in the top story. However, the probabilities of occurrence for the case of brittle braces are very large. Moreover, it is seen from Table II that element end 15 is a critical brittle one whose failure triggers a chain-reaction failure resulting in a total collapse.

Transverse Ring of Ships

Fig. 4 shows a plane frame structure which is modelled for a transverse ring of a medium size tanker under four notional load conditions. The probabilistic analysis of plastic collapse is carried out for the numerical data of
the structure given in Fig. 4, Tables IV
and V. The applied loads are estimated,
based on the load condition for the
direct calculation suggested by the
Japan Classification Society of Ships
(NK), and the lengths of rigid bodies
are estimated, using "the span point for
bending" given in Ref. (22). It is
assumed that the strengths of the nodes
and the applied loads are mutually
independent normal random variables.

The results for the loading condi-
tion "case 1-1" are listed in Table VI.
The first column indicates the selected
failure paths. In the second column,
probabilities of occurrence of the failure
paths are given when the combined
effect of bending moment, shearing
force and axial force (a=1, b=0.5, c=1)
is considered. The numbers in brackets are
those of the selected failure paths. The
third column shows those corresponding
to the case where the combined effect
of bending moment, axial moment and axial
force is considered (a=1, b=0, c=1). The fourth
column shows those corresponding to the
case where only bending moment effect is
considered (a=0, b=0, c=1). Further, collapse
mode are given in the fifth column.
In each column the number in paren-
theses indicates the central safety
factor corresponding to each failure path:

\[ SF = \left( \frac{P_k}{R_k} \right)^{\frac{1}{3} \frac{1}{3} \frac{1}{3}} \left( \frac{a_{ik} P_k}{\sum_{i=1}^{n} b_i P_i} \right) \]  (32)

It is seen from the table that the
dominant failure mode of each case is
essentially similar, which is formed by
failure of wing tank. However, the prob-
abilities of occurrence with combined
effect considered are very large, as
seen in the failure paths A-1 and A-3 of
the table. Moreover, it is seen from
comparison between the safety factor and
failure probabilities having the same
failure path, that the deterministically
dominant collapse is not always stochas-
tically relevant.

Finally, Table VII shows the most
dominant collapse mode based on probabi-
listic analysis for the transverse ring
of the tanker under some notional load
conditions. It is seen from this table
that full load condition with empty
centre tank "case 1-1" is the severest.
In Table VII, the dominant failure modes
are also given for two other types of
ships, i.e., a tanker(DW 240,000) and
an oce carrier(DW 50,000), which con-
figurations and numerical data are shown
in Fig. 5 and Tables VII, and Fig. 6
and Table IX, respectively.

CONCLUDING REMARKS

The methods are presented for the
reliability assessment of marine struc-
tures, which are modeled as relatively
stiff frames and subjected to quasi-
static extreme loads, based on the utti-
mate collapse analysis. The methods are
applied to the offshore jacket plat-
form with brittle members and the
transverse ring of ships under some
notional load conditions. For the latter
example, the effect of the brittle mem-
ber on probabilistic collapse of the
jacket platform is discussed. For the
latter example, effects of combined
loads and notional load conditions on
probabilistic properties of the plastic
collapse are discussed.

Although this paper is concerned
with the case where the structural
system is idealized as plane frame struc-
tures, it is possible for this method
to be extended to reliability analysis
of spatial frame structures by incorpo-
rating the terms of biaxial bending mo-
ment, torsional moment, etc. in the
plasticity condition of the equation
(1).

ACKNOWLEDGEMENTS

The authors give their appreciation
to Dr. M. Kishi and Dr. S. Matsuzaaki
for their help in numerical calculation.

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siens, J. and Vrouwenvelde, A., "Method-
ologies for Ultimate Limit State Relia-
bility Analysis of Offshore Jacket Plat-
Fig. 4 Transverse ring of tanker 1 (DW 60,000t)

Table IV Numerical data of tanker 1

<table>
<thead>
<tr>
<th>Element end sectional number</th>
<th>Cross sectional area $A_{le}$ $m^2$</th>
<th>Cross sectional area of web $A_{w}$ $m^2$</th>
<th>Moment of inertia $I_{le}$ $m^4$</th>
<th>Mean value of yield stress of reference body $\overline{\sigma}_{le}$ $kN/m^2$</th>
<th>Length of body $s_{le}$ $m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1, 2</td>
<td>0.126</td>
<td>0.047</td>
<td>0.187</td>
<td>38950.0</td>
<td>1.0 4.2</td>
</tr>
<tr>
<td>3, 4</td>
<td>0.114</td>
<td>0.037</td>
<td>0.111</td>
<td>25640.0</td>
<td>2.5 2.5</td>
</tr>
<tr>
<td>5, 6</td>
<td>0.088</td>
<td>0.024</td>
<td>0.042</td>
<td>12850.0</td>
<td>3.1 1.1</td>
</tr>
<tr>
<td>7, 8</td>
<td>0.088</td>
<td>0.024</td>
<td>0.042</td>
<td>12850.0</td>
<td>1.1 2.4</td>
</tr>
<tr>
<td>9, 10</td>
<td>0.100</td>
<td>0.024</td>
<td>0.043</td>
<td>12910.0</td>
<td>2.4 2.3</td>
</tr>
<tr>
<td>11, 12</td>
<td>0.100</td>
<td>0.025</td>
<td>0.044</td>
<td>12700.0</td>
<td>2.4 0.0</td>
</tr>
<tr>
<td>13, 14</td>
<td>0.078</td>
<td>0.026</td>
<td>0.040</td>
<td>13490.0</td>
<td>2.4 1.1</td>
</tr>
<tr>
<td>15, 16</td>
<td>0.088</td>
<td>0.026</td>
<td>0.043</td>
<td>13540.0</td>
<td>1.1 6.0</td>
</tr>
<tr>
<td>17, 18</td>
<td>0.033</td>
<td>0.019</td>
<td>0.013</td>
<td>6730.0</td>
<td>1.8 1.9</td>
</tr>
</tbody>
</table>

Young's modulus $E = 210$ GPa
Mean value of yield stress $\overline{\sigma}_{le} = 353$ MPa
Coeff. of variation of yield stress $CV_{\sigma_{le}} = 0.05$
Table V: Notional load conditions for the transverse ring of tanker 1

<table>
<thead>
<tr>
<th>Load no.</th>
<th>Case 1-1 (Full loaded conditions)</th>
<th>Case 1-2</th>
<th>Case 1-3</th>
<th>Case 1-4 (Ballast conditions)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Empty centre tank</td>
<td>Empty wing tanks</td>
<td>Empty centre tank</td>
<td>Empty wing tanks</td>
</tr>
<tr>
<td>$L_1$</td>
<td>2800.0</td>
<td>-1070.0</td>
<td>1610.0</td>
<td>-2270.0</td>
</tr>
<tr>
<td>$L_2$</td>
<td>-2970.0</td>
<td>3070.0</td>
<td>-2970.0</td>
<td>3070.0</td>
</tr>
<tr>
<td>$L_3$</td>
<td>1910.0</td>
<td>1520.0</td>
<td>-390.0</td>
<td>-780.0</td>
</tr>
<tr>
<td>$L_4$</td>
<td>820.0</td>
<td>-2150.0</td>
<td>1570.0</td>
<td>-1400.0</td>
</tr>
<tr>
<td>$L_5$</td>
<td>-884.0</td>
<td>2600.0</td>
<td>-1940.0</td>
<td>-1490.0</td>
</tr>
<tr>
<td>$L_6$</td>
<td>590.0</td>
<td>-2970.0</td>
<td>2120.0</td>
<td>-1450.0</td>
</tr>
<tr>
<td>$L_7$</td>
<td>-1010.0</td>
<td>-550.0</td>
<td>450.0</td>
<td>0.0</td>
</tr>
<tr>
<td>$L_8$</td>
<td>-450.0</td>
<td>500.0</td>
<td>-450.0</td>
<td>500.0</td>
</tr>
<tr>
<td>$L_9$</td>
<td>-3570.0</td>
<td>3750.0</td>
<td>-3570.0</td>
<td>3750.0</td>
</tr>
</tbody>
</table>

* These values denote the mean values of loads. Coefficients of variation of loads $CV_{L_j}=0.30$ ($j=1,2,...,9$).

Table VI: Failure modes and their probabilities of occurrence for the transverse ring of tanker 1 in the notional load condition "case 1-1"

$\gamma_1 = \gamma_2 = \gamma_4 = 1.0, \ c=0.001, CV_{Rj}/CV_{Lj}=0.05/0.3$

<table>
<thead>
<tr>
<th>No.</th>
<th>Failure paths</th>
<th>Bending moment, axial force and shearing force interaction considered ($a=1,b=0$)</th>
<th>Bending moment and axial force interaction only considered ($a=1,b=0$)</th>
<th>Collapse mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>A-1</td>
<td>(17,18,19,20,21,22,23,24,25,26)</td>
<td>7) $0.3495 \times 10^{-2}$ (38) 7,13 0.1193 $10^{-6}$ (68) 7) $0.1250 \times 10^{-9}$ (118)</td>
<td>(2.730) (2.433) (1.614)</td>
<td>—</td>
</tr>
<tr>
<td>-2.</td>
<td>—</td>
<td>15) $0.3463 \times 10^{-2}$ (75) 7) $0.7950 \times 10^{-7}$ (69)</td>
<td>(2.454) (1.614)</td>
<td>—</td>
</tr>
<tr>
<td>-3.</td>
<td>(17,18,19,20,21,22,23,24,25,26)</td>
<td>15) $0.2760 \times 10^{-2}$ (1) 7) $0.1282 \times 10^{-7}$ (42) 7,13 0.8129 $10^{-11}$ (7)</td>
<td>(2.564) (1.614) (1.736) (2.582)</td>
<td>—</td>
</tr>
<tr>
<td>-4.</td>
<td>—</td>
<td>7) $0.2617 \times 10^{-2}$ (38) 13,15 0.1376 $10^{-10}$ (5)</td>
<td>(2.582) (1.614) (1.736) (2.582)</td>
<td>—</td>
</tr>
<tr>
<td>-5.</td>
<td>(17,18,19,20,21,22,23,24,25,26)</td>
<td>7) $0.1395 \times 10^{-2}$ (22)</td>
<td>(1.798)</td>
<td>7) $0.4844 \times 10^{-11}$ (39)</td>
</tr>
<tr>
<td>-6.</td>
<td>others</td>
<td>$&lt; 0.11 \times 10^{-2}$ (107)</td>
<td>$&lt; 0.25 \times 10^{-7}$ (109)</td>
<td>(1.700)</td>
</tr>
<tr>
<td>B-1</td>
<td>(17,18,19,20,21,22,23,24,25,26)</td>
<td>15,16 0.6399 $10^{-6}$ (1)</td>
<td>—</td>
<td>1,16 0.1001 $10^{-16}$ (6)</td>
</tr>
<tr>
<td>-2.</td>
<td>—</td>
<td>7) $0.5683 \times 10^{-6}$ (1)</td>
<td>(2.001)</td>
<td>—</td>
</tr>
<tr>
<td>-3.</td>
<td>(17,18,19,20,21,22,23,24,25,26)</td>
<td>—</td>
<td>—</td>
<td>1,13 0.1465 $10^{-14}$ (2)</td>
</tr>
<tr>
<td>-4.</td>
<td>(17,18,19,20,21,22,23,24,25,26)</td>
<td>—</td>
<td>—</td>
<td>7) $0.2184 \times 10^{-16}$ (19)</td>
</tr>
<tr>
<td>C-1</td>
<td>(17,18,19,20,21,22,23,24,25,26)</td>
<td>—</td>
<td>—</td>
<td>7) $0.4884 \times 10^{-11}$ (13)</td>
</tr>
</tbody>
</table>

Total number of selected paths | 292 | 233 | 294 |
Computation time (sec) | 102.2 | 88.2 | 85.6 |

* Central safety factor: $\phi_{P_k} = \left( \frac{P_0^{-1} \prod_i p_{R_k} R_{k_i}}{\sum_{j=1}^{m} \prod_{i=1}^{n} R_{k_i}^j} \right)^{\frac{1}{m}}$
<table>
<thead>
<tr>
<th>Type of ships</th>
<th>Type of structures and notional load conditions</th>
<th>The most dominant collapse mode and its probability of occurrence</th>
<th>Coeff. of axial and shearing force effect</th>
<th>Failure probability</th>
<th>The most dominant collapse mode</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Tanker 1</strong></td>
<td>Full loaded conditions</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Case 1-1</td>
<td></td>
<td>[0.3495] × 10^-2 (1.71)</td>
<td>0.1193 × 10^-5 (2.44)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Case 1-2</td>
<td></td>
<td></td>
<td>&lt; 0.1 × 10^-18</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Case 1-3</td>
<td></td>
<td></td>
<td>0.2808 × 10^-16</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Case 1-4</td>
<td></td>
<td></td>
<td>0.1057 × 10^-5</td>
<td></td>
</tr>
<tr>
<td><strong>Tanker 2</strong></td>
<td>Full loaded conditions</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Case 2-1</td>
<td></td>
<td>[0.3886] × 10^-4</td>
<td>0.1640 × 10^-10</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Case 2-2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Oil carrier</strong></td>
<td>Full loaded conditions</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Case 3-1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Case 3-2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*The value in parentheses indicates the safety factor given in Eq. (22).*
Table VIII Numerical data of tanker 2

<table>
<thead>
<tr>
<th>Element number</th>
<th>Cross sectional area $A_t$ m$^2$</th>
<th>Cross sectional area of web $A_{wt}$ m$^2$</th>
<th>Moment of inertia $I_t$ m$^4$</th>
<th>Mean value of reference strength $R_t$ kN</th>
<th>Length of rigid body $s_t$ m</th>
</tr>
</thead>
<tbody>
<tr>
<td>1, 2</td>
<td>0.183</td>
<td>0.064</td>
<td>0.438</td>
<td>59200.0</td>
<td>2.8 4.2</td>
</tr>
<tr>
<td>3, 4</td>
<td>0.176</td>
<td>0.054</td>
<td>0.255</td>
<td>43100.0</td>
<td>2.8 2.9</td>
</tr>
<tr>
<td>5, 6</td>
<td>0.135</td>
<td>0.045</td>
<td>0.156</td>
<td>29400.0</td>
<td>3.1 1.0</td>
</tr>
<tr>
<td>7, 8</td>
<td>0.135</td>
<td>0.045</td>
<td>0.156</td>
<td>29400.0</td>
<td>1.0 1.0</td>
</tr>
<tr>
<td>9, 10</td>
<td>0.126</td>
<td>0.035</td>
<td>0.147</td>
<td>25900.0</td>
<td>1.0 3.0</td>
</tr>
<tr>
<td>11, 12</td>
<td>0.136</td>
<td>0.030</td>
<td>0.076</td>
<td>14600.0</td>
<td>3.0 2.9</td>
</tr>
<tr>
<td>13, 14</td>
<td>0.137</td>
<td>0.030</td>
<td>0.079</td>
<td>15100.0</td>
<td>2.2 0.0</td>
</tr>
<tr>
<td>15, 16</td>
<td>0.089</td>
<td>0.035</td>
<td>0.117</td>
<td>24800.0</td>
<td>3.0 0.9</td>
</tr>
<tr>
<td>17, 18</td>
<td>0.099</td>
<td>0.035</td>
<td>0.127</td>
<td>25700.0</td>
<td>0.9 1.0</td>
</tr>
<tr>
<td>19, 20</td>
<td>0.116</td>
<td>0.045</td>
<td>0.142</td>
<td>29200.0</td>
<td>1.0 5.1</td>
</tr>
<tr>
<td>21, 22</td>
<td>0.043</td>
<td>0.018</td>
<td>0.016</td>
<td>6650.0</td>
<td>2.3 2.3</td>
</tr>
<tr>
<td>23, 24</td>
<td>0.056</td>
<td>0.025</td>
<td>0.026</td>
<td>9700.0</td>
<td>2.3 2.3</td>
</tr>
</tbody>
</table>

Young's modulus $E = 210$ GPa
Mean value of yield stress $f_y = 276$ MPa
Table IX Numerical data of the ore carrier

<table>
<thead>
<tr>
<th>Element number</th>
<th>Cross sectional area of web $A_{\text{w}}$ (m$^2$)</th>
<th>Cross sectional area of flange $A_{\text{fl}}$ (m$^2$)</th>
<th>Moment of inertia $I$ (m$^4$)</th>
<th>Mean value of yield stress $f_Y$ (MPa)</th>
<th>Length of rigid body $L$ (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1, 2</td>
<td>0.133</td>
<td>0.044</td>
<td>0.045</td>
<td>13500.0</td>
<td>1.3</td>
</tr>
<tr>
<td>3, 4</td>
<td>0.133</td>
<td>0.044</td>
<td>0.045</td>
<td>13500.0</td>
<td>1.7</td>
</tr>
<tr>
<td>5, 6</td>
<td>0.158</td>
<td>0.046</td>
<td>0.035</td>
<td>12300.0</td>
<td>2.0</td>
</tr>
<tr>
<td>7, 8</td>
<td>0.158</td>
<td>0.046</td>
<td>0.035</td>
<td>12300.0</td>
<td>0.5</td>
</tr>
<tr>
<td>9, 10</td>
<td>0.158</td>
<td>0.046</td>
<td>0.035</td>
<td>12300.0</td>
<td>0.5</td>
</tr>
<tr>
<td>11, 12</td>
<td>0.166</td>
<td>0.044</td>
<td>0.027</td>
<td>10500.0</td>
<td>1.6</td>
</tr>
<tr>
<td>13, 14</td>
<td>0.055</td>
<td>0.017</td>
<td>0.013</td>
<td>4400.0</td>
<td>1.4</td>
</tr>
<tr>
<td>15, 16</td>
<td>0.080</td>
<td>0.024</td>
<td>0.019</td>
<td>6400.0</td>
<td>0.5</td>
</tr>
<tr>
<td>17, 18</td>
<td>0.080</td>
<td>0.024</td>
<td>0.019</td>
<td>6400.0</td>
<td>0.5</td>
</tr>
<tr>
<td>19, 20</td>
<td>0.130</td>
<td>0.047</td>
<td>0.031</td>
<td>10600.0</td>
<td>1.6</td>
</tr>
<tr>
<td>21, 22</td>
<td>0.046</td>
<td>0.015</td>
<td>0.059</td>
<td>4300.0</td>
<td>1.3</td>
</tr>
<tr>
<td>23, 24</td>
<td>0.050</td>
<td>0.017</td>
<td>0.066</td>
<td>4700.0</td>
<td>1.3</td>
</tr>
<tr>
<td>25, 26</td>
<td>0.098</td>
<td>0.024</td>
<td>0.013</td>
<td>5350.0</td>
<td>1.4</td>
</tr>
<tr>
<td>27, 28</td>
<td>0.041</td>
<td>0.011</td>
<td>0.023</td>
<td>1750.0</td>
<td>1.2</td>
</tr>
</tbody>
</table>

Young's modulus $E = 210$ GPa
Mean value of yield stress $f_Y = 276$ MPa

Fig. 6 Transverse structure of an ore carrier (DW 50,000~)

Case 3-1: Wing tank fully loaded
Case 3-2: Main tank fully loaded

232