THE SOCIETY OF NAVAL ARCHITECTS AND MARINE ENGINEERS
601 Pavonia Avenue, Suite 400, Jersey City, New Jersey 07306 USA
Paper presented at the Marine Structural Inspection, Maintenance, and Monitoring Symposium
Sheraton National Hotel, Arlington, Virginia, March 18-19, 1991

Probability-Based Cost Benefit Analysis of Fatigue Design, Inspection and Maintenance

H.O. Madsen, Det norske Veritas, Copenhagen, Denmark
R. Torhaug, A.S. Veritas Research, Høvik, Norway
E. H. Cramer, University of California, Berkeley, California

ABSTRACT

Safety of marine structures against fatigue failure is achieved through design of individual elements, utilization of structural redundancy, and inspection for fatigue cracks with subsequent repair of detected cracks. Each safety item has a certain cost and it is of importance to minimize the total expected cost for the life time of the structure. Four different repair strategies are compared and the total expected cost of design, inspection, repair and failure is minimized. The optimization parameters are a stress related design parameter, inspection times, and inspection qualities.

1. INTRODUCTION

Safety of marine structures against fatigue failure is an important design consideration. Sufficient safety is achieved through the use of several safety items: design of individual elements, utilization of structural redundancy, and inspection for fatigue cracks with subsequent repair of detected cracks. Each safety item has a certain cost and it is of importance to minimize the total expected cost for the life time of the structure. The optimization must be carried out with the at any time available information. At the design stage the system configuration is decided, the sizing of the individual elements and joints is performed, and the initial inspection plan is prepared. The cost considered in the optimization at this time is cost related to structural parameters, cost of inspection, expected cost of repair, and expected cost of failure. After fabrication and installation, new information about the as-built configuration and from fabrication control becomes available. With this information an updated initial inspection plan can be determined. The cost considered in the optimization at this time is related to cost of inspection, expected repair cost and expected failure cost. The first inspection may result in the detection and also possible repair of a crack, or no crack may be detected. With this additional information an updated inspection plan is prepared. The cost considered at this time is the inspection cost for the remaining inspections, the expected repair cost and the expected failure cost. After the next inspection a new optimization is done and so on. Although a full inspection plan is determined at each step, it is thus only the first inspection which is actually carried out according to the plan.

In the optimization a strategy for repair is necessary, and four different strategies are considered here.

- all detected cracks are repaired by welding,
- only detected cracks larger than a certain size are repaired (by welding),
- all detected cracks are repaired. Cracks smaller than a certain size are repaired by grinding, while cracks larger than this size are repaired by welding,
- all detected cracks are repaired by replacement of the element.

The system reliability aspects, i.e. the effect of redundancy, should be treated by considering the changes in load paths when large cracks develop. Another system aspect concerns the updating of the reliability for one part of the structure based on inspection results for another part of the structure. Such analyses capabilities are in principle simple extensions of the present analysis, but may cause large computational difficulties.

This paper gives a contribute to mathematical modeling of the design, inspection and maintenance optimization. A number of more practical aspects for inspection planning have not been included, while the paper attempts to be rigorous on crack geometry modeling, reliability and optimization analysis. The concern is on the crack growth phase with little emphasis on the crack initiation phase. As such the analysis is more relevant to structural than mechanical parts.

The paper first presents the applied fatigue crack growth model based on a fracture mechanics analysis. The necessary input for the loading, the geometry and the material properties are identified. Corrosion is included through a reduction in plate thickness with time. The four repair criteria are presented, and associated with each repair criterion is an event tree giving the possible events from design until the end of the design life time. The various safety and event margins for the different branches of one of the defined event trees are formulated. The associated failure and repair probabilities are computed by first-order reliability methods. In the event margins a smallest detectable crack size or crack detection threshold appears. This crack size is specific for each inspection method and its reliability. Inspection reliability in terms of a probability of detection curve is treated. Modeling of the various cost items is described and the optimization problem is formulated for optimization at the time of initial design. The optimization variables are the number, quality and times of inspection, and a structural design parameter. The objective function giving the total expected cost is derived, and constraints on the reliability as well as simple constraints on the optimization variables are formulated. The focus is next on the optimization for structures in service. The structural design parameters are then fixed and some inspection/repair results may be available. The optimization problem is formulated and the objective function expresses the total expected cost for the remaining of the design life time.

Some example results are presented comparing the four strategies for one case and considering one strategy in more detail for another case. Results are presented for two different classes of POD curves, inspection quality models. Both a constant geometry function and a transverse stiffener weld geometry function are applied. A parameter study is presented both for the crack size limit for which weld repair is performed and for the expected cost of failure.

2. CRACK GROWTH MODEL AND CORROSION MODEL

A one-dimensional description of crack size is employed. Crack growth is described by Paris' equation with the stress intensity factor calculated by linear elastic fracture mechanics

$$\frac{da}{dN} = C(\Delta K)^m, \quad \Delta K > \Delta K_{thr}, \quad a(N=0)=a_0 \tag{2.1}$$

The left hand side gives the crack size increment in one stress cycle with stress intensity factor range ΔK. a is the crack size, N is the number of cycles and C and m are material constants. ΔK is expressed as

$$\Delta K = Y(a)\sqrt{\pi a}\, S \tag{2.2}$$

where $Y(a)$ is the geometry function depending on the overall geometry of the detail including the presence of the weld, and S is the range of a far-field reference stress.

Although a one-dimensional description of crack size is employed above, a two dimensional description can easily be used as well. Instead of solving one differential equation (2.1), it is necessary to solve two coupled differential equations.

$$\begin{cases} \dfrac{da}{dN} = C(\Delta K_a)^m, \quad \Delta K_a > \Delta K_{thr}, \quad a(N=0)=a_0 \\ \dfrac{dc}{dN} = C(\Delta K_c)^m, \quad \Delta K_c > \Delta K_{thr}, \quad c(N=0)=c_0 \end{cases} \tag{2.3}$$

where the first equation describes the growth in depth a and the second equation describes the growth in length $2c$ of a semi-elliptical surface crack. When $\Delta K_{thr}=0$ the two equations are conveniently rewritten as

$$\begin{cases} \dfrac{dc}{da} = \left(\dfrac{\Delta K_c}{\Delta K_a}\right)^m, \quad c(a=a_0)=c_0 \\ \dfrac{dN}{da} = \dfrac{1}{C}(\Delta K_a)^{-m}, \quad N(a=a_0)=0 \end{cases} \tag{2.4}$$

The differential equations are coupled since ΔK_a and ΔK_c both depend on (a,c). With ΔK_a from (2.2) and a similar expression for ΔK_c, the first equation in (2.4) becomes

$$\frac{dc}{da} = \left(\frac{Y_c(a,c)}{Y_a(a,c)}\frac{\sqrt{c}}{\sqrt{a}}\right)^m, \quad c(a=a_0)=c_0 \tag{2.5}$$

This equation does not involve the loading and can be solved to give the crack length as a function of depth for given geometry functions and initial condition. The result can be inserted into the second equation in (2.4) which is then simply an equation for the growth in crack depth identical to (2.1). Alternatively, a differential equation for the aspect ratio a/c can be formulated with its initial condition and the result for the aspect ratio be inserted in (2.4). Both two-dimensional crack size analyses require more computer time than the one-dimensional analysis, but the extension is necessary in many cases in particular when the inspection result is on crack length without depth information.

It is also possible to include more complicated crack growth descriptions than the semi-elliptical surface crack, e.g. crack growth of a semi-elliptical surface crack through the thickness followed by further crack growth of the through-thickness crack.

A crack initiation period is not included in the formulation above. This is easily done by changing the initial condition $N(a_0)=0$ to $N(a_0)=N_0$. A separate stochastic model for N_0 can then be formulated. Alternatively a_0 can be considered as an equivalent initial crack size as is commonly done within analysis of aircraft structures.

A Weibull distribution is often applied to express the long term stress range distribution for marine structures. Here a Weibull distribution with random scale parameter A and shape parameter B is used.

$$F_S(s) = 1-\exp(-(s/A)^B), \quad s>0 \tag{2.6}$$

The number of stress cycles per unit time is ν, and a joint normal distribution is assumed for $(\ln A, 1/B)$. The uncertainty in the Weibull parameters is a lumped representation of the uncertainties in the long term characterization of the environmental conditions, in the load models, in the global response analysis, and in the calculation of the local reference stress.

A structural design parameter z is introduced later in the formulation of the optimization problem. For a ship structure, this structural design parameter could typically represents the hull thickness or the spacing between stiffeners. The base value of z is z_0 for which the Weibull parameters A and B have been determined. When z varies from z_0 each stress range is multiplied by the factor s_z which is selected of the form

$$s_z = \left(c_z\frac{z_0}{z}+(1-c_z)\left(\frac{z_0}{z}\right)^2\right), \quad z^{min}\le z\le z^{max}, \quad 0\le c_z\le 1 \tag{2.7}$$

This function is assumed to be able to model stress variations well in all cases.

Due to corrosion the thickness may decrease with time. The following liear model is introduced for the thickness at time t

$$z(t) = z-k_z t \tag{2.8}$$

where k_z is the (random) corrosion rate.

3. STRATEGIES FOR REPAIR

The optimization is carried out without knowledge of the actual outcome of future inspections. For an optimization carried out either at the design state or in-service, it is thus necessary to consider all possible outcomes of future inspections, repairs and possible failure. If crack sizes are measured in an inspection, the number of possible outcomes becomes infinite, thus making the optimization extremely complicated. To overcome this problem, a finite set of possible outcomes must be defined. This can be done by only referring crack sizes to a finite number of intervals. In the strategies considered here, this number of intervals is limited to two or three. The limiting crack sizes can be random.

Four different strategies are considered.

STRATEGY-1 all detected cracks are repaired by welding.

STRATEGY-2 only detected cracks larger than a certain size are repaired (by welding),

STRATEGY-3 all detected cracks are repaired. Cracks smaller than a certain size are repaired by grinding, while cracks larger than this size are repaired by welding,

STRATEGY-4 all detected cracks are repaired by replacement of the element.

In the first strategy, only one limiting crack size is included corresponding to the smallest detectable crack size. At each inspection a crack may either be detected and repaired or no crack may be detected. An event tree for this strategy is illustrated in Fig.1. The number of inspections is n and these are performed at times $T_1,\ldots,T_n$, where $0=T_0 \le T_1 \le \cdots \le T_n \le T_{n+1}=T$. The total number of different courses is 2^n, see Fig. 1. This event tree in fact only illustrates a sub-optimization, as it is not necessary to choose the same time for the second inspection independent of the outcome of the first inspection.

In the second strategy two limiting crack sizes are included, corresponding to the smallest detectable crack size and a size which governs whether or not repair is done. Small cracks may be due to weld defects which do not grow, and in this strategy a crack is only repaired if its size is larger than a limiting value. At each inspection three possibilities then exist: a crack may be detected and repaired by welding, a crack may be detected but not repaired, or no crack may be detected. An event tree for this strategy is illustrated in Fig.2.

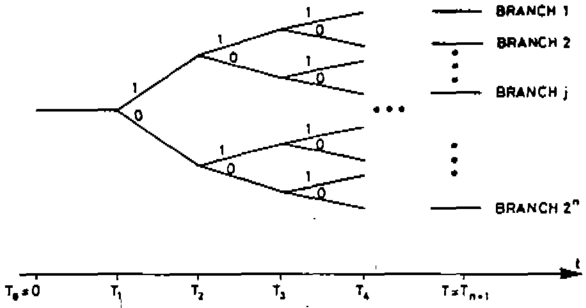

Fig.1: Illustration of event tree with repair of all detected cracks. 0 denotes no repair, while 1 denotes repair.

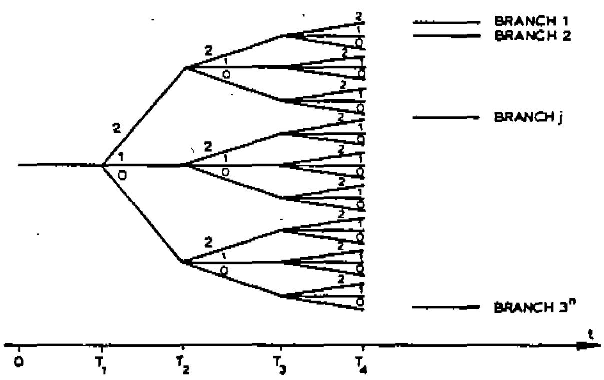

Fig.2: Illustration of event tree with no repair of small detected cracks and weld repair of large detected cracks. 0 denotes no crack detection, 1 denotes no repair of a detected crack, and 2 denotes weld repair.

The n inspections are performed at times $T_1,\ldots,T_n$, where $0=T_0 \le T_1 \le \cdots \le T_n \le T_{n+1}=T$, and the total number of different courses is 3^n, see Fig.2. Also the event tree in Fig.2 illustrates a sub-optimization as it is again not necessary to choose the same time for the second inspection independent of the outcome of the first inspection.

The third strategy also includes two limiting crack sizes, corresponding to the smallest detectable crack size, and a size which governs whether repair is by grinding or by welding. When the crack depth is small compared to the thickness, repair by grinding is often preferred to repair by welding due to the significantly smaller cost, and because the reduction in cross sectional area is so small that is has no significant effect on the static strength. The limiting size could depend on the remaining life time. At each inspection a crack may thus either be detected and repaired by grinding or welding or no crack may be detected. An event tree for this strategy is as in Fig.2, except that 0 denotes no repair, 1 denotes grind repair, and 2 denotes weld repair. The total number of inspections is n and these are performed at times $T_1,\ldots,T_n$, where $0=T_0 \le T_1 \le \cdots \le T_n \le T_{n+1}=T$. The total number of different repair courses is less than 3^n as two successive repairs by grinding are not allowed. These branches should thus be deleted from Fig.2.

The fourth strategy is similar to the first strategy, except that the element is replaced when a crack is detected. This strategy may not be very relevant for ship structures, but for other structures a replacement may be easier and less costly than a repair. The structural parameters for the replacing element may be different from the properties for the original element. The n inspections are performed at times $T_1,\ldots,T_n$, where $0=T_0 \le T_1 \le \cdots \le T_n \le T_{n+1}=T$, and the total number of different repair courses is 2^n. An event tree for this strategy is identical to that in Fig.1, except that 0 denotes no replacement and 1 denotes replacement.

4. SAFETY AND EVENT MARGINS

Failure is defined as crack growth beyond a critical crack size a_c. This size is often selected as corresponding to the element/hull thickness, but can also refer to a size for which brittle or ductile failure of the remaining cross section takes place for a specified extreme loading. The limit state function g for failure before a time t is therefore

$$g = a_c - a(t) \tag{4.1}$$

When (2.2) is inserted in (2.1), this equation may be written as

$$\frac{da}{dN} = C\, Y(a)^m (\pi a)^{m/2} S^m \,\mathbf{1}\!\left(S > \frac{\Delta K_{thr}}{Y(a)\sqrt{\pi a}}\right) \tag{4.2}$$

where $\mathbf{1}(\,)$ denotes an indicator function which takes the value one when the inequality in the paranthesis is valid and zero otherwise. Due to the generally large number of cycles to failure, the two terms containing the stress range can be approximated by their expected value. With Weibull distributed stress ranges this gives

$$E[S^m \,\mathbf{1}(S > \frac{\Delta K_{thr}}{Y(a)\sqrt{\pi a}})] = A^m\, \Gamma(1+\frac{m}{B})\, G(a) \tag{4.3}$$

where the auxiliary G-function is

$$G(a)=\frac{\Gamma(1+\frac{m}{B};(\frac{\Delta K_{thr}}{A\,Y(a)\sqrt{\pi a}})^{B})}{\Gamma(1+\frac{m}{B})} \tag{4.4}$$

In (4.2) the variables can now be separated and both sides of the equation be integrated

$$\int_{a_0}^{a(t)}\frac{dx}{Y(x)^m(\pi x)^{m/2}G(x)}=C\nu t A^m\Gamma(1+\frac{m}{B}) \tag{4.5}$$

where the initial condition $N(a_0)=0$ has been inserted. The left hand side of this equation is an increasing function in the upper integration limit $a(t)$. From the failure criterion in (4.1) the safety margin for failure before time t can then be formulated as

$$M=\int_{a_0}^{a_c}\frac{dx}{Y(x)^m(\pi x)^{m/2}G(x)}-C\nu t A^m\Gamma(1+\frac{m}{B}) \tag{4.6}$$

The first inspection at time T_1 leads to a crack detection or no crack detection. An event margin is defined as

$$H=\int_{a_0}^{a_{d1}}\frac{dx}{Y(x)^m(\pi x)^{m/2}G(x)}-C\nu T_1 A^m\Gamma(1+\frac{m}{B}) \tag{4.7}$$

The event margin is negative when a crack is detected, i.e. when the crack is larger than the smallest detectable crack size and is otherwise positive. a_{d1} is the smallest detectable crack size as described in detail later. If a crack is detected in the first inspection, and if the decision about repair (strategy 2) or repair method (strategy 3, welding or grinding) depends on a measurement of the crack size, an event margin can be formulated for the event that the crack is larger than a_{gr} and therefore should be repaired by welding

$$H_{gr}=\int_{a_0}^{a_{gr}}\frac{dx}{Y(x)^m(\pi x)^{m/2}G(x)}-C\nu T_1 A^m\Gamma(1+\frac{m}{B}) \tag{4.8}$$

where a_{gr} is a random variable to account for crack size measurement uncertainty. The distribution of a_{gr} is discussed later. The event that a detected crack is repaired by welding is $\{H_{gr}\le 0\}$.

When a crack is detected and repaired at time T_1, the safety margin after weld repair is

$$M^1(t)=\int_{a_R}^{a_c}\frac{dx}{Y(x)^m(\pi x)^{m/2}G(x)}-C\nu(t-T_1)A^m\Gamma(1+\frac{m}{B}) \tag{4.9}$$

where $t>T_1$. The geometry function is modelled as identical before and after a repair. The material parameter C is fully dependent before and after a repair when grind repair has been performed, and independent before and after repair when a weld repair or replacement has been selected. With weld repair, the crack size after repair a_R is the crack size after welding and inspection has been carried out. With grind repair a_R is replaced by a_G, which is an equivalent initial flaw size. Finally, with replacement of an element with a detected crack, a_R is replaced by a_0 since the distribution of the initial crack size is assumed identically distributed for the original and replacing element. In the examples presented later, it is assumed that crack sizes a_R, a_G and a_0 for different inspections are mutually independent.

It follows from the description above that a crack is assumed present initially and after each repair. This is perhaps a dubious assumption after a repair, if this is performed by grinding. The idea behind this repair method is exactly to remove the crack and to introduce a long crack initiation period. The geometry function is also likely to change due to grinding. It is, however, possible to select an equivalent initial crack size to account for the initiation period. This is a common practice for design of aircrafts where procedures for determining the equivalent flaw size from experiments are available. This approach also solves the complication with the change in geometry function, because the crack looses the memory of the grinding once it has started propagating and has propagated a short distance. It then acts as if no grinding had been performed.

A notation is introduced to describe the sequence of repair/no repair events in each branch. As an example, with repair at times T_1 and T_2 and no repair at T_3, the safety margin for failure before t, where $T_3<t\le T_4$ is

$$M^{110}(t)=\int_{a_R}^{a_c}\frac{dx}{Y(x)^m(\pi x)^{m/2}G(x)}-C\nu(t-T_2)A^m\Gamma(1+\frac{m}{B}) \tag{4.10}$$

The event margin for crack detection at time T_4 is

$$H^{110}=\int_{a_R}^{a_{d4}}\frac{dx}{Y(x)^m(\pi x)^{m/2}G(x)}-C\nu(T_4-T_2)A^m\Gamma(1+\frac{m}{B}) \tag{4.11}$$

and the event margin for weld repair at time T_4 (strategies 2 and 3) is

$$H_{gr}^{110}=\int_{a_R}^{a_{gr}}\frac{dx}{Y(x)^m(\pi x)^{m/2}G(x)}-C\nu(T_4-T_2)A^m\Gamma(1+\frac{m}{B}) \tag{4.12}$$

Safety and event margins are defined similarly for the other branches.

A structural design parameter z is introduced in the optimization, typically representing the hull thickness or the stiffener spacing. The base value of z is z_0 for which the Weibull long term stress parameters A and B have been determined. When z varies from z_0 each stress range is multiplied by the factor s_z from (2.7). When z is varied from its base value, the safety margin in (4.6) is changed as

$$M(t)=\int_{a_0}^{a_c}\frac{dx}{Y(x)^m(\pi x)^{m/2}G(x)} \tag{4.13}$$
$$-C\nu t A^m(c_z\frac{z_0}{z}+(1-c_z)(\frac{z_0}{z})^2)^m\Gamma(1+\frac{m}{B})$$

The event margins H and H_{gr} in (4.7) and (4.8) and the other safety and event margins are changed correspondingly.

Due to corrosion the thickness may decrease with time. A linear model for the thickness at time t was introduced in (2.8) in terms of the is the (random) corrosion rate k_z. A damage amplification factor $F_k(t)$ can then be defined as described in Madsen[1] in terms of $z/(z-k_z t)$ and $(1-c_z)z_0/(c_z z)$. When crack growth is from time T_1 to time T_2 the factor (T_2-T_1) in a safety or event margin is replaced by $(T_2F_k(T_2)-T_1F_k(T_1))$.

5. FAILURE AND REPAIR PROBABILITIES

The probability of failure before time t is $P_F(t)$. The corresponding reliability index is

$$\beta(t)=-\Phi^{-1}(P_F(t)) \tag{5.1}$$

Expressions for the failure probability and expected numbers of repair are here presented for strategy 1 only. For the other strategies similar although slightly more involved expressions are valid.

Expressions for the failure probability are given.

For $0 \le t \le T_1$:

$$P_F(t) = P(M(t) \le 0) \tag{5.2}$$

For $T_1 < t \le T_2$:

$$\begin{aligned} P_F(t) &= P_F(T_1) + \Delta P_F(T_1,t) \qquad (5.3) \\ &= P_F(T_1) + \Delta P_F^0(T_1,t) + \Delta P_F^1(T_1,t) \\ &= P_F(T_1) \\ &\quad + P(M(T_1)>0 \cap H>0 \cap M^0(t) \le 0) \\ &\quad + P(M(T_1)>0 \cap H \le 0 \cap M^1(t) \le 0) \end{aligned}$$

For $T_2 < t \le T_3$:

$$\begin{aligned} P_F(t) &= P_F(T_2) + \Delta P_F(T_2,t) \qquad (5.4) \\ &= P_F(T_2) + \Delta P_F^{00}(T_2,t) + \Delta P_F^{01}(T_2,t) + \Delta P_F^{10}(T_2,t) + \Delta P_F^{11}(T_2,t) \\ &= P_F(T_2) \\ &\quad + P(M(T_1)>0 \cap H>0 \cap M^0(T_2)>0 \cap H^0>0 \cap M^{00}(t) \le 0) \\ &\quad + P(M(T_1)>0 \cap H>0 \cap M^0(T_2)>0 \cap H^0 \le 0 \cap M^{01}(t) \le 0) \\ &\quad + P(M(T_1)>0 \cap H \le 0 \cap M^1(T_2)>0 \cap H^1>0 \cap M^{10}(t) \le 0) \\ &\quad + P(M(T_1)>0 \cap H \le 0 \cap M^1(T_2)>0 \cap H^1 \le 0 \cap M^{11}(t) \le 0) \end{aligned}$$

and so on for each inspection time and the life time. With n inspections between 0 and T, $2^{n+1}-1$ parallel systems are analysed to compute the failure probabilities.

The variation of the reliability index with time is shown in Fig.3 in sketch form.

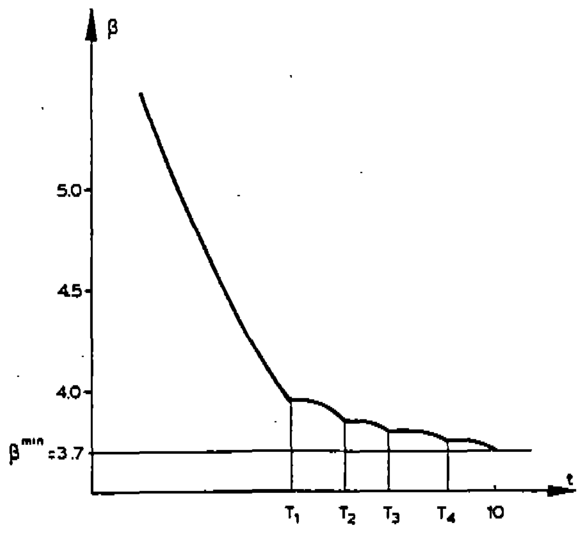

Fig.3: Variation of reliability index with time.

The reliability index decreases with time corresponding to an increasing probability of failure with time. The curves for the reliability index have a change in slope after an inspection and are close to having a horizontal tangent. This is so because the failure rate immediately after an inspection is very small, since the inspection has either not revealed a crack or repair has taken place. A crack of size close to the critical size is detected with a very large probability, and when no crack is detected there is a very small probability that the crack can grow to the critical size within a small time period after the inspection. If a crack is repaired the crack size after repair is expected to be much smaller than the critical size, and also in this case is there a very small probability that the crack can grow to the critical size within a small time period after the inspection.

The expected number of repairs $E[R_i]$ at time T_i is identical to the probability of repair at time T_i. It is

$$E[R_1] = P(M(T_1)>0 \cap H \le 0) \tag{5.5}$$

$$\begin{aligned} E[R_2] &= E[R_2^0] + E[R_2^1] \qquad (5.6) \\ &= P(M(T_1)>0 \cap H>0 \cap M^0(T_2)>0 \cap H^0 \le 0) \\ &\quad + P(M(T_1)>0 \cap H \le 0 \cap M^1(T_2)>0 \cap H^1 \le 0) \end{aligned}$$

$$\begin{aligned} E[R_3] &= E[R_3^{00}] + E[R_3^{01}] + E[R_3^{10}] + E[R_3^{11}] \qquad (5.7) \\ &= P(M(T_1)>0 \cap H>0 \cap M^0(T_2)>0 \cap H^0>0 \\ &\quad \cap M^{00}(T_3)>0 \cap H^{00} \le 0) \\ &\quad + P(M(T_1)>0 \cap H>0 \cap M^0(T_2)>0 \cap H^0 \le 0 \\ &\quad \cap M^{01}(T_3)>0 \cap H^{01} \le 0) \\ &\quad + P(M(T_1)>0 \cap H \le 0 \cap M^1(T_2)>0 \cap H^1>0 \\ &\quad \cap M^{10}(T_3)>0 \cap H^{10} \le 0) \\ &\quad + P(M(T_1)>0 \cap H \le 0 \cap M^1(T_2)>0 \cap H^1 \le 0 \\ &\quad \cap M^{11}(T_3)>0 \cap H^{11} \le 0) \end{aligned}$$

and so on for each inspection time. With n inspections between 0 and T, 2^n-1 parallel systems are analysed to compute repair probabilities.

PROBAN, see Tvedt [2], can be used for the analysis of the parallel systems to determine the expected number of repairs at each inspection time and the probability of failure. The FORM option for parallel systems with inactive constraints included is applied, and this is consistent with the manner in which the sensitivity factors are computed. To check the accuracy of the failure probability in the life time a SORM analysis has also been performed. The SORM result has not been found to deviate significantly from the FORM result. For a general reference on FORM/SORM methods, see e.g. Madsen et al [3].

6. DEFINITION OF INSPECTION QUALITY

The inspection quality is related to the probability of detecting a crack of a given size and the accuracy in sizing a detected crack. The probability of detecting a crack depends on the crack size, the inspection method and the inspection team. The reliability with respect to the probability of detecting a crack is defined by the POD (probability of detection) curve for which a shifted exponential form is used here.

$$p(a) = 1-\exp(-\frac{a-a_{\min}}{\lambda}), \quad a>a_{\min} \tag{6.1}$$

The smallest detectable crack size is denoted by a_d. The probability of detecting a crack of size a is equal to the probability that a_d is smaller than a. The following identity therefore holds:

$$F_{a_d}(a) = p(a) \tag{6.2}$$

showing that the POD curve is identical to the distribution function of the smallest detectable crack size. Other functions than the exponential function for the POD curve can easily be used. Values for the smallest detectable crack size in different inspections are assumed mutually independent.

The inspection quality is characterized by the parameter λ, which is the mean size above $a_{\min}$ of the smallest detectable crack. λ can take values between 0 and ∞, and a small λ signifies a high inspection quality while a large λ-value signifies a poor inspection quality. In the optimization an auxiliary measure of inspection quality q is introduced. q can take values in the interval $[0;\infty>$.

$$q = \frac{1}{\lambda} \tag{6.3}$$

q=0 corresponds to no inspection, while q=∞ corresponds to a perfect inspection where all cracks larger than $a_{\min}$ are found.

Simple constraints on q are introduced in the optimization

$$q^{\min} \le q \le q^{\max} \tag{6.4}$$

where $q^{\min}$ and $q^{\max}$ correspond to the poorest and best possible inspection quality, respectively.

Data on inspection qualities are scarse. For MPI a quality corresponding to a 90% probability of detecting a 40.0 mm long crack may be reasonable. Since MPI recognizes the crack length, some (random) relation between crack length and crack depth is necessary as described in the previous sections.

The POD curve has a finite probability of not detecting a crack which has grown through the thickness. This is in some situations not reasonable as this event is detected by other means, e.g. by oil spill. It may thus be relevant to modify the POD curve to yield a probability of one for detecting cracks larger than a specific size.

The value of λ as described above refers to the average performance of the inspection equipment handled by different operators. The variation from operator to operator should, however, also be included. The form of the POD curve in (6.1) can be maintained, but the parameter λ should then be random.

The reliability with respect to the accuracy in sizing a detected crack is relevant for the two repair strategies 2 and 3. The limiting size is a_{gr} but due to measurement uncertainty a random variable a_{gr} is used.

The inspection quality is a continuous variable in the optimization. In reality only a discrete number of inspection qualities are available. The optimization can then be performed in two steps. POD first step the quality is continuous and based on the result, fixed inspection qualities are selected for the second optimization, which is then only for inspection intervals. The same applies for the inspection intervals as inspections can only be performed during certain periods of the year, and the same may apply for hull thickness or spacing between stiffeners.

7. COST MODELING

The following cost items are included in the modeling of the total expected cost C_T:

Initial cost	$C_I = C_I(z)$
Inspection cost	$C_{IN} = C_{IN}(q)$
Cost of grind repair	C_G
Cost of weld repair	C_R
Cost of replacement	$C_{RE} = C_{RE}(z)$
Cost of failure	$C_F = C_F(t)$

The mean values of all cost items are assumed to increase with the rate of inflation. The difference between the rate of return for the project and the rate of inflation is assumed to be a constant r. An example of cost modeling is shown in the example.

8. FORMULATION OF OPTIMIZATION PROBLEM

8.1 Optimization at design

The number of inspections n during the life time T is selected beforehand. This is done to avoid an optimization with a mixture of integer and real valued optimization variables. The analysis is repeated for several values of n and the resulting optimal cost values are compared. The value of n with the smallest total expected cost is the optimal value. The optimization variables are the inspection times and qualities together with the structural design parameter z.

The optimization is now formulated for strategy 1 as:

$$\min_{\substack{z \\ t_1,\dots,t_n \\ q_1,\dots,q_n}} \quad C_I + \sum_{i=1}^{n} (C_{IN}(q_i)(1-P_F(T_i)) + C_R E[R_i]) \frac{1}{(1+r)^{T_i}} \tag{8.1}$$

$$+ \sum_{i=1}^{n+1} C_F(T_i)\,(P_F(T_i)-P_F(T_{i-1}))\,\frac{1}{(1+r)^{T_i}}$$

$$\begin{aligned} s.t\ & \beta(T)\ge\beta^{\min} \\ & t^{\min}\le T-\sum_{i=1}^{n} t_i \le t^{\max} \\ & t^{\min}\le t_i \le t^{\max}, \quad i=1,2,\dots,n \\ & q^{\min}\le q_i \le q^{\max}, \quad i=1,2,\cdots,n \\ & z^{\min}\le z \le z^{\max} \end{aligned}$$

The possibility of predetermining one or more of the inspection times and qualities as well as z is available.

The constraint on the minimum reliability is somewhat superfluous as the effect of the reliability is already included in the objective function. The constraint is solely included to allow for an optimization also in cases where authorities or others have defined limiting values for the failure probability. If the optimization without this constraint leads to a design and inspection procedure with an intuitively too small reliability, this most likely indicates that an error in the cost modeling has been made. Instead of giving the requirement on the failure probability in the life time it would also be possible to give a requirement on the failure rate, i.e. the probability of failure per time unit, but this would cause serious computational difficulties in the optimization. Another possibility is to limit the failure probability within each inspection interval rather than for the life time.

For strategies 2-4 the optimization is formulated similarly. For strategy 2 the optimization is formulated identically to strategy 1. For strategy 3 the only change is that the expected cost of repair contains two terms corresponding to the two repair methods, while for strategy 4 the cost of replacement depends on the structural parameter for the replacing element, and the simple constraint on the structural parameter is extended to the structural parameter for all replacing elements also.

8.2 Optimization after inspection

The inspection plan is optimized at the design stage as formulated in the previous section. When the result of the first inspection is known, a new optimal inspection plan can be determined applying this information in addition to the information available at the design stage. The time of the second inspection therefore depends on the result of the first inspection, i.e. whether or not a crack was detected and possibly also repaired. The first inspection plan as illustrated in Figs. 1-2 has the same time for the second inspection in the two or tree branches after the first inspection. Clearly the optimization as formulated in the previous section therefore only represents a suboptimization. If the number, times and qualities of inspections in each branch are included as optimization variables, the number of such variables increases drastically and the optimization becomes impracticable. The major contribution to the total expected cost is for well designed ship structures generally from the branch without any crack detections, i.e. involves inspection cost and expected repair cost for this branch. If this branch has a dominant importance, this indicates that the suboptimization results in a choice of design and inspection parameters which are also globally near optimal.

With information about inspection results and repair at one or more inspection times, the various failure probabilities and probabilities of repair are conditional probabilities, conditioned upon the result of the inspections. For each inspection result being available, the trees of remaining possibilities in Figs. 1-2 are reduced to one half or third of their size as the selected branch at each performed inspection time is known. This is illustrated in Fig.4 for strategy 1 with two inspection results available. In the first inspection no crack was detected, while a crack was detected and repaired in the second inspection. With the notation from the previos sections, this information is expressed as the event I

$$I = \{M(T_1)>0 \cap H>0 \cap M^0(T_2)>0 \cap H^0 \le 0\} \tag{8.2}$$

The probability of failure given the event I is then for $T_2 < t \le T_3$

$$\begin{aligned} P_F(t\,|\,I) &= \Delta P_F^{01}(T_2,t\,|\,I) \\ &= P(M(T_1)>0 \cap H>0 \cap M^0(T_2)>0 \cap H^0 \le 0 \cap M^{01}(t) \le 0\,|\,I) \\ &= \frac{P(M(T_1)>0 \cap H>0 \cap M^0(T_2)>0 \cap H^0 \le 0 \cap M^{01}(t) \le 0 \cap I)}{P(I)} \\ &= \frac{P(M(T_1)>0 \cap H>0 \cap M^0(T_2)>0 \cap H^0 \le 0 \cap M^{01}(t) \le 0)}{P(I)} \\ &= \frac{\Delta P_F^{01}(T_2,t)}{P(I)} \end{aligned} \tag{8.3}$$

where $P(I)$ is the probability that the event I occurs. For $T_3 < t \le T_4$ the failure probability is similarly

$$\begin{aligned} P_F(t\,|\,I) &= P_F(T_3\,|\,I) + \Delta P_F(T_3,t\,|\,I) \\ &= \Delta P_F^{01}(T_2,T_3\,|\,I) + \Delta P_F^{010}(T_3,t\,|\,I) + \Delta P_F^{011}(T_3,t\,|\,I) \\ &= \frac{\Delta P_F^{01}(T_2,T_3)}{P(I)} + \frac{\Delta P_F^{010}(T_3,t)}{P(I)} + \frac{\Delta P_F^{011}(T_3,t)}{P(I)} \end{aligned} \tag{8.4}$$

and so on for the remaining inspection times. The expected number of repairs is at times T_3 and T_4

$$E[R_3\,|\,I] = E[R_3^{01}\,|\,I] = \frac{E[R_3^{01}]}{P(I)} \tag{8.5}$$

$$\begin{aligned} E[R_4\,|\,I] &= E[R_4^{010}\,|\,I] + E[R_4^{011}\,|\,I] \\ &= \frac{E[R_4^{010}]}{P(I)} + \frac{E[R_4^{011}]}{P(I)} \end{aligned} \tag{8.6}$$

and so on for the other inspection times. It follows that the updated repair and failure probabilities are simply computed as ratios of probabilities which are already formulated for the inspection optimization at the design state. The same is true for the derivatives of these probabilities with respect to the optimization parameters.

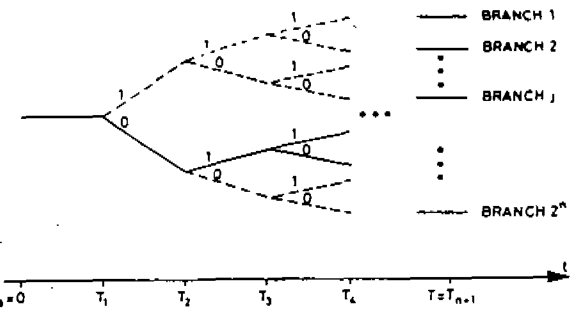

Fig.4: Illustration of event tree with repair of all detected cracks and two inspection results available. 0 denotes no repair, while 1 denotes repair.

With inspection results available at times $T_1, \cdots, T_{j-1}$, the optimization problem in (8.1) is modified as

$$\min_{\substack{t_j,\ldots,t_n \\ q_j,\ldots,q_n}} \sum_{i=j}^{n} (C_{IN}(q_i)(1-P_F(T_i\,|\,I)) + C_R E[R_i\,|\,I]) \frac{1}{(1+r)^{T_i}} + \sum_{i=j}^{n+1} C_F(T_i)\,(P_F(T_i\,|\,I) - P_F(T_{i-1}\,|\,I)) \frac{1}{(1+r)^{T_i}} \tag{8.7}$$

$$\begin{aligned} s.t.\ & \beta(T\,|\,I) \ge \beta^{min} \\ & q^{min} \le q_i \le q^{max}, \quad i=j,\ldots,n \\ & t^{min} \le T - \sum_{i=1}^{n} t_i \le t^{max} \\ & t^{min} \le t_i \le t^{max}, \quad i=j,\ldots,n \end{aligned}$$

where failure and repair probabilities are computed conditioned upon the results of the first $j-1$ inspections. All terms in the objective function are determined from the formulas above. The factor $1/P(I)$ is a common factor for all terms. For the minimization this factor is therefore only entering in the constraint on the reliability index. Without this constraint, only the history, i.e. the selected branches in the event tree, is of importance, not the probability of the history.

8.3 Optimization methods

Two methods have been considered for solving the optimization problems. Both methods solve the optimization for a fixed value of the number of inspections n and the minimum total expected cost by varying n can then be determined.

One optimization method uses the NLPQL algorithm as implemented by Schittkowski(4). Each step in this method consists of two steps. The first step is a determination of a search direction by solving a quadratic optimization problem formed by a quadratic approximation of the Lagrangian function of the non-linear optimization problem and a linearization of the constraints. The second step is a line search with an augmented Lagrangian merit function and a stopping criterion based on the Goldstein-Armijo principle. The second optimization method is similar to the NLPQL implementation, but the line search is somewhat different.

The values of the objective function, the constraints and their partial derivatives with respect to the optimization parameters are computed in a separate routine. This routine calls upon PROBAN for analysis of $2^{n+1}-1$ parallel systems for calculation of failure probabilities and 2^n-1 parallel systems for calculation of expected repair cost for strategies 1 and 4 and an even larger number of parallel systems for strategies 2 and 3. PROBAN provides a reliability index and probability for each parallel system together with partial derivatives of the reliability index or probability with respect to λ_i, T_i and z. From these partial derivatives the partial derivatives with respect to q_i, t_i and z are easily derived, and the gradients of the objective function and constraints can be determined. The Hessian for the Lagrangian function is approximated based on gradient information.

9. EXAMPLE

An examples is investigated with data which are fairly realistic and to some extent represent results from an analysis of a non-load-carrying stiffener weld in a tanker. The stiffener is analysed both with a constant geometry function and a more refined geometry function.

The initial crack size a_0 is taken as exponentially distributed with a mean value of 0.1 mm. The crack size after repair a_R is taken as independent of and identically distributed as the initial crack size. This has been done both for weld repair and grind repair. Crack sizes after repair are assumed to be mutually independent from repair to repair. The critical crack size a_C is taken as the hull thickness, 30 mm, defining leakage as failure.

The nominel long term stress range distribution is modeled as a Weibull distribution. The distribution parameters $\ln A$ and $1/B$ are assumed to follow a two-dimensional normal distribution with parameters

$$E[\ln A] = 1.6;\ D[\ln A] = 0.16,\ \rho[\ln A, 1/B] = -0.8, \qquad (9.1)$$
$$E[1/B] = 1.2;\ D[1/B] = 0.15$$

For the constant geometry function the Weibull parameter lnA represents local stress and the parameter is scaled to give the same mean local stress as the more realistic and complicated non-load-carrying stiffener weld geometry function, see Fig. 5.

$$E[\ln A] = 2.3;\ D[\ln A] = 0.20,\ \rho[\ln A, 1/B] = -0.8, \qquad (9.1b)$$
$$E[1/B] = 1.2;\ D[1/B] = 0.15$$

The mean frequency of stress cycles is 5 million cycles per year which represent a mean stress cycle period of 6.3 seconds.

The parameter c_z is taken as 1.0, and the corrosion rate is taken as 0, i.e. k_z=0.

The stress intensity factor for the transverse non-load-carrying fillet weld is estimated from a superposition of influence functions. The stress intensity factor is expressed as Almar-Næss et al (8)

$$\Delta K = \bar{Y}\sqrt{\pi a}\,\Delta\sigma \qquad (9.2)$$

where: $\Delta\sigma$ is the nominel stress range and $\bar{Y} = Y_E\, Y_S\, Y_T\, Y_W\, Y_G$:

Y_E: Crack shape factor

Y_S: Front face factor

Y_T: Finite thickness factor

Y_W: Finite width factor

Y_G: Stress gradient factor

A semi-elliptical shape of the crack described by the aspect ratio a/c is assumed. A simple empirical $a-c$ approximation for the stiffener specimen is assumed

$$2c = 2.59\, a^{0.946} \quad [mm]$$

Y_E: The crack shape factor takes into account the effect of the crack shape and is here approximated by

$$Y_E = (1.0 + 4.59\,(a/2c)^{1.65})^{-0.5}$$

Y_S: The front face factor accounts for the free surface at the front of the crack and depends on the crack opening stress distribution, the aspect ratio and the free

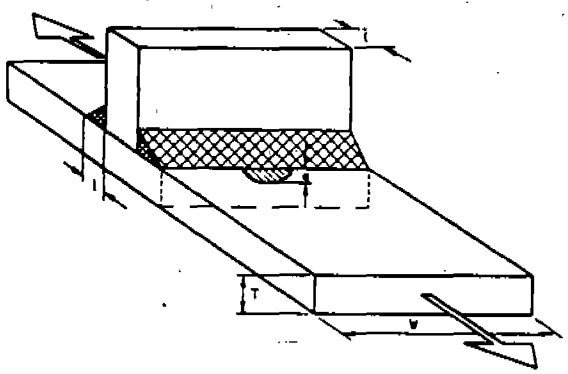

Fig.5: Transverse non-load-carrying stiffener weld.

surface shape. The correction factor is approximated as:

$$Y_S = (0.98 - 0.16\,(a/2c))$$

Y_T: The finite thickness correction factor accounts for the effect of free surface ahead of the crack front and depends on the crack opening stress distribution and the aspect ratio. The finite thickness correction factor is here approximated by a second order polynomium.

$$Y_T = 1.0 + 0.21\,(a/T) + 0.14\,(a/T)^2$$

Y_W: The finite width correction factor is only of interest for a through crack, here Y_W=1.0.

Y_G: The stress gradient factor accounts for the non-uniform crack opening stresses at the crack locus. An approximation for estimating Y_G for transverse non-load carrying fillet welds is

$$Y_G = SCF\,(1 + \frac{1}{Y_3}\,(a/T)^{Y_4})^{-1}$$

where SCF is the elastic stress consentration factor at the weld toe, modeled as.

$$SCF = 1.621\,\log_{10}(\,Y_2/Y_1\,) + 3.963$$

The geometry function parameters Y_1 and Y_2 represents the hull thickness and the hight of the weld while Y_3 and Y_4 are decaying coefficients for the stress gradient factor. In the analysis Y_1=30.0, Y_2=15.0, Y_3=0.360 and Y_4=0.249 .

For the constant geometry function $\bar{Y}$ is modeled as 1.0, eliminating the need for applying numerical integrations techniques for estimating the crack growth with time.

The geometry function parameters can be modeled as stochastic variables representing the uncertainty in the geometry function calculations.

The material crack growth parameter m is taken as a fixed value of 3.0. The material crack growth parameter C is taken as a lognormal variable. The mean value of $\ln C$ is taken as -29.9 and the standard deviation as 0.5. It is required that units N/mm^2 and mm are used for stresses and crack sizes. The threshold value ΔK_{thr} for the stress intensity factor is taken as zero.

The life time of the joint is taken as 30 years. The maximum time interval between successive inspection is taken as 30 years and the minimum interval as 1 year. The quality q of an inspection can vary between 0.23 (roughly corresponding to a visual inspection) and 1.3 (roughly corresponding to a very careful MPI inspection). Results have been computed both for an unshifted (a_{min}=0.0) and a shifted (a_{min}=1.0) POD curve, see eq.(6.1). The required reliability index is 3.70.

The cost of an inspection is taken as

$$C_{IN}(q) = C_{IN0} + C_{IN1}\,q + C_{IN2}\,q^2 = 0.1 + 0.0\,q + 0.4\,q^2 \qquad (9.3)$$

The cost of the design is taken as

$$C_I(z) = C_{I0} + C_{I00}(z - z_0) = 0.0 + 0.01(z - z_0) \qquad (9.4)$$

The value of C_{I0} is taken as zero, but this value has no influence on the values of the optimization parameters at the optimal point. The cost of repair is taken as

$$C_R = \begin{cases} C_W = 5.0, & \text{weld repair} \\ C_G = 0.2, & \text{grind repair} \end{cases} \qquad (9.5)$$

The cost of replacement is taken as (repair strategy 4).

$$C_{RE}(z) = C_{R1} + C_{R2}\,z = 5.0 + 0.1(z - z_0) \qquad (9.6)$$

The cost of failure is taken as

$$C_F = C_{F0} = 8000 \qquad (9.7)$$

and the rate of interest is taken as 4%. All costs are given as relative values only and may not be very representative.

Results for a constant geometry function are shown in Table 1 and Table 2 for the unshifted and shifted POD-curves, respectively. The physical interpretation of the shift in the POD-curve is that cracks smaller than 1.0 mm are not detected. The repair, failure and total cost are all expected values.

It is observed that the minimum is rather flat as a function of the number of inspections. It is also seen that the design parameter in all cases is at its maximum allowable value at the solution point. For the selected cost functions it thus appears to be more economic to put more effort into the design to make the hull almost certain to cause no fatigue problems, rather than to design with e.g. a smaller thickness and maintain the reliability through inspection and possible repair.

It is observed that the total cost is reduced significantly by introducing the more realistic shifted POD curve; this is because an unshifted POD curve gives detection of small cracks with little effect on the fatigue reliability. Since, according to the strategy, all detected cracks have to be repaired, this results in a high repair cost. It is also observed that the optimal solution is even more flat as a function of the number of inspections than for an unshifted POD-curve.

The effect of the four different inspection strategies are next compared for two inspections in a 25 year period. The input data are as above, except that no limits on the reliability index are defined, the cost of failure is taken as 1000, and the mean value of $\ln A$ is increased to 2.4.

TABLE 1: Optimal solution for a constant geometry function and varying number of inspections. Strategy 1: all detected cracks are weld repaired. Unshifted POD-curve.

No. of insp.	2	3	4	5	6	7
Initial cost	0.10	0.10	0.10	0.10	0.10	0.10
Inspect. cost	0.53	0.30	0.28	0.31	0.35	0.40
Weld rep. cost	0.48	0.37	0.33	0.34	0.35	0.37
Failure cost	0.39	0.39	0.37	0.30	0.25	0.20
Total cost	1.50	1.16	1.08	1.05	1.05	1.07
Time of insp. 1	14.6	14.3	14.0	13.1	12.6	12.6
Time of insp. 2	21.7	19.1	17.7	16.3	15.4	15.1
Time of insp. 3	-	24.1	21.5	19.3	18.1	17.4
Time of insp. 4	-	-	25.5	22.7	20.8	19.9
Time of insp. 5	-	-	-	26.3	23.7	22.1
Time of insp. 6	-	-	-	-	26.8	24.8
Time of insp. 7	-	-	-	-	-	27.4
Qual. of insp. 1	1.00	0.50	0.32	0.28	0.24	0.23
Qual. of insp. 2	1.11	0.52	0.34	0.28	0.26	0.23
Qual. of insp. 3	-	0.56	0.36	0.29	0.24	0.23
Qual. of insp. 4	-	-	0.38	0.30	0.24	0.23
Qual. of insp. 5	-	-	-	0.28	0.25	0.23
Qual. of insp. 6	-	-	-	-	0.25	0.23
Qual. of insp. 7	-	-	-	-	-	0.23
Design param.	70.0	70.0	70.0	70.0	70.0	70.0
Minimum β	3.70	3.70	3.72	3.77	3.82	3.89

For repair strategy 2 with constant geometry function and unshifted POD curve the total expected cost is shown for some values of the mean value of a_{gr}. The smallest total expected cost is obtained with a mean value of 1.0 mm.

$E[a_{gr}]$ *(mm)*	3.0	2.0	1.5	1.3	1.0	0.9
Total expected cost	.610	.575	.565	.560	.557	.559

For strategy 3 with constant geometry function and unshifted POD curve the smallest total expected cost is obtained for a_{gr} larger than 4 mm. The cost at the solution point for the different strategies is shown in Table 3.

Results for the stiffener weld geometry function is shown in table 4 and 5 for unshifted and shifted POD-curves, respectively. The results indicate an optimal inspection plan based on three inspecions for both the POD modeling alternatives, but with large reduction in the estimated repair cost for the shifted POD curve.

TABLE 2: Optimal solution for a constant geometry function and varying number of inspections. Strategy 1: all detected cracks are weld repaired. Shifted POD-curve.

No. of insp.	3	4	5	6	7
Initial cost	0.10	0.10	0.10	0.10	0.10
Inspect. cost	0.38	0.37	0.40	0.43	0.45
Weld rep. cost	0.01	0.01	0.01	0.01	0.01
Failure cost	0.21	0.19	0.16	0.14	0.13
Total cost	0.70	0.67	0.67	0.68	0.69
Time of insp. 1	12.9	12.3	12.1	11.5	11.2
Time of insp. 2	17.8	16.5	15.4	14.7	14.1
Time of insp. 3	23.5	20.6	18.9	17.4	16.4
Time of insp. 4	-	25.1	22.3	20.5	19.2
Time of insp. 5	-	-	25.8	23.6	21.7
Time of insp. 6	-	-	-	26.5	24.5
Time of insp. 7	-	-	-	-	27.4
Qual. of insp. 1	0.53	0.39	0.37	0.31	0.28
Qual. of insp. 2	0.66	0.49	0.40	0.39	0.31
Qual. of insp. 3	0.70	0.52	0.41	0.33	0.29
Qual. of insp. 4	-	0.52	0.42	0.33	0.29
Qual. of insp. 5	-	-	0.42	0.36	0.30
Qual. of insp. 6	-	-	-	0.36	0.31
Qual. of insp. 7	-	-	-	-	0.28
Design param.	70.0	70.0	70.0	70.0	70.0
Minimum β	3.87	3.90	3.94	3.98	4.00

TABLE 3: Optimal solution for a constant geometry function and two inspections. Unshifted POD-curve. Comparison between strategies.

Strategy	1	2	3	4
Initial cost	0.10	0.10	0.10	0.10
Replacement cost	-	-	-	0.20
Inspection cost	0.14	0.19	0.19	0.13
Weld repair cost	0.18	0.02	0.01	-
Grind repair cost	-	-	0.01	-
Failure cost	0.33	0.25	0.24	0.35
Total cost	0.75	0.56	0.55	0.78
Minimum β	3.19	3.29	3.30	3.18

Table 6 shows a parameter study of the failure cost, results are shown for two inspections with failure cost at 5.000, 8.000 and 10.000, applying the stiffener weld geometry function.

Examples of optimization results with inspection informaton available can be found in Holck et al[5] and Madsen et al[6].

TABLE 4: Optimal solution for a stiffener weld geometry function and varying number of inspections. Strategy 1: all detected cracks are weld repaired. Unshifted POD-curve.

No. of inspections	1	2	3	4
Initial cost	0.10	0.10	0.10	0.10
Inspection cost	0.145	0.140	0.174	0.224
Weld repair cost	0.213	0.215	0.256	0.269
Failure cost	0.50	0.30	0.218	0.139
Total cost	0.958	0.761	0.718	0.732
Time of insp. 1	18.1	17.0	15.5	14.4
Time of insp. 2	-	23.0	19.7	18.4
Time of insp. 3	-	-	24.4	21.8
Time of insp. 4	-	-	-	25.4
Quality of insp. 1	0.70	0.36	0.25	0.23
Quality of insp. 2	-	0.37	0.24	0.23
Quality of insp. 3	-	-	0.26	0.23
Quality of insp. 4	-	-	-	0.23
Design parameter	70.0	70.0	70.0	70.0
Minimum β	3.70	3.75	3.84	3.97

TABLE 5: Optimal solution for a stiffener weld geometry function and varying number of inspections. Strategy 1: all detected cracks are weld repaired. Shifted POD-curve. amin=1.0 mm

No. of inspections	1	2	3	4
Initial cost	0.10	0.10	0.10	0.10
Inspection cost	0.213	0.212	0.227	0.265
Weld repair cost	0.010	0.013	0.014	0.015
Failure cost	0.279	0.117	0.088	0.064
Total cost	0.602	0.443	0.430	0.444
Time of insp. 1	16.5	14.2	13.3	12.5
Time of insp. 2	-	21.3	18.5	16.9
Time of insp. 3	-	-	23.7	21.2
Time of insp. 4	-	-	-	25.2
Quality of insp. 1	0.87	0.48	0.33	0.29
Quality of insp. 2	-	0.58	0.37	0.33
Quality of insp. 3	-	-	0.41	0.30
Quality of insp. 4	-	-	-	0.29
Design parameter	70.0	70.0	70.0	70.0
Minimum β	3.75	4.01	4.08	4.16

TABLE 6: Optimal solution for a stiffener weld geometry function and two inspections. Shifted POD-curve Parameter study on different failure costs

COST OF FAILURE	5.000	8.000	10.000
Initial cost	0.10	0.10	0.10
Inspection cost	0.187	0.212	0.228
Weld repair cost	0.013	0.013	0.014
Failure cost	0.093	0.117	0.133
Total cost	0.393	0.443	0.475
Minimum β	3.94	4.01	4.02

10. CONCLUSIONS

A procedure for optimal design, inspection and repair of a fatigue sensitive element has been presented. Fatigue crack growth has been described by Paris' equation and failure been defined as growth to a critical size. Reliability calculations and associated sensitivity calculations have been performed by a first-order reliability method. Inspection times and qualities as well as structural design parameters are the optimization variables. A standard non-linear optimization routine is used. The optimization is first carried out at the design stage and later updated each time new inspection information becomes available. Four different repair strategies are presented with different criteria for repair method.

ACKNOWLEDGEMENT

The work of Espen H. Cramer has been supported by the Danish Research Academy.

The opinions stated in the paper are those of the authors and do not necessarily reflect the opinion of Det norske Veritas.

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DISCUSSION

E. Nikolaidis

I have talked with sponsors from FAA and NASA Langley about probabilistic optimization. One thing that they are interested to see is a method where experimentally you can verify that the optimum that you have found is an actual optimum and not something that is working on a piece of paper only. I wonder if you have any comments on that and if you have anything in mind to verify that your final design is the actual optimal one.

H.O. Madsen

I think one thing that is reassuring about this is that the reliability levels we've achieved in this optimization are very close to the reliability levels that we actually decide from. So probably practice, so far, has not been that far off. Besides that, I don't think I can give you a real answer.